A Robust Constrained Set-Membership Affine-Projection Adaptive-Filtering Algorithm

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Outline

- Set-Membership (SM) Adaptive Filters

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- Simulation Results
- Conclusions

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- The *solution set* can be expressed as

$$\Theta = \cap_{(\mathbf{x},\ d) \in S} \{\mathbf{w} \in \mathcal{R}^M :\ |d - \mathbf{w}^T \mathbf{x}| \leq \gamma\}$$

- The *constraint* or *observation set* at iteration k is the set of weights that satisfy the prespecified bound at iteration k and is given by

$$H_k = \{\mathbf{w} \in \mathcal{R}^M : \ |d_k - \mathbf{w}_k^T \mathbf{x}_k| \leq \gamma\}$$

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- The exact membership set over the P most recent iterations is given by

$$\Psi_k^P = \cap_{i=k-P+1}^{k} H_i$$

Constrained SMAP Adaptive Filters

- In the constrained SMAP algorithm, whenever the weight vector $\mathbf{w}_k$ satisfies the constraint $\mathbf{f} = \mathbf{C}\mathbf{w}_k$ where $\mathbf{f} \in \mathcal{R}^K$ is the constraint vector, $\mathbf{C} \in \mathcal{R}^{K\times M}$ is a constraint matrix with $K < M$ and is not a member of the set Ψ_k^P, an update is performed by solving the optimization problem

$$\underset{\mathbf{w}_{k+1}}{\text{minimize}} \; J_{\mathbf{w}_{k+1}} = \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2$$

subject to the constraints

$$\begin{aligned} \mathbf{f} &= \mathbf{C}\mathbf{w}_{k+1} \\ \mathbf{g}_k &= \mathbf{d}_k - \mathbf{X}_k^T \mathbf{w}_{k+1} \end{aligned}$$

where $\mathbf{g}_k$ is the error-bound vector.

- If the error-bound vector is chosen as $\mathbf{g}_k = [\gamma \text{sign}(e_k)\ \epsilon_{k-1} \cdots\ \epsilon_{k-P+1}]^T$, the weight-vector update formula of the constrained SMAP algorithm becomes

$$\mathbf{w}_{k+1} = \mathbf{Z}\left[\mathbf{w}_k + \alpha_k \mathbf{X}_k(\mathbf{X}_k^T \mathbf{Z}\mathbf{X}_k)^{-1} e_k \mathbf{u}_1\right] + \mathbf{F}$$

where

$$\mathbf{u}_1 = [1_k\ 0_{k-1} \cdots\ 0_{k-P+1}]^T$$

$$\alpha_k = \begin{cases} 1 - \dfrac{\gamma}{|e_k|} & \text{if } |e_k| > \gamma \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \mathbf{Z} &= \mathbf{I} - \mathbf{C}^T\left(\mathbf{C}\mathbf{C}^T\right)^{-1}\mathbf{C} \\ \mathbf{F} &= \mathbf{C}^T\left(\mathbf{C}\mathbf{C}^T\right)^{-1}\mathbf{f} \end{aligned}$$

Proposed Robust CSMAP Algorithm

- The proposed robust constrained SMAP (PCSMAP) adaptation algorithm essentially solves the optimization problem

$$\underset{\mathbf{w}_{k+1}}{\text{minimize}} \; J_{\mathbf{w}_{k+1}} = \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2$$

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- The PCSMAP algorithm uses the error-bound vector $\mathbf{g}_k$ as

$$\mathbf{g}_k = \gamma \left[\frac{e_k}{|e_k|}\ \frac{\epsilon_{k-1}}{|e_k|}\ \cdots\ \frac{\epsilon_{k-P+1}}{|e_k|} \right]^T$$

instead of $\mathbf{g}_k = [\gamma \text{sign}(e_k)\ \epsilon_{k-1}\ \cdots\ \epsilon_{k-P+1}]^T$.

- The weight-vector update formula in the PCSMAP algorithm becomes

$$\mathbf{w}_{k+1} = \mathbf{Z}\left[\mathbf{w}_k + \alpha_k \mathbf{X}_k (\mathbf{X}_k^T \mathbf{Z} \mathbf{X}_k)^{-1} \mathbf{e}_k\right] + \mathbf{F}$$

where

$$\alpha_k = \begin{cases} 1 - \dfrac{\gamma}{|e_k|} & \text{if } |e_k| > \gamma \\ 0 & \text{otherwise} \end{cases}$$

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- The error-bound γ in the PCSMAP algorithm is obtained as

$$\gamma = \begin{cases} \|\mathbf{e}_k\|_\infty - \nu\theta_k & \text{if } \|\mathbf{e}_k\|_\infty > \theta_k \\ \gamma_c & \text{otherwise} \end{cases}$$

where $0 < \nu \ll 1$, $\theta_k = \vartheta\sigma_{e,k}$ and ϑ is chosen such that $\vartheta\sigma_{e,k} < \gamma_c$.

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- The variance of the error signal is estimated as

$$\sigma_{e,k}^2 = \lambda\sigma_{e,k-1}^2 + (1-\lambda)\text{median}(e_k^2, \ldots, e_{k-m+1}^2)$$

  where the *forgetting factor*, λ, assumes values in the range 0 to 1.

Proposed Robust CSMAP Algorithm, Cont'd...

- The initial value $\sigma^2_{e,0}$ is chosen to be large to ensure that $\|\mathbf{e}_k\|_\infty < \theta_k$ during transience in which case the algorithm would work with the error bound $\gamma = \gamma_c$: This leads to a faster convergence.

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- At steady state, we have $\sigma_{e,k}^2 \approx \sigma_v^2$ and hence $\|\mathbf{e}_k\|_\infty > \theta_k$ and, as a result, the algorithm would work with error bound $\|\mathbf{e}_k\|_\infty - \nu\theta_k$: This leads to robustness with respect to impulsive noise and low steady-state misalignment.

Proposed Robust CSMAP Algorithm, Cont'd...

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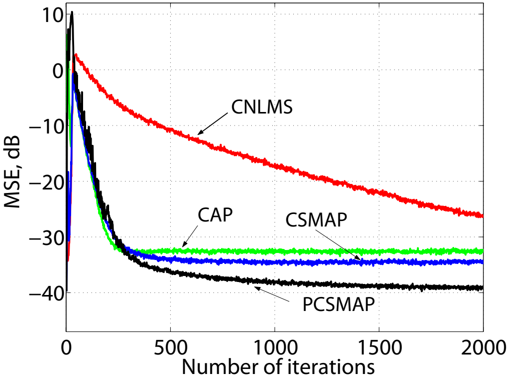
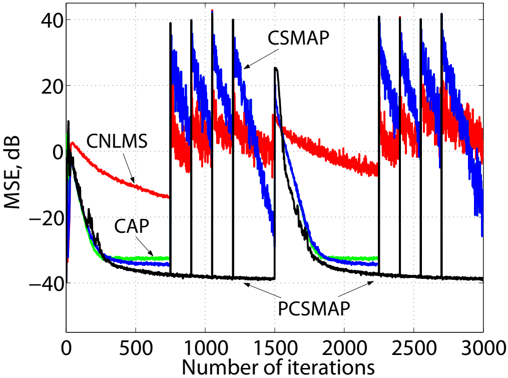
- At steady state, we have $\sigma^2_{e,k} \approx \sigma^2_v$ and hence $\|\mathbf{e}_k\|_\infty > \theta_k$ and, as a result, the algorithm would work with error bound $\|\mathbf{e}_k\|_\infty - \nu\theta_k$: This leads to robustness with respect to impulsive noise and low steady-state misalignment.
- During sudden system disturbances, the estimate $\sigma^2_{e,k}$ tends to grow in which case the algorithm would again work with the error bound γ_c which would thus lead to fast tracking.

Simulation Results

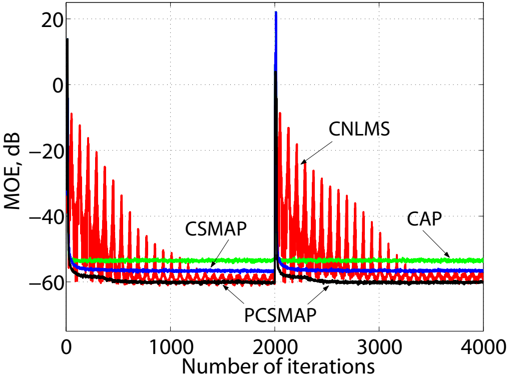
- Learning curves for a linear-phase system identification application for the CNLMS, CAP, CSMAP, and PCSMAP algorithms:



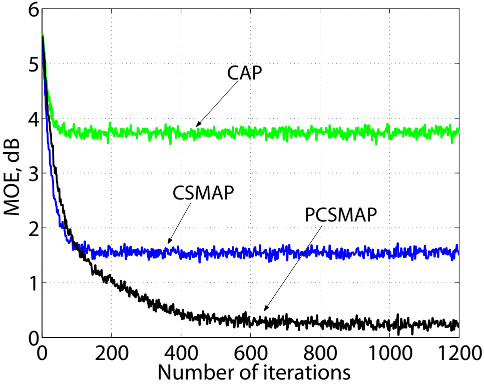
- Learning curves for a linear-phase system identification application in an impulsive-noise environment for the CNLMS, CAP, CSMAP, and PCSMAP algorithms:



- Learning curves for a time-series filtering application for the CNLMS, CAP, CSMAP, and PCSMAP algorithms:



- Learning curves for an interference-cancellation application in an direct-sequence code-division multiple access communication system for the CNLMS, CSMAP, and PCSMAP algorithms:



Conclusions

- A new constrained robust set-membership affine-projection algorithm has been proposed.

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- A new constrained robust set-membership affine-projection algorithm has been proposed.
- The proposed algorithm yields
  - a significantly reduced steady-state misalignment,
  - better tracking,
  - faster convergence, and
  - robust performance

  relative to several known competing adaptation algorithms of the SM family.

Thank you for your attention.