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New least squares solutions for estimating the average centre of rotation and the axis of rotation

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Abstract

A new method is proposed for estimating the parameters of ball joints, also known as spherical or revolute joints and hinge joints with a fixed axis of rotation. The method does not require manual adjustment of any optimisation parameters and produces closed form solutions. It is a least squares solution using the whole 3D motion data set. We do not assume strict rigidity but only that the markers maintain a constant distance from the centre or axis of rotation. This method is compared with other methods that use similar assumptions in the cases of random measurement errors, systematic skin movements and skin movements with random measurement noise. Simulation results indicate that the new method is superior in terms of the algorithm used, the closure of the solution, consistency and minimal manual parameter adjustment. The method can also be adapted to joints with translational movements.

Keywords: Joint kinematics; Centre of rotation; Axis of rotation; Geometric algebra

1. Introduction

The study and measurement of human joint kinematics have many applications in biomechanics, motion synthesis and analysis. Articulated kinematic chain models of varying sophistication are used in modelling human joint movements. In this context, estimation of the centre of rotation (CoR) and the axis of rotation (AoR) for limb complexes relative to various observation frames is often important as an aid to gait analysis and treatment of anatomical defects. It can also be indirectly important in building better models for motion capture systems used in biomedical research. CoR arises with ball joints and AoR with hinge joints.

The literature on estimation of CoR has mainly divided into finding the instantaneous CoR, as in Woltring (1990) and the average CoR as in Halvorsen et al. (1999) and Silaghi et al. (1998), over the given time interval. The instantaneous CoR has zero angular velocity in the observe's coordinate system and is mostly estimated using two time instances, in practice, although smoothing is possible. Woltring (1990) estimates the so-called instantaneous helical axis (IHA) assuming the joints have both translational and rotational components. A least squares method is applied to this set of IHAs to calculate the average CoR or the average AoR where appropriate. Halvorsen et al. (1999) use a least squares cost function formed by the vector differences of the points traced out by each marker. The solutions are in closed form. The lengths of these difference vectors or, equivalently, the choice of the time lags, are crucial for the performance of the method. Silaghi et al. (1998) use a cost function based on the fact that the markers should be on a sphere centred on the CoR in the case of a stationary joint. But Silaghi et al. (1998) do not give a closed form solution and use a weighted average of the CoRs given by individual markers.

We assume that the 3D coordinates of markers attached to the moving anatomical parts are given in an arbitrary coordinate system which is the camera coordinate frame if acquired by a motion capture system. In this paper, we present new least squares solutions to the estimation of the CoR or the AoR which are in closed form and require no 'tuning' parameters. It should also be noted that the CoR or

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AoR is estimated using the whole data set. Also we do not depend on the method used to acquire the data as we accommodate for errors in a least squares sense. Since the least squares estimation is only optimal in the case of Gaussian errors (see, e.g. Anderson and Moore, 1979), the performance of the method may vary for systems with different error distributions. In this context, we consider the effect of skin movements as reported by Cappozzo et al. (1996). In our derivation, we do not assume strict rigidity of the markers in relation to the joint but instead assume that a given marker traces out a sphere centred at the CoR or a circle around the AoR. Thus, substantial and systematic radial movements will affect the performance of the method. We compare the new method with the methods of Halvorsen et al. (1999) and Silaghi et al. (1998).

2. Centre of rotation estimation

Here we assume that a set of vectors on a body rotates around a time varying AoR with the CoR fixed. The 'tips' of the vectors should then lie on co-centric spheres. If $\mathbf{v}_k^p$ represents the pth vector in the kth time instance, the centre of rotation is $\mathbf{m}$, and the radius of the sphere marked out by the pth vector is r^p (see Fig. 1a), it is possible to form a least squares cost function,

$$C = \sum_{p=1}^{P} \sum_{k=1}^{N} [(\mathbf{v}_k^p - \mathbf{m})^2 - (r^p)^2]^2 \tag{1}$$

assuming there are P markers and N frames. This means that we do not assume that the vectors at different time instances or frames are related by a rigid body rotation (i.e. all the P vectors are not assumed to have the same amount of rotation) but each pth vector is individually rotated around the CoR. Note that the rigid body rotation is a subset of the above assumption. Hence the method is also applicable in the case of rigid body rotations. To estimate the r^p and $\mathbf{m}$ that minimise the cost function, we can first differentiate with respect to (wrt) the scalar quantity r^p and arrive at the result

$$r^p = \sqrt{\frac{1}{N}\sum_{k=1}^{N} (\mathbf{v}_k^p - \mathbf{m})^2}. \tag{2}$$

Also, using the definition of vector differentiation (see, e.g. Therrien, 1992), we can differentiate wrt vector $\mathbf{m}$ to obtain the result

$$\sum_{p=1}^{P} \sum_{k=1}^{N} [(\mathbf{v}_k^p - \mathbf{m})\{(\mathbf{v}_k^p - \mathbf{m})^2 - (r^p)^2\}] = 0. \tag{3}$$

Substituting Eq. (2) into (3) and applying simple algebraic manipulations yields

$$\sum_{p=1}^{P} [\overline{(\mathbf{v}^p)^3} - \overline{\mathbf{v}^p}\,\overline{(\mathbf{v}^p)^2}]$$

$$= 2\sum_{p=1}^{P} \left[\frac{1}{N}\sum_{k=1}^{N} \mathbf{v}_k^p(\mathbf{v}_k^p \cdot \mathbf{m}) - \overline{\mathbf{v}^p}(\mathbf{m} \cdot \overline{\mathbf{v}^p})\right], \tag{4}$$

where $\overline{(\mathbf{v}^p)^3} = 1/N \sum_{k=1}^{N} (\mathbf{v}_k^p)^3$, $\overline{(\mathbf{v}^p)^2} = 1/N \sum_{k=1}^{N} (\mathbf{v}_k^p)^2$ and $\overline{\mathbf{v}^p} = 1/N \sum_{k=1}^{N} \mathbf{v}_k^p$; note that $(\mathbf{v}_k^p)^3 \equiv (\mathbf{v}_k^p)^2 \mathbf{v}_k^p$. Since we can write $\mathbf{a}^i\,(\mathbf{b} \cdot \mathbf{c}) = \mathbf{a}^i \mathbf{b}^j \mathbf{c}^j = M\mathbf{c}^i$, where matrix $M = (\mathbf{a}\mathbf{b}^{\mathrm{T}})$ and $\mathbf{a}^k$, $\mathbf{b}^k$, $\mathbf{c}^k$ are the vector components (assuming column vectors), it is possible to put Eq. (4) into the form

$$A\mathbf{m} = \mathbf{b}, \tag{5}$$

where

$$A = 2\sum_{p=1}^{P} \left[\left\{\frac{1}{N}\sum_{k=1}^{N} \mathbf{v}_k^p(\mathbf{v}_k^p)^{\mathrm{T}}\right\} - \overline{\mathbf{v}^p}(\overline{\mathbf{v}^p})^{\mathrm{T}}\right]$$

$$\mathbf{b} = \sum_{p=1}^{P} [\overline{(\mathbf{v}^p)^3} - \overline{\mathbf{v}^p}\,\overline{(\mathbf{v}^p)^2}].$$

We have derived the above result using geometric algebra (see, e.g. Hestenes and Sobczyk, 1984) although standard techniques can also be used. The complete derivation is given in Hiniduma Udugama Gamage and Lasenby (2001). It should be noted that Eq. (5) is linear and a robust solution can be obtained using linear algebra. Also note that this method does not rely on any particular time (frame) difference as in Halvorsen et al. (1999) or on the method of averaging the P sphere centres adopted by Silaghi et al. (1998). In addition, the solution is in closed-form and needs no manual adjustment of weights unlike the method suggested by Silaghi et al. (1998).

3. Axis of rotation estimation

When the joint is modelled as a hinge (e.g. knee joint), the concept of CoR does not apply. In this case, it is not possible to find a unique CoR since each point on the axis is stationary. Hence we find a single point on the AoR and, in addition, the direction of the AoR using another least squares cost function. These two define the axis of rotation.

We assume that a set of vectors on a body rotates around a fixed AoR. Therefore, the 'tips' of the vectors are on circles with their centres on a straight line, where the line is the rotational axis. Note that this implicitly assumes that minimum distances from the markers to the axis are fixed. If $\mathbf{v}_k^p$ represents the pth vector in the kth time instant, $\mathbf{m}^p$ any point on the plane traced out by the pth vector 'tip' and $\mathbf{n}$ the unit vector in the direction of rotational axis, (see Fig. 1b) then it is possible to form a least squares type cost function as

$$C = \sum_{p=1}^{P} \sum_{k=1}^{N} [(\mathbf{v}_k^p - \mathbf{m}^p) \cdot \mathbf{n}]^2$$

assuming there are P markers and N frames. This is based on the fact that the vector components $\mathbf{v}_k^p - \mathbf{m}^p$ should ideally be on a plane perpendicular to the AoR. In the noisy case, the sum of the magnitudes of components parallel to the AoR is minimised. Note that this is similar to the cost function used by Halvorsen et al. (1999) except that we do not assume any particular displacement of $\mathbf{v}_k^p$. Similar to the method adopted in Section 2, we first differentiate the cost function wrt to $\mathbf{n}$ and set the result to zero to give

$$\sum_{p=1}^{P} \sum_{k=1}^{N} \{(\mathbf{v}_k^p - \mathbf{m}^p) \cdot \mathbf{n}\}(\mathbf{v}_k^p - \mathbf{m}^p) = 0. \tag{6}$$

Differentiating wrt $\mathbf{m}^p$ results in

$$\mathbf{m}^p \cdot \mathbf{n} = \left(\frac{1}{N} \sum_{k=1}^{N} \mathbf{v}_k^p \right) \cdot \mathbf{n} = \overline{\mathbf{v}^p} \cdot \mathbf{n}. \tag{7}$$

By substituting for $\mathbf{m}^p \cdot \mathbf{n}$ from Eq. (7) into Eq. (6), we get

$$\sum_{p=1}^{P} \sum_{k=1}^{N} \{\mathbf{v}_k^p \cdot \mathbf{n} - \overline{\mathbf{v}^p} \cdot \mathbf{n}\}\mathbf{v}_k^p = 0. \tag{8}$$

Again, writing $\mathbf{a}(\mathbf{b} \cdot \mathbf{c})$ in matrix form, Eq. (8) can be simplified to

$$\sum_{p=1}^{P} \left[\left\{ \frac{1}{N} \sum_{k=1}^{N} \mathbf{v}_k^p (\mathbf{v}_k^p)^{\mathrm{T}} \right\} - \overline{\mathbf{v}^p} (\overline{\mathbf{v}^p})^{\mathrm{T}} \right] \mathbf{n} = 0,$$

$$A\mathbf{n} = 0. \tag{9}$$

Note that A takes the same form as in the case of estimating the CoR. The solution to this linear equation is the eigenvector corresponding to the eigenvalue with the smallest magnitude since A is symmetric. The complete derivation is given in Hiniduma Udugama Gamage and Lasenby (2001). This can be efficiently evaluated using the singular value decomposition (SVD) (see, e.g. Press et al., 1992).

In order to define the axis of rotation, a point on the axis is also required. Take any point on the axis as $\mathbf{m}$, and the distance from that point to the circular arc as r^p (which marks out a section of a cone), it is then possible to form a least squares cost function as

$$C = \sum_{p=1}^{P} \sum_{k=1}^{N} [(\mathbf{v}_k^p - \mathbf{m})^2 - (r^p)^2]^2.$$

This is identical to the cost function used in Section 2 (Eq. (1)). Hence Eq. (5) can be used to find the vector $\mathbf{m}$. But note that in the no-noise case, the pseudo-inverse of A has to be taken since A is singular according to Eq. (9). In practice, we perform an SVD on matrix A, taking the vector corresponding to the smallest singular value as an approximation to $\mathbf{n}$ and use the remaining singular values to calculate the pseudo-inverse (see, e.g. Strang, 1980). This solution of $\mathbf{n}$ is an approximation to the null space of A. Also note that the value for $\mathbf{m}$ is the solution that is entirely in the row space of A since the pseudo-inverse is the minimum-norm solution. Therefore, the equation of the AoR,

$$\mathbf{x}_{\mathrm{AoR}} = \mathbf{m} + \tau \mathbf{n},$$

is a straight line passing through $\mathbf{m}$ in the direction of $\mathbf{n}$, parameterised by a scalar τ. Since the direction and the location of the axis are known, it is possible to find the centres of the circles traced out by each marker, $\mathbf{m}_c^p$, (see Fig. 1) using

$$\mathbf{m}_c^p = \mathbf{m} + \tau^p \mathbf{n} \tag{10}$$

where τ^p is a constant for each set of p vectors. Taking the inner product wrt to $\mathbf{n}$ in Eq. (10) gives

$$\tau^p \mathbf{n} \cdot \mathbf{n} = \mathbf{m}_c^p \cdot \mathbf{n} - \mathbf{m} \cdot \mathbf{n}.$$

Since $\mathbf{m}^p$ is also on the plane of the pth circle, we can use Eq. (7) and the fact that $\mathbf{n} \cdot \mathbf{n} = 1$ to give

$$\tau^p = (\overline{\mathbf{v}^p} - \mathbf{m}) \cdot \mathbf{n}.$$

4. Simulations and results

In the first set of simulations, we estimate the AoR and have chosen the same points as Halvorsen et al. (1999). We have also compared our method with the Halvorsen et al. (1999) method using different choices of time interval. The marker coordinates relative to a frame fixed in the femur with the x-axis in the anterior direction, the y-axis in the medial direction and the z-axis in the superior direction, are $(0, 0, -5)$, $(0, 0, -30)$, $(0, -5, -15)$, $(0, 5, -15)$ in cm (see Fig. 2a). The points were rotated 1° per frame for 60 frames around an axis parallel to the y-axis and going through the point $(0, 0, 5)$. Three sets of simulations were carried out. First with the addition of pseudo-random Gaussian error sequences. Second with added skin movements as observed by Cappozzo et al. (1996). Third with the more realistic situation of random errors plus skin movements. The standard deviation of the added noise was chosen to be 0.01. Note here that the marker positions were chosen in order to make direct use of the skin movements observed by Cappozzo et al. (1996) as in Halvorsen et al. (1999). The random errors are intended to account for observation errors (calibration, reconstruction, tracking, etc.).

Another set of simulations was carried out to evaluate the validity of the methods of finding the CoR. The orientation of the coordinate frame was chosen as shown in Fig. 2b according to the scheme given in

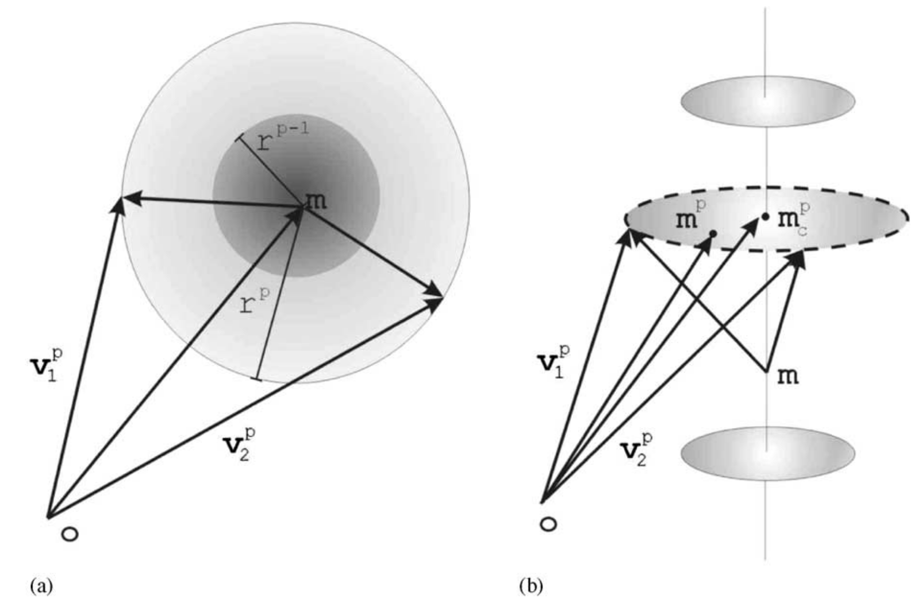

Fig. 1. (a) Assumption of spherical marker movement in a ball joint. r^p is the radius of the pth sphere, $\mathbf{m}$ is the centre of rotation and $\mathbf{v}_k^p$ is the observed vector at the kth time instant. (b) Assumption of circular marker movement in the case of a hinged joint. $\mathbf{m}_c^p$ is the centre of the pth circle, $\mathbf{m}^p$ is an arbitrary point on the plane containing the pth circle, $\mathbf{m}$ is an arbitrary point on the axis of rotation and $\mathbf{v}_k^p$ is the observed vector at the kth time instant.

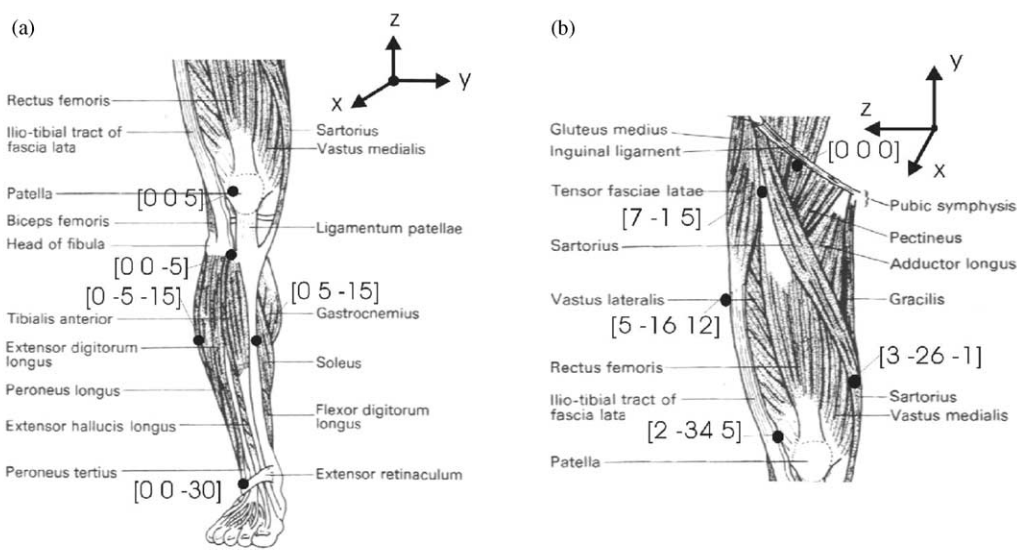

Fig. 2. Marker placements for the simulations. (a) AoR estimation. (b) CoR estimation. (The leg anatomy was taken from Green and Silver, 1981.)

Cappozzo et al. (1996) and the origin was chosen to be the femoral head (CoR). The axis of rotation at the initial frame was along the z-axis. With each frame, the AoR was allowed to change direction by adding a vector with each component drawn from a pseudo-random Gaussian sequence of standard deviation of 0.1, and re-normalising, for 100 frames. The marker positions were $(7, -1, 5), (5, -16, 12), (3, -26, -1), (2, -34, 5)$ in cm. In this set of simulations, we have repeated the experiments using both the methods of Halvorsen et al. (1999) and Silaghi et al. (1998). In the Silaghi et al. (1998) method, we have used Eq. (5) instead of the numerical search algorithm as proposed by Silaghi et al. (1998) for a single marker since the new method is the closed-form solution to their cost function in this context.

In both sets of simulations, the cases which involve the addition of Gaussian noise were repeated 30 times and the values averaged to give the final result. The results are presented in Figs. 3–5.

4.1. Interpretation of results

From Fig. 3, it is evident that the performance of the Halvorsen et al. (1999) method is extremely sensitive to the choice of number of frames used to calculate the vector differences, in contrast, the new method is free from this defect. Note that the choice of frame difference as half the total sequence length is not 'near optimal' in general; depending on the nature of the motion, the link can be moving in such a way as to make the effective vector lengths small. A strength of the new method is that it can be applied straightforwardly to find the direction of the rotational axis without manually considering the 'optimal' choice for the frame difference, while having similar performance to the best choice of frame difference.

It can be seen from Figs. 4a and b that with the effect of systematic skin displacements, the new method performs comparably with the best case of Halvorsen et al. (1999). Note that with a bad choice of frame difference, the Halvorsen et al. (1999) method performs very poorly. Since the new method does not use the frame differencing, its performance does not degrade. With the systematic skin error and random measurement noise as in Figs. 4c and d the results are similar, but deterioration of the Halvorsen et al. (1999) method is worse than that for the pure skin displacement errors.

We see similar behaviour for estimation of the CoR—see Fig. 5. The method proposed by Silaghi et al. (1998) under-performs consistently when compared to the new method since it averages the estimates of the centres of rotations for each marker, as opposed to finding the CoR from all the data simultaneously. In general, the Silaghi et al. (1998) method should do worse than that depicted in the graphs since it uses the Levenberg–Marquardt method (see, e.g. More, 1977) to achieve the least squares minimisation instead of the closed-form solution presented in Eq. (5). Effectively, the closed-form solution to the cost function of Silaghi et al. (1998) is a subset of Eq. (5). In addition, the

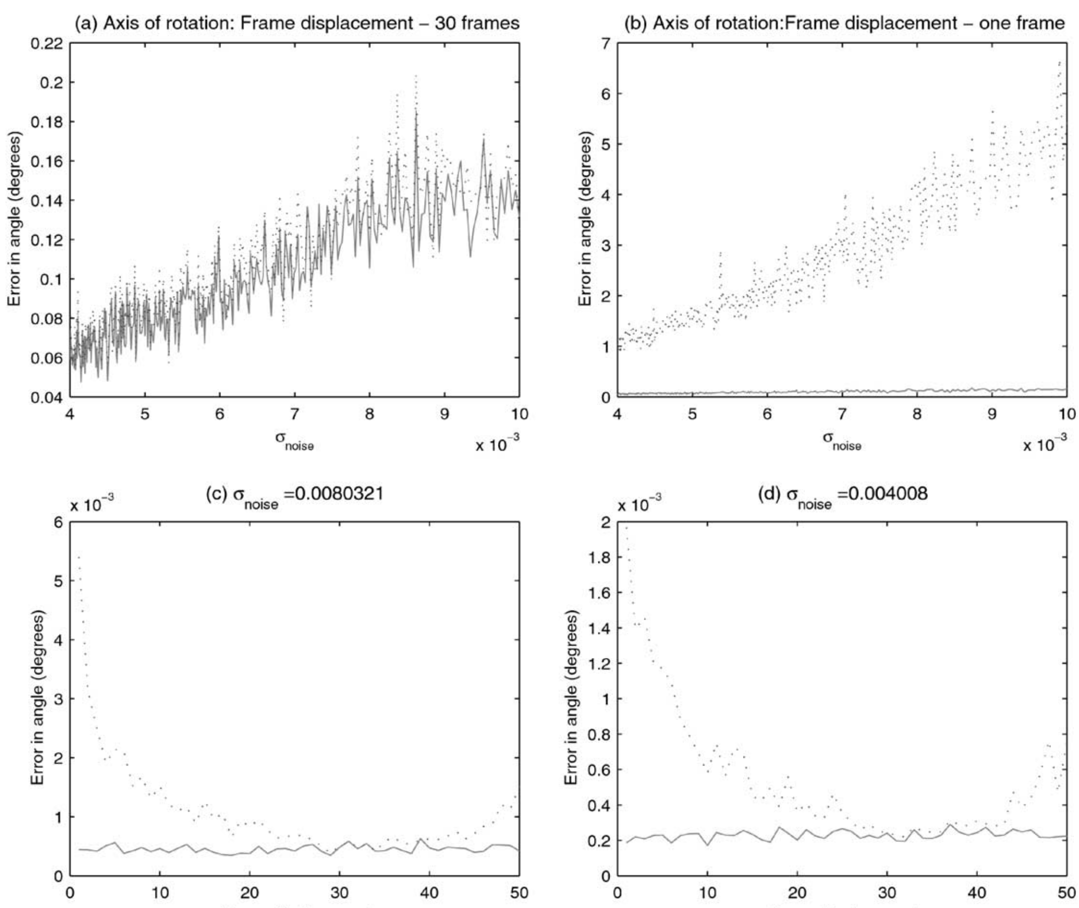

Fig. 3. Error in the direction of the axis of rotation in the case of random errors compared with Halvorsen et al. (1999). The solid line is the new method and the dotted line is the Halvorsen et al. (1999) method. Total number of frames used is 60. Here the frame displacement is the frame difference used to calculate the vector difference in the Halvorsen et al. (1999) method. Hence it only effects the Halvorsen et al. (1999) method. (a) and (b) show the error in the angle of the estimated axis of rotation when 30 frames and 1 frame respectively are used in the Halvorsen method compared to the new method. (c) and (d) show how the errors in the two methods vary with number of frames used for two values of the noise.

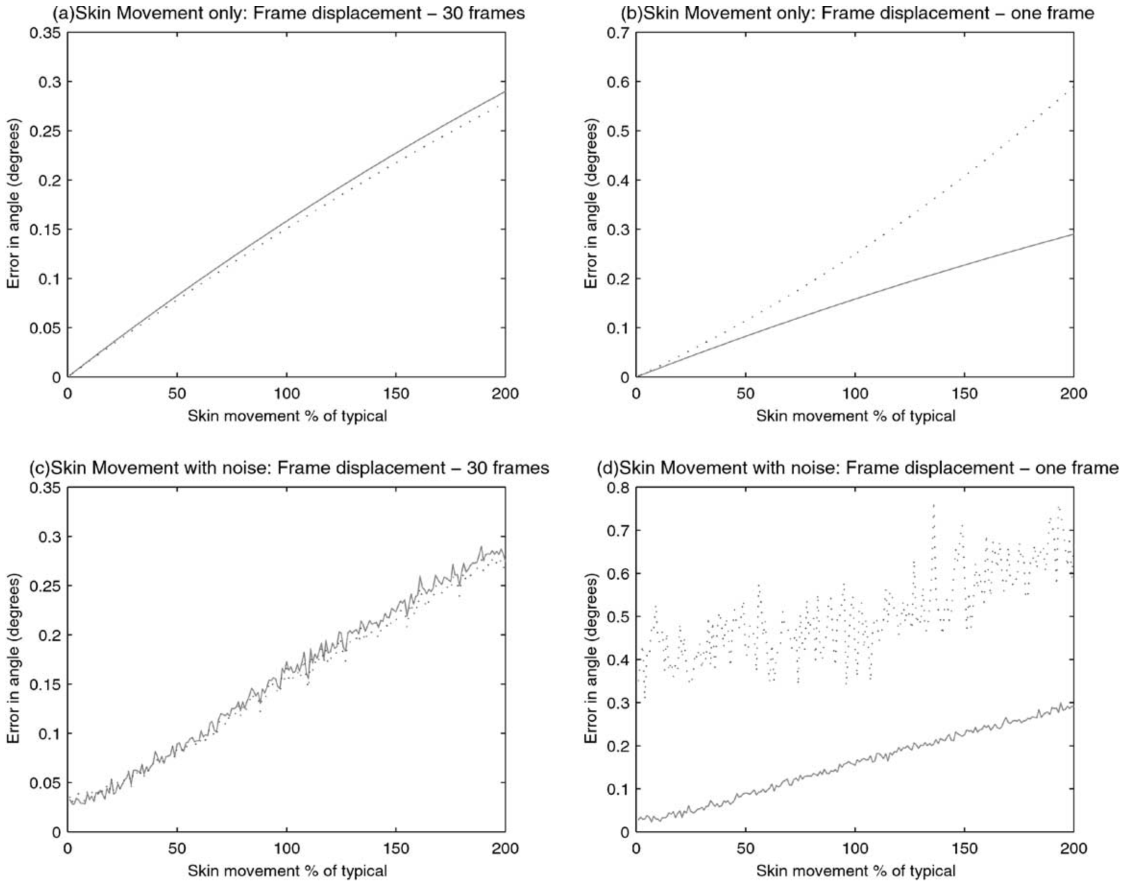

Fig. 4. Error in the direction of the axis of rotation in the case of skin movement compared with the Halvorsen et al. (1999) method. The x-axis is the percentage of errors introduced as described by Cappozzo et al. (1996). The solid line is the new method and the dotted line is the Halvorsen et al. (1999) method. (a) and (b) show how the methods compare when 30 frames and 1 frame respectively are used in the Halvorsen method; only systematic skin movements are included. (c) and (d) show similar results when both skin movements and noise are added. In (c) and (d) the added Gaussian noise has a variance of 0.01. Total number of frames used is 60. The frame displacement is the frame difference used to calculate the vector difference in Halvorsen et al. (1999) method. Hence it only effects the Halvorsen et al. (1999) method.

solution of Silaghi et al. (1998) requires adjustment of weights manually to achieve good results because of the weaknesses mentioned above. The new method requires no subjective adjustments of weights.

5. Discussion and conclusion

It should be noted that the methods proposed here do not assume strict rigidity of the movements since the constraining cost function only assumes each vector relative to the centre or the axis of rotation to trace out a sphere or a circle. These methods would not perform well if there is significant radial displacement from the centre or the axis of rotation. But note that methods that assume rigid body motion such as Söderkvist and Wedin (1993) would also suffer in this case.

In this paper, we have presented a new method of estimating the CoR of a stationary ball joint (i.e. without translation of the CoR) as well as estimating the direction and the plane centres in the case of a stationary hinge joint. This method does well when compared to the best case of the Halvorsen et al. (1999) method and has the advantage of not requiring manual adjustment of the unknown 'optimal' frame displacement. Also it can be used to estimate the instantaneous centre or axis of rotation if the number and the configuration of markers are sufficient (at least three non-planar markers in general). The new method outperforms the method outlined in Silaghi et al. (1998) being superior in the areas of the methodology (i.e. CoR from the whole data set vs. the average of individual CoRs), the solution (i.e. closed-form vs. numerical search) and the necessity of manual intervention (*without* vs. *with* weight adjustment). These three facts were outlined in Section 4.1. The new method can also be applied to joints with translational movements by referring the marker positions to a coordinate frame fixed in one of the links as done by Silaghi et al. (1998) and would be superior due to the reasons given above.

We envisage that this method can be used straightforwardly and automatically for biomedical purposes

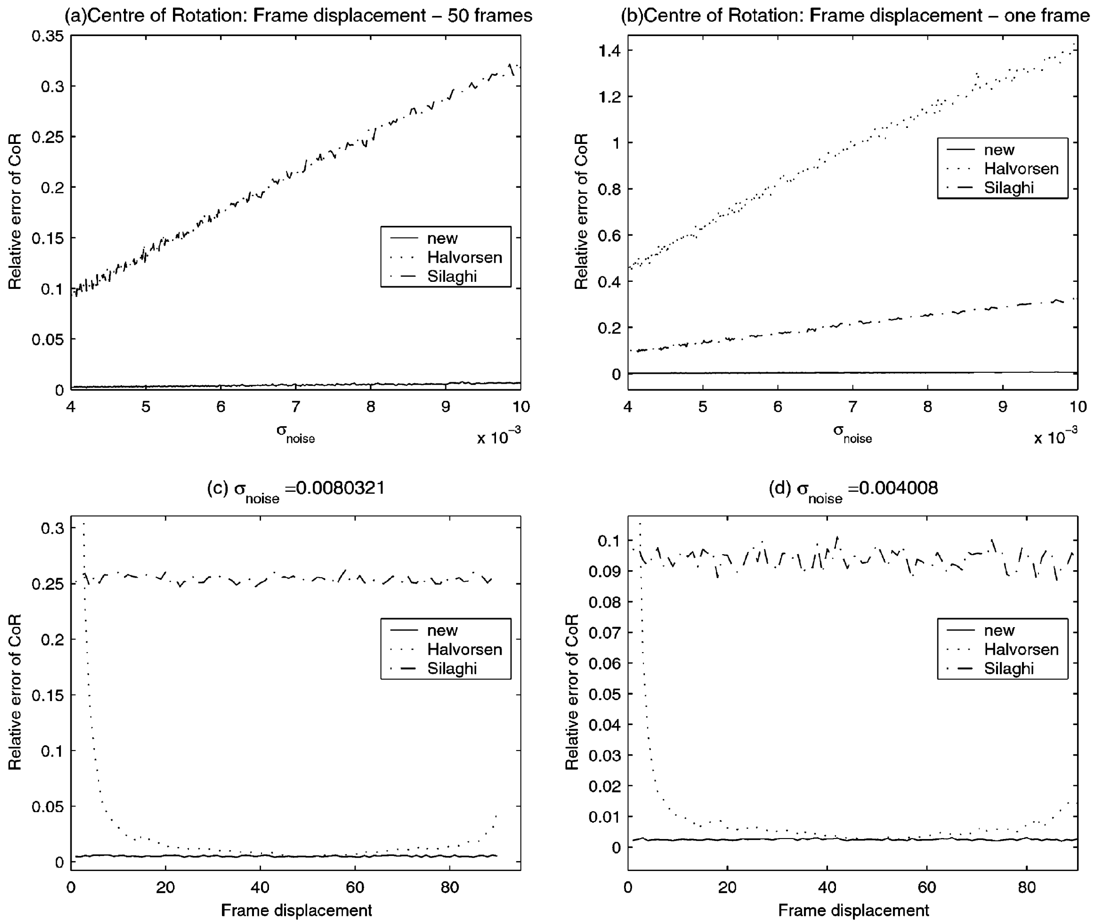

Fig. 5. Error in the location of the centre of rotation in the case of random errors compared with Halvorsen et al. (1999) and Silaghi et al. (1998) methods. The relative error (y-axis) is the absolute error divided by the minimum marker distance. Total number of frames used is 100. The frame displacement only effects the Halvorsen et al. (1999) method. (a),(b),(c) and (d) are as given in Fig. 3. In (a) the new method and the Halvorsen et al. (1999) method are very similar.

directly as well as in extracting CoR and AoR data from optical motion capture systems.

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