New Constrained Affine-Projection Adaptive-Filtering Algorithm

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ISCAS 19-23 May 2013
Objectives
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Two versions of a new constrained affine-projection (CAP) algorithm, PCAP-I and PCAP-II, are proposed as follows:

- derivation of PCAP-I algorithm
- derivation of the PCAP-II algorithm
- discussion on proposed and conventional CAP algorithms.
Outline

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- Simulation results
  - system identification application
  - interference-suppression application for direct-sequence code-division multiple access (DS-CDMA) communication systems
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compared to the constrained normalized least-mean square (CNLMS), known CAP, and set-membership CAP (CSMAP) algorithms.
The two versions of the CAP algorithm are obtained by solving the minimization problem

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subject to the constraint

\[
\mathbf{C} \mathbf{w} = \mathbf{f}
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The two versions of the CAP algorithm are obtained by solving the minimization problem

$$\min_w J(w) = 0.5 \|X_k^T w - d_k\|^2$$

subject to the constraint

$$Cw = f$$

where $d_k \in \mathbb{R}^{l \times 1}$ is the desired signal vector, $X_k \in \mathbb{R}^{m \times l}$ is the input signal matrix, $C \in \mathbb{R}^{p \times m}$ is a constraint matrix, and $f \in \mathbb{R}^{p \times 1}$ is a constraint vector.
New CAP Algorithm

- The two versions of the CAP algorithm are obtained by solving the minimization problem

\[
\text{minimize } J(w) = 0.5\|X_k^Tw - d_k\|^2
\]

subject to the constraint

\[
Cw = f
\]

where \(d_k \in \mathbb{R}^{l\times1}\) is the desired signal vector, \(X_k \in \mathbb{R}^{m\times l}\) is the input signal matrix, \(C \in \mathbb{R}^{p\times m}\) is a constraint matrix, and \(f \in \mathbb{R}^{p\times1}\) is a constraint vector.

- The gradient of \(J(w)\) at points \(w_k\) and \(w_{k-1}\) satisfies the inequality

\[
\|\nabla J(w_k) - \nabla J(w_{k-1})\|_2 \leq \lambda_{k,\text{max}}(X_kX_k^T)\|w_k - w_{k-1}\|_2
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where $\lambda_{k,\text{max}}$ is the maximum eigenvalue of $X_k X_k^T$. 
We can, therefore, approximate \( J(w) = 0.5\|X_k^T w - d_k\|^2 \) at \( w_k \) as

\[
J(w) \approx \hat{J}(w_k) = J(w_{k-1}) + (w_k - w_{k-1})^T \nabla J(w_{k-1}) + \frac{1}{2\mu_k} \|w_k - w_{k-1}\|^2
\]

where \( J(w) < \hat{J}(w_k) \) and \( \mu_k \) is called the **Lipschitz constant**.
We can, therefore, approximate $J(w) = 0.5\|X_k^T w - d_k\|^2$ at $w_k$ as

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$$+ \frac{1}{2\mu_k} \|w_k - w_{k-1}\|_2^2$$

where $J(w) < \hat{J}(w_k)$ and $\mu_k$ is called the Lipschitz constant.

The PCAP-I algorithm can now be obtained by solving the minimization problem

$$\minimize_{w_k} J(w_{k-1}) + (w_k - w_{k-1})^T \nabla J(w_{k-1}) + \frac{1}{2\mu_k} \|w_k - w_{k-1}\|_2^2$$
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subject to the constraint

$$Cw_k = f$$
The solution of the problem at hand can be obtained by using the *Lagrange* multiplier method as

\[ w_k = Z [w_{k-1} + \mu_k X_k e_k] + t \]
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where $Z \in \mathcal{R}^{m \times m}$ is a matrix given by

$$Z = I - C^T (CC^T)^{-1} C$$

and $t \in \mathcal{R}^{m \times 1}$ is a vector given by

$$t = C^T (CC^T)^{-1} f$$
For the PCAP-I algorithm, we solve the minimization problem

$$\min_{\mu_k} F(\mu_k) = 0.5\|X_k^T w_k - d_k\|^2$$

to obtain

$$\mu_k = \frac{e_k^T X_k^T Z X_k (d_k - X_k^T [Z w_{k-1} + t])}{e_k^T X_k^T Z X_k X_k^T Z X_k e_k}$$

which can be used in the update formula

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With the initialization $w_0 = t$ the update formula of the PCAP-I algorithm simplifies to

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New PCAP-I Algorithm, Cont’d…

- The solution of the system of equations $Cw = f$ can be obtained as

$$w = V_r\omega + C^+f$$

where $C^+$ denotes the Moore-Penrose pseudo-inverse of $C$, $V_r \in \mathbb{R}^{m \times r}$ is a matrix consisting of the last $m = r - p$ columns of $V$ which is obtained by using the singular-value decomposition of $C$, and $\omega \in \mathbb{R}^{r \times 1}$ is a vector.
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With this solution, the optimization problem

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With this solution, the optimization problem

$$\min_w J(w) = 0.5 \|X_k^T w - d_k\|^2$$

subject to the constraint

$$Cw = f$$

can be expressed as

$$\min_\omega J(\omega) = 0.5 \|X_k^T V_r \omega + X_k^T C^+ f - d_k\|^2$$
The **PCAP-II** algorithm is obtained by solving the minimization problem

$$
\text{minimize } J(\omega) = 0.5\|X_k^T V_r \omega + X_k^T C^+ f - d_k\|^2
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by using the same steps as for the **PCAP-I** algorithm.
The **PCAP-II** algorithm is obtained by solving the minimization problem

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by using the same steps as for the **PCAP-I** algorithm.

The update formula of the **PCAP-II** algorithm becomes

$$\omega_k = \omega_{k-1} + \mu_k V_r X_k e_k$$

where

$$\mu_k = \frac{e_k^T X_k^T V_r V_r^T X_k e_k}{e_k^T X_k^T V_r V_r^T X_k X_k^T V_r V_r^T X_k e_k}$$
The PCAP-I and PCAP-II algorithms do not require the inverse of $X_k^T ZX_k$ and hence they require less computation than the conventional CAP algorithm.
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The PCAP-II algorithm requires reduced computation as compared to the PCAP-I algorithm due to the reduced dimensions of $\omega_k$ and $V_r^T$. 
Simulation Results

- MSD learning curves for system identification application:

![Graph showing MSD learning curves for system identification application with different algorithms: CNLMS, CAP, SMCAP, PCAP-I, PCAP-II. The x-axis represents the number of iterations, and the y-axis represents MSD in dB. The graph compares the performance of these algorithms over iterations.]
Simulation Results, Cont’d...

**Table: Average CPU Time, in Microseconds**

<table>
<thead>
<tr>
<th></th>
<th>CNLMS</th>
<th>CAP</th>
<th>SMCAP</th>
<th>PCAP-I</th>
<th>PCAP-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>27</td>
<td>61</td>
<td>8</td>
<td>38</td>
<td>36</td>
</tr>
</tbody>
</table>
Simulation Results, Cont’d…

- Learning mean output-error (MOE) curves for DS-CDMA interference suppression application:

![Graph showing MOE curves for different algorithms](image-url)

- CNLMS
- CAP
- SMCAP
- PCAP-I
- PCAP-II
Two new closely related CAP adaptive-filtering algorithms, the PCAP-I and PCAP-II, that use a new step size have been proposed. The new algorithms

- produce an unbiased output in applications where the desired signal is unavailable or not required
Conclusions

Two new closely related CAP adaptive-filtering algorithms, the PCAP-I and PCAP-II, that use a new step size have been proposed. The new algorithms

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- yield reduced steady-state misalignment relative to the CNLMS algorithm as well as some recent CAP algorithms
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- produce an unbiased output in applications where the desired signal is unavailable or not required
- require reduced computational effort than the conventional CAP algorithm
- yield reduced steady-state misalignment relative to the CNLMS algorithm as well as some recent CAP algorithms
- offer faster convergence than the CNLMS algorithm.