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PATH CONSTRAINTS ON POINT-TO-POINT ARM MOVEMENTS IN THREE-DIMENSIONAL SPACE

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Abstract—In this paper data are presented concerning the kinematic and dynamic characteristics of point-to-point arm movements which are inwardly or outwardly directed in three-dimensional space. Elbow and wrist position as well as elbow angle of extension were measured. From these data, other angles were computed trigonometrically and elbow and shoulder torques were calculated. Some of the angles describing arm and forearm motion were found to be linearly related for any given movement. Changes in shoulder and elbow torque were found to be similar to those described for movements restricted to one degree of freedom. Shoulder and elbow motions were not affected when it was required that the orientation of the hand in space remain constant.

These observations were taken to indicate that shoulder and elbow motions are tightly coupled for movements in three-dimensional space and that wrist motion has no influence on this coupling. Linear relations between angles express such coupling. They are taken to result from functional constraints and may facilitate the mapping between extrinsic and intrinsic coordinate systems. Some of the observations pertaining to the torque lead to the hypothesis of a further constraint limiting the number of possible trajectories in a point-to-point movement.

In the preceding paper[16] kinematic and dynamic characteristics of closed and highly constrained trajectories, namely circles and ellipses, were described. Such repetitive movements avoid complications arising when open, point-to-point trajectories are considered, since the latter also involve the initiation and arrest of the movement. However, in everyday life such arm movements are the most commonly used. They are also employed routinely in neurological examination.[7] Therefore it seems appropriate to extend the study of motor coordination in three-dimensional space to point-to-point movements.

Previously, for point-to-point movements restricted to the sagittal plane, it was found that angular elevation at the elbow and shoulder covaried, being linearly related during the deceleratory phase of the movement independently from movement speed, target location and load.[9,14,15] This invariance was taken to be the expression of a control law[14] and not as the result of a functional constraint in the sense detailed in the preceding paper.[16] A study of the kinematic and dynamic aspects of point-to-point movements not confined to the sagittal plane was thought useful to see which of these interpretations of the described invariances was more appropriate.

METHODS

Motor tasks

Right-handed subjects were asked to make pointing movements to targets which consisted of metal rods protruding from a pegboard located in the frontal plane. The targets were located in front of the subject and were placed at varying heights and at different distances to the right and left of the subject's midline. At the onset of each movement the right arm was in the sagittal plane, the forearm approximately horizontal and the upper arm close to vertical. This initial position of the arm was marked by a rod which extended from the floor. The subjects were asked to begin the movements upon presentation of an audible signal.

In a second series of experiments, we asked subjects to touch their nose with the index finger, starting from the same initial position as before. We also asked them to make the movement in the opposite direction, namely from the nose downward. The same subjects were also asked to carry a cup of water to their mouth. The latter task required the subjects to maintain a constant orientation of the hand in space to keep the cup level. Since the trajectory of the wrist for these self-directed movements showed appreciable curvature (see Figs 7 and 8) a string was stretched from the initial position to the nose to confine the movement to rectilinear motion at the wrist.

Recording system and data analysis

Experimental procedures have been described in detail in the preceding paper.[16] The positions of the elbow and wrist in space were measured, as was the anatomical angle of elbow flexion–extension (ϕ). From these data, the angular elevations of the upper arm and forearm (θ, β) and the yaw angles of the two limb segments (η, α) were calculated. Torques at the shoulder and elbow joints required to produce the movement were computed following smoothing and numerical differentiation of the orientation angles.[12] Electromyographic activity of some of the muscles acting at the shoulder and elbow joints was recorded by means of surface electrodes. Kinematic data were sampled at 100 or 125 Hz, electromyographic (EMG) activity at 500 Hz.

Trials in which the maximum velocity at the wrist was similar were averaged. Individual trials were aligned relative to movement onset, which was defined as the time at which velocity began to exceed 10% of its maximum value. For purposes of averaging, EMG activity was full-wave rectified.

This report summarizes the results of 10 experiments involving five subjects.

Abbreviation: EMG, electromyographic.

RESULTS

Kinematic characteristics of pointing movements to targets in extrapersonal space

In a first series of experiments, we examined the trajectories of pointing movements of the right arm to a set of targets. Subjects were instructed to point to one of three targets located in front of the subject which were aligned and separated vertically in height by 20 cm. Movements were alternated at random between the three targets and after 6–10 movements to each target had been obtained, target location was changed by moving the targets medially or laterally to the subject.

Figure 1 shows typical results. The trajectories described by the elbow and the wrist are shown; each trace depicts the results of one trial. The traces in Fig. 1A are for movements to a target located in front of and to the left of the subject, those in Fig. 1B for a target located to his right. The traces depict the movement in three-dimensional space, the X-axis corresponding to the anterior–posterior direction, Y to the medial–lateral direction and Z to the vertical. The dashed lines indicate the projection of the trajectories of the elbow and of the wrist onto the horizontal (X–Y) and the sagittal (X–Z) planes. The direction of the movement is indicated by arrows.

Some features of the trajectories of these movements agree with those described for planar movement in the sagittal[14] or horizontal[3,6,11] plane. There is little trial-to-trial variability of the path described by the wrist and by the elbow. While the path of the wrist approximates a straight line in some cases (e.g. Fig. 1A), other movements exhibit appreciable curvature. For example, in Fig. 1B, the maximum deviation of the wrist from a straight line is about 4.5 cm. The movements illustrated in Fig. 1 were all performed at about the same speed, a maximum wrist velocity of 150–180 cm/s and a movement duration of 500–600 ms. When movement speed was varied, we could find no path dependence on speed, in agreement with previous observations.[14] Furthermore, also in agreement with previous observations,[5,13] the velocity profile at the wrist was approximately bell-shaped, with a single maximum.

The changes in the angular orientation of the upper

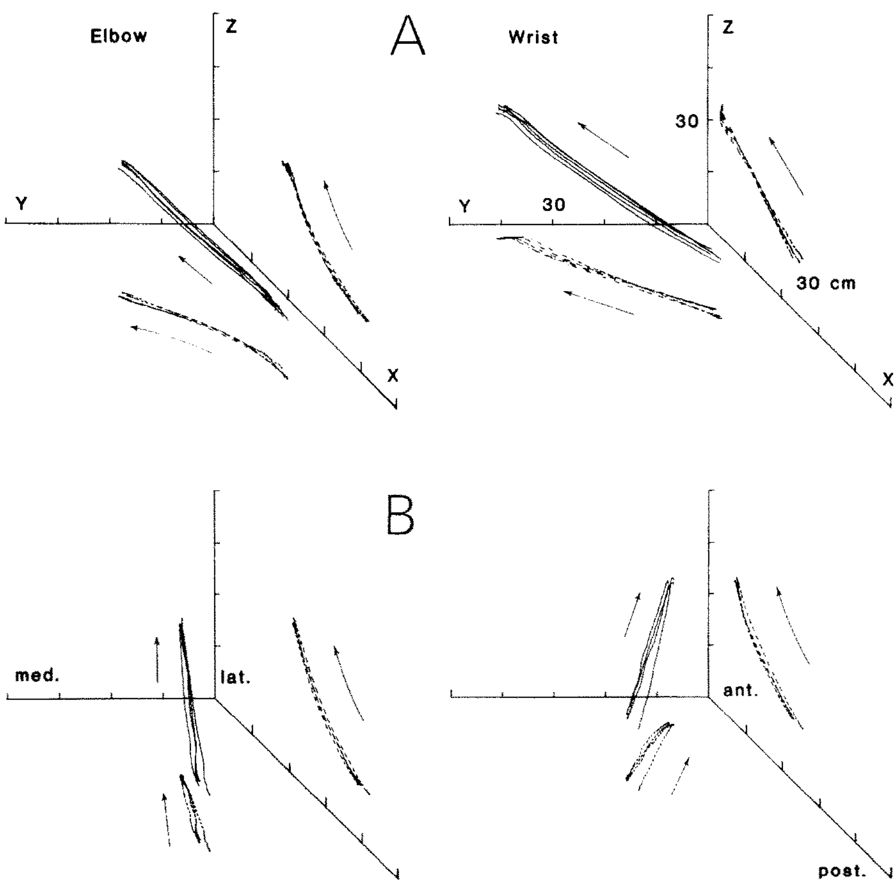

Fig. 1. Trajectories of wrist and elbow during pointing movements to targets located anteriorly and to the left (A) and to the right (B). Each solid trace depicts the path taken by the elbow (left column) and the wrist during one movement in three-dimensional space. The dashed lines depict the projection of the trajectories onto the horizontal (XY) and sagittal (XZ) planes. The arrows denote the direction of the movement.

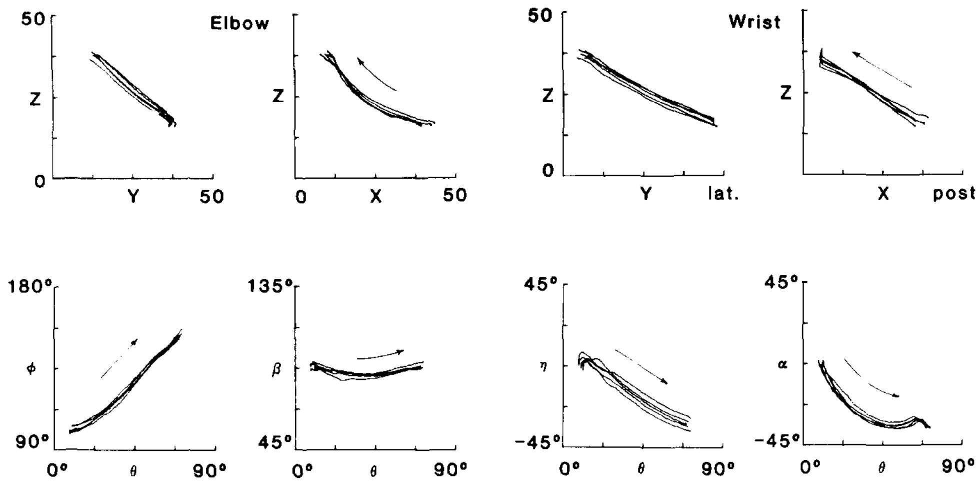

Fig. 2. Pointing movements to a target located to the left of the mid-sagittal plane. The upper row shows the projection of elbow and wrist trajectories onto the frontal (YZ) and sagittal (XZ) planes, the lower row changes in elbow angle of extension (ϕ), forearm elevation (β) and upper arm (η) and forearm (α) yaw as a function of changes in upper arm elevation (θ). The data are for the same trials as those depicted in Fig. 1A.

arm and the forearm during the movements shown in Fig. 1 are described in Figs 2 and 3. Figure 2 refers to movements to the target located to the left of the subject (medially directed movements, as in Fig. 1A), while the angular motions corresponding to the rightward directed movements (Fig. 1B) are described in Fig. 3. Our choice of angular coordinates to describe the motion, based on a psychophysical determination of preferred angular coordinates,[17] is the same as those used in the preceding paper[16] where they are defined in Fig. 1. They may well be referred to as orientation angles and comprise: the angular elevation of the upper arm (θ) and of the forearm (β) and upper arm and forearm yaw (η, α). In addition, changes in the anatomical angle of forearm flexion–extension (ϕ), which were measured directly, are shown.

The top rows in Figs 2 and 3 illustrate the projection of the trajectories of the elbow and wrist onto the frontal (YZ) and the sagittal (XZ) planes, while the lower rows show changes in the angles ϕ, β, η and α plotted as a function of upper arm elevation (θ). Given the initial orientation of the arm, the two yaw angles η and α are close to 0° at the onset of the movement, while β is about 90°. Medially directed rotations, as in Fig. 2, correspond to negative yaw angles, while a laterally directed rotation (Fig. 3) corresponds to an increase in η and α.

Since there is little variability in the trajectories of the elbow and the wrist, there is also little variability in the angle–angle plots in Figs 2 and 3. Furthermore, rectilinear relations can be seen between some of the angles as well as between components of wrist and elbow motions in Cartesian coordinates (X, Y, Z). In the following, we shall concentrate first on those angles which were found to be consistently linearly related throughout the movement. Thus, in Fig. 2 the

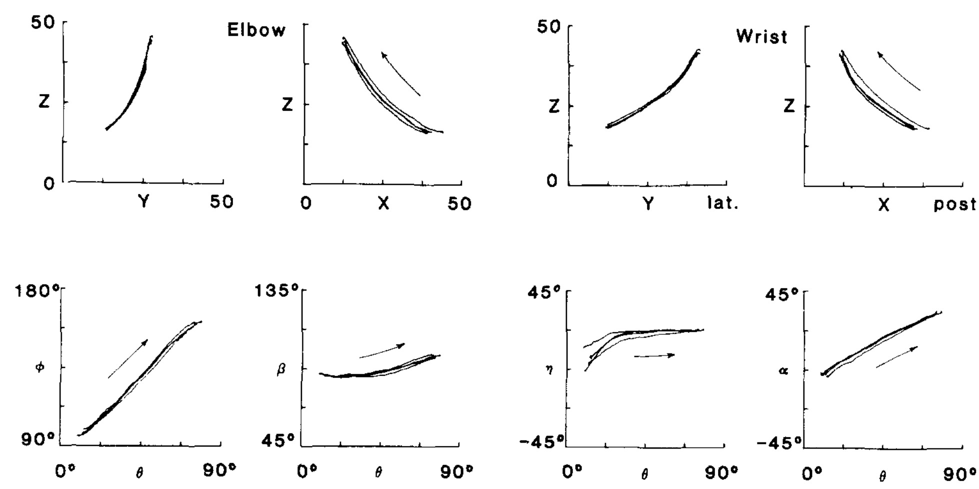

Fig. 3. Pointing movements to a target located to the right of the mid-sagittal plane. The data are for the same trials as those depicted in Fig. 1B.

variation of the yaw angle (η) of the upper arm with upper arm elevation (θ) is well approximated by a linear relation, except for a small portion at the onset of the movement. Instead, for the laterally directed movements in Fig. 3 forearm yaw (α) covaries linearly with θ. This was true for all the target locations examined for this subject; namely, η and θ were linearly related for movements to the left of the mid-sagittal plane, while α and θ covaried linearly for rightward, laterally directed movements. The relationship between θ and the other yaw angle in each case cannot be approximated by a straight line. For example, the α–θ plot in Fig. 2 is clearly curvilinear, while η initially increases rapidly and then remains approximately constant in Fig. 3.

For the drawing movements described in the preceding paper,[16] β and θ were found to be linearly related throughout the movement, while for pointing movements confined to the sagittal plane such a relationship had previously been found to hold true only in the deceleratory portion of the movement.[14] As can be seen in Figs 2 and 3, such restricted linear relationships are observed also for movements directed obliquely to the sagittal plane. The relationship between β and θ is close to linear in the later portion of the movements ($\theta > 55°$), but it is not as strict as was found previously for movements in the sagittal plane. In those instances, even when one angular velocity was plotted as a function of the other (thus accentuating small deviations from linearity) the angular velocities of θ and β were linearly related in the deceleratory phase of the movement. Such was not always the case for the movements described here, since there were cases when the angular velocities fluctuated about a straight-line relation. This can already be appreciated in the plots of β vs θ in Fig. 2.

The plots presented in Figs 1–3 are representative of the behavior of all three subjects examined on this task. Figures 4 and 5 present data from the other two subjects. In these figures, which have the same format as Figs 2 and 3, we present average trajectories for movements to several different target locations. Part A of each figure shows data for movements which are primarily in the sagittal plane to three targets separated vertically by about 20 cm. Part B shows averages of movements to targets located anteriorly to

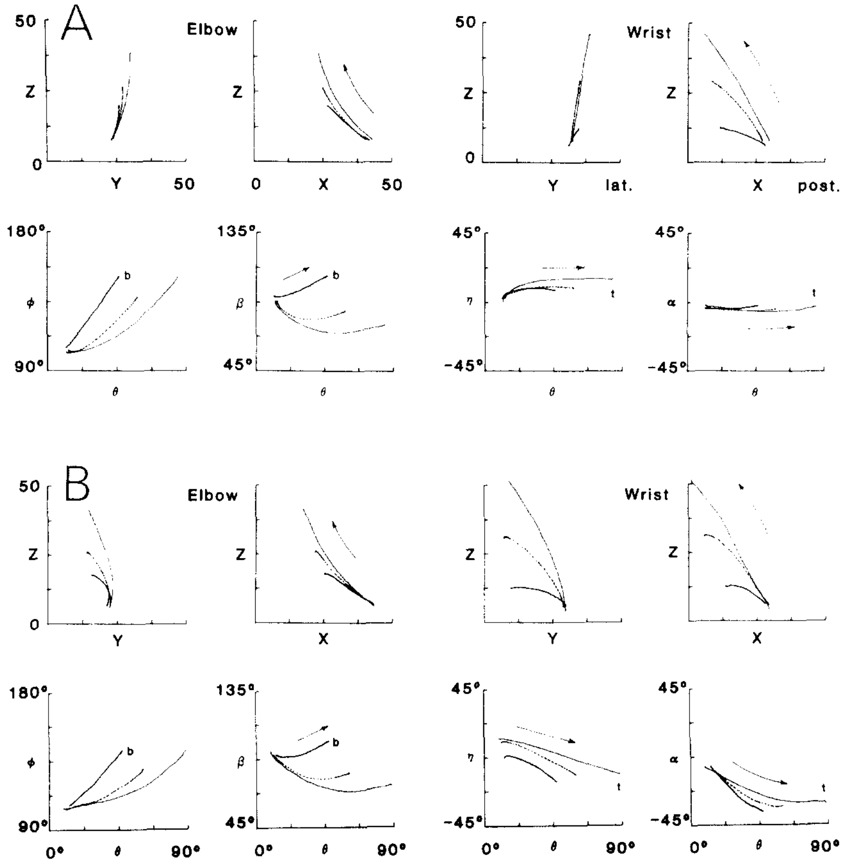

Fig. 4. Average trajectories of pointing movements performed in a sagittal plane (A) and to targets located medially, to the left of the mid-sagittal plane. Each trace denotes the average trajectory of the wrist and elbow (upper row of each part of the figure) and the average changes in elbow and shoulder angles (lower row) for movements to one of three targets aligned vertically and separated by about 20 cm. Traces labeled t refer to movements to the top-most target, those labeled b to movements to the bottom target.

the left of the subject (Fig. 4B) and to the right in Fig. 5B. Traces corresponding to movements to the top-most target are labeled t, those to the lowest target, b. Note that as in Fig. 2, η and θ covary linearly when movements are directed to the left (Fig. 4), while α and θ are linearly related when the movements are in the right anterior quadrant (Fig. 5). In all cases, the two angular elevations β and θ are linearly related toward the end of the movement and sometimes throughout.

In summary then, the kinematic analysis of pointing movements did reveal linear relations between some of the orientation angles, as was found to be the case for drawing movements examined in the preceding paper[16] and for point-to-point movements in the sagittal plane.[14] The details of such invariant relations were different, however, as pointed out above.

Dynamic characteristics of pointing movements

Electromyographic activity of some of the muscles acting at the shoulder joint (anterior and posterior deltoid, pectoralis) and at the elbow joint (biceps) was also recorded. The torque at the two joints required to produce the observed movements was computed from the kinematic data. The variation of these variables during movements to different target locations is shown in Fig. 6. The panels in the upper row (Fig. 6A–C) are for movements to the top-most target and those in the lower row to the lowest target for each of three locations: the left anterior quadrant (Fig. 6A and D), midsagittal plane (Fig. 6B and E) and the right anterior quadrant (Fig. 6C and F). The direction of the motion of the wrist is indicated schematically by the arrow at the top of each panel.

Torque at the shoulder is represented in a Cartesian coordinate system fixed to the upper arm and rotating with it. In such a coordinate system, the lines of action of the muscles inserting on the humerus, and thus the direction of the torques they exert, are approximately constant. When the arm lies in the sagittal plane and the upper arm is vertical (the starting position for the movements we have investigated, corresponding to θ, η and α equal to 0), the axes of this coordinate system coincide with the principal directions (X, Y, Z as in Figs 1–5). In this orientation, a positive torque at the shoulder about the Y-axis (T_{sy}) would produce forward flexion of the arm. Similarly, positive T_{sx} would result in abduction while positive T_{sz} would give outward (lateral) rotation of the arm about the axis of the humerus. Positive torque at the elbow (T_e) is defined to yield elbow flexion.

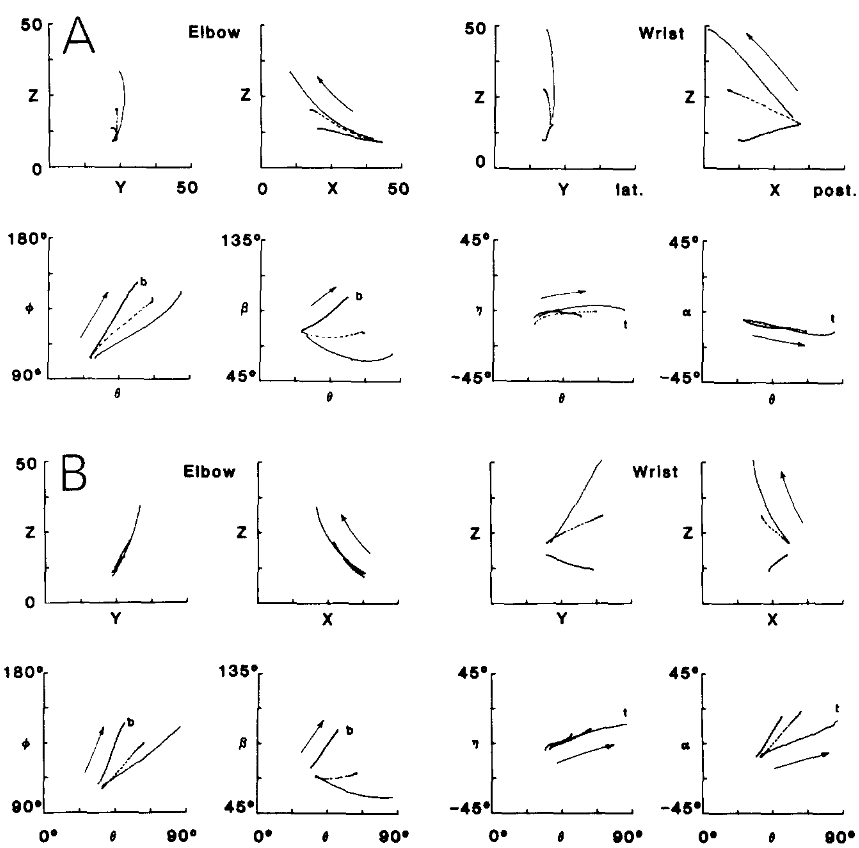

Fig. 5. Average trajectories of pointing movements performed in a sagittal plane (A) and to targets located laterally, to the right of the mid-sagittal plane.

Note that there is reasonable qualitative agreement between the changes in EMG activity and the changes in torque which were calculated. For instance biceps activation is most pronounced in Fig. 6A, and a large increase in flexor torque (T_e) at the elbow is present. When the initial change in torque is in the extensor direction (Fig. 6F), biceps initially becomes silent. Similarly, there is reasonable correlation between activity of anterior deltoid and torque producing forward flexion at the shoulder (T_{sy}), the increase in anterior deltoid activity being most pronounced in Fig. 6A, and least pronounced in Fig. 6D–F. Note that changes in anterior deltoid and biceps activity need not covary. While activity in both muscles does increase in Fig. 6A, biceps pauses as anterior deltoid activity increases in Fig. 6F. The changes in activity of pectoralis are correlated with changes in T_{sx} and T_{sz}. The action of this muscle is more complicated since it can produce both internal rotation (negative T_{sz}) and adduction (negative T_{sx}). Pectoralis activity is largest in Fig. 6D, where the decrease in T_{sx} and T_{sz} is largest. Activity in posterior deltoid is modest in all cases.

The pattern of the changes in torque and the patterns of EMG activities we have just described are in general very simple and similar to those which have been described for movements restricted to one[2,4,10] or two[6,14] degrees of freedom. In most instances an initial accelerative increase in torque is followed by a change in torque in the opposite direction, tending to decelerate the movement. This is the pattern which markedly characterizes the behavior of elbow torque (T_e) in Fig. 6A and B, T_{sy} in Fig. 6A, B and D, and T_{sx} in Fig. 6F. In other cases, only an initial accelerative component is obvious; thereafter torque remains constant, as for T_{sx} in Fig. 6D and T_{sz} in Fig. 6D and F. Similarly, agonist activity exhibits either an initial burst, followed by a depression and

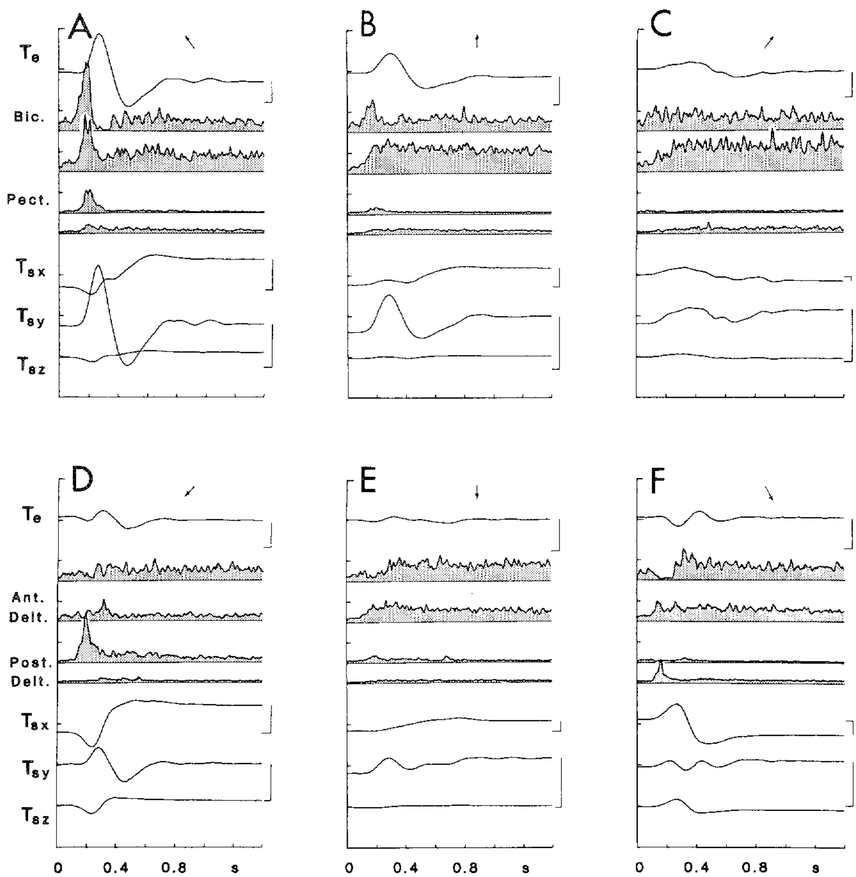

Fig. 6. Changes in torque and electromyographic activity of shoulder and elbow muscles during point-to-point movements in three-dimensional space. The traces depict the calculated changes in torque at the elbow (T_e) and in the X, Y and Z components in torque at the shoulder (T_s) required to produce the observed movements to targets located anteriorly and medially (left-most column), straight ahead (center column) or laterally to the subject. Data for movements to the uppermost target are shown in (A)–(C), those for movements to the lowest targets in (D)–(F). Averaged, full-wave rectified EMG activity of biceps (Bic.), anterior (Ant.) and posterior (Post.) deltoid (Delt.) and pectoralis (Pect.) is also shown. The sign convention adopted for torque is described in the text. One division equals 5 nm for elbow torque and 10 nm for the shoulder torque.

a plateau (biceps and anterior deltoid in Fig. 6A) or a more or less gradual increase to a plateau (biceps in Fig. 6B and C).

As regards timing, the maxima and minima of elbow torque and of the Y-component of shoulder torque (T_{sy}) tended to coincide. In those cases where clear maxima and minima could be identified, the maximum in elbow torque occurred 23 ± 33 ms after that in T_{sy} and the minima differed by 26 ± 24 ms (calculated on the basis of averages of 18 sets of data). The peaks of the initial changes in T_{sx} and T_{sz}, instead, preceded that in T_{sy} by about 70 ms. In those instances where the timing of the maximum of the initial changes in these components of shoulder torque (T_{sx} and T_{sz}) could be measured (for example, Fig. 6A, D and F), they were coincident.

Kinematics and dynamics of self-directed movements

We also investigated movements directed to points on the subjects' body surface. Specifically, we asked them to touch the tip of their nose with their index finger, starting from the same initial arm position as before (arm in the sagittal plane, forearm horizontal and upper arm close to vertical). Movements in the opposite direction, that is from the nose downwards, were also studied.

Figures 7 and 8 illustrate the performance of one subject on these tasks. As with movements directed to targets in extrapersonal space, there was little trial-to-trial variability in the trajectories described by the wrist or the elbow. As is apparent in Fig. 7, the trajectory of the wrist had appreciable curvature in these tasks. The deviation from straightness was quantified by interpolating a straight line between the initial and final points and measuring the perpendicular distance from the actual path to the straight line. According to this criterion, the maximum deviation averaged 5.0 cm (14% of the movement distance of 35 cm) for the five subjects who performed this task. Despite the appreciable curvature, wrist velocity exhibited an approximately bell-shaped profile with a single maximum.[1,13]

The path of the wrist was not always identical for the movements in the two directions. Thus, in the example shown in Fig. 8, the curvature of the trajectory of the wrist was much greater on the downward movement (Fig. 8B). One of the other subjects showed the opposite behavior, namely a greater deviation from a straight-line path on the upward movement. In other instances, there was little difference in the paths described by the wrist for the movements in the two directions.

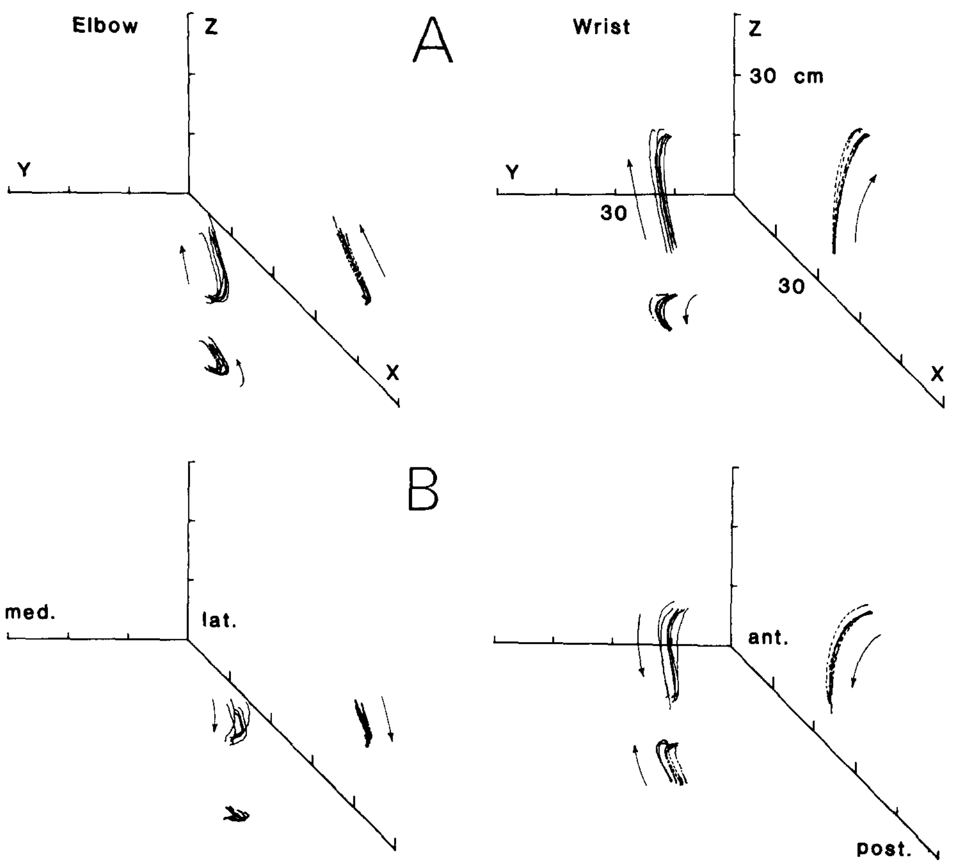

Fig. 7. Elbow and wrist trajectories for pointing movements to the nose (A) and away from it (B). Each trace depicts the trajectory of wrist and elbow in three-dimensional space or its projection onto the horizontal and sagittal planes.

Linear relations between some of the orientation angles were found also for these movements. For example, in Fig. 8A, the yaw angle α of the forearm varies linearly with upper arm elevation, while yaw of the upper arm (η) and forearm elevation (β) show curvilinear relations to θ. There was some inter-subject variability, however. For one other subject the relationships among the orientation angles were similar to those shown in Fig. 8A. For two other subjects, the relationship between α and θ was clearly curvilinear; instead, β and θ were linearly related to each other. One example, for trials in which the subject touched the nose with his finger, is given in Fig. 9A. The fifth subject showed one type of behavior for movements in one direction, the other type when movements were oppositely directed. The two kinds of relations among the angles correlated with the motion of the elbow. In Fig. 8A, there is appreciable lateral motion (Y direction) of the elbow and the final value of upper arm yaw is about 35°. In Fig. 9A, instead, it is apparent that elbow motion was primarily in the sagittal plane and η remained close to zero throughout the motion.

In agreement with previous observations on planar movements,[8,13] we found that task requirements on the orientation of the hand in space did not affect motion at the shoulder and elbow joints. The experiment required subjects to transport a cup filled with water to their mouth. The initial and final positions of the hand were thus approximately the same as when the subject was asked to touch his nose with his index finger. However, successful completion of the former task requires the orientation of the hand in space to remain constant (for the cup to remain level), while obviously no such constraint is imposed in the latter task. Results of one such experiment are shown in Fig. 9. In the trials depicted in Fig. 9A, the subject made a pointing movement to his nose; those in Fig. 9B are instances in which a cup was transported. Except for the not unexpected finding that the latter movements are appreciably slower (maximum wrist velocity of 44 cm/s and movement duration of 1240 ms on average, compared to 104 cm/s and 660 ms), there were no consistent differences between the arm trajectories for the two tasks. In particular, the trajectory of the wrist still exhibited appreciable curvature when a constraint on hand orientation was imposed.

The dynamic characteristics for pointing movements towards and away from the nose are shown in Fig. 10. In agreement with observations reported in Fig. 6, elbow (T_e) and shoulder torque (T_{sf}) show a single maximum and minimum, whose times did not differ significantly. On average, the accelerative peak

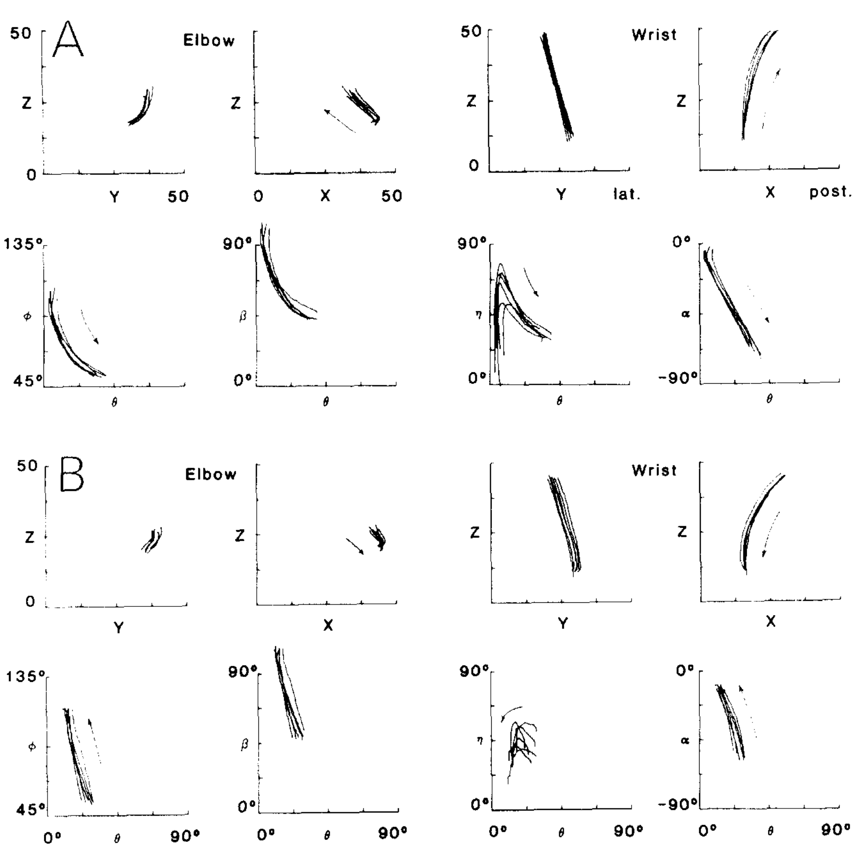

Fig. 8. Pointing movements directed to the nose (A) and away from it (B). The data in (A) and (B) are for the same trials as those depicted in Fig. 7A and B, respectively.

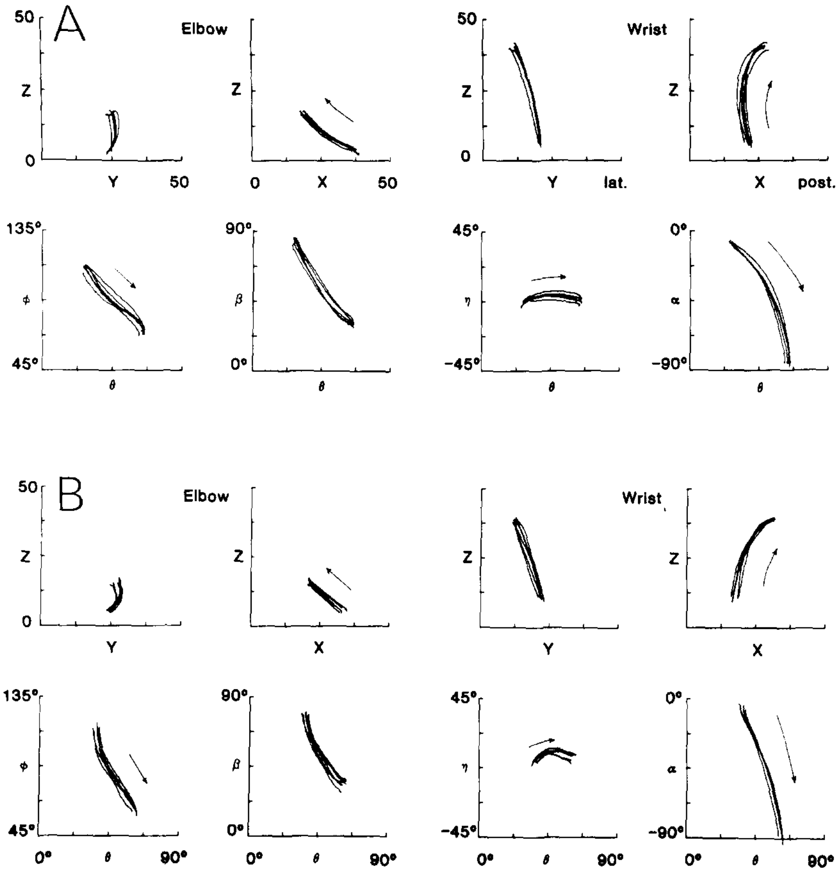

Fig. 9. Effect of task requirements concerning hand orientation on shoulder and elbow motion. Data shown in (A) are for trials in which the subject touched the nose with his finger, those in (B) for trials in which the subject transported a cup of water to his mouth.

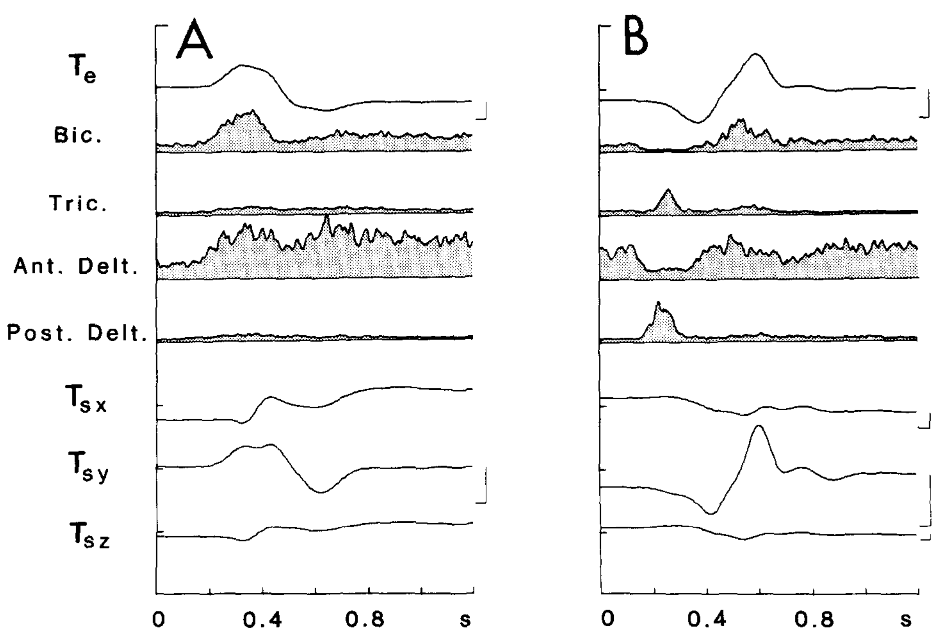

Fig. 10. Changes in torque and electromyographic activity during movements directed toward the nose (A) and away from it (B). Averaged, full-wave rectified EMG activity from biceps (Bic.), triceps (Tric.) and anterior (Ant.) and posterior (Post.) deltoid (Delt.) are shown together with torque at the elbow (T_e) and at the shoulder (T_s) required to produce the observed movements. One division equals 10 nm.

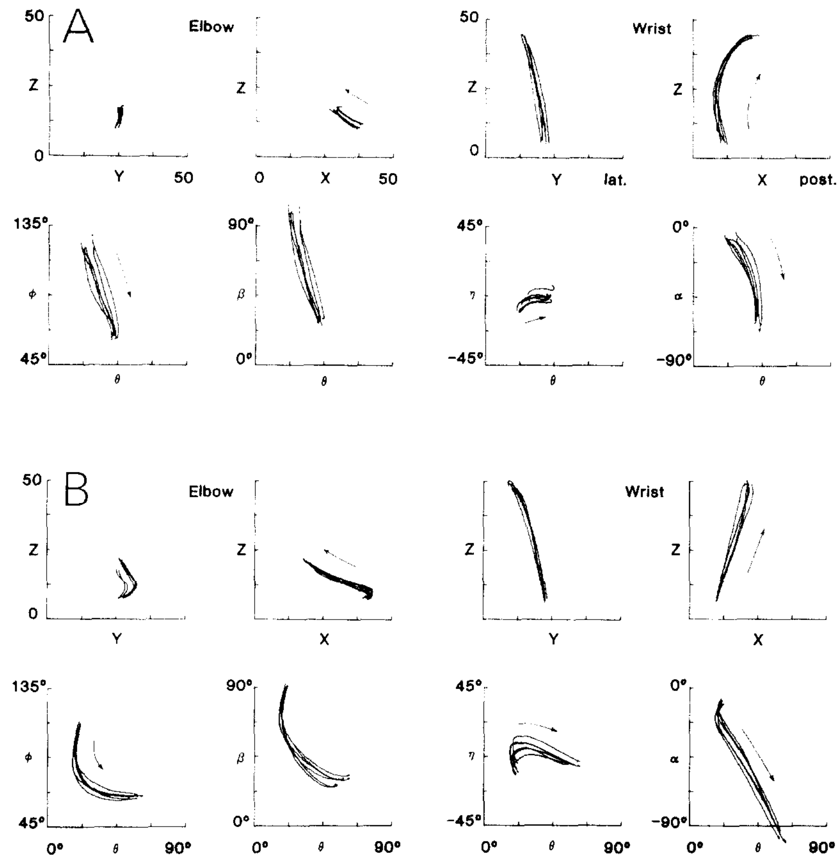

Fig. 11. Kinematics of free and guided movements to the nose. In the trials illustrated in (A) the subject touched the nose with his finger; in (B) the same movement was performed by following a string stretched from the initial position of the hand to the nose.

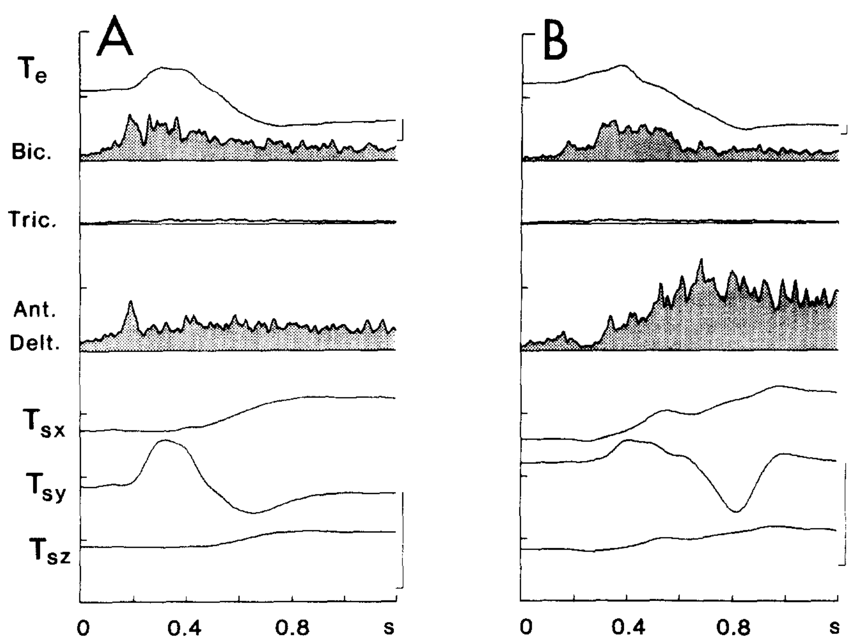

Fig. 12. Dynamics of free and guided movements to the nose. (A) shows averaged data for the same trials as illustrated in Fig. 11A, (B) those corresponding to Fig. 11B. One division equals 5 nm.

of T_e preceded that of T_{sy} by 4 ± 27 ms, the decelerative peak of T_e lagged by 41 ± 63 ms. Also, EMG activity of shoulder and elbow muscles shown is in reasonable agreement with the calculated torques.

Since some investigators have emphasized straight-line trajectories in the context of the planning of movements involving motion at more than one joint,[6,11] we also performed some experiments in order to see why these particular movements might show such appreciable curvature. Specifically, we stretched a string from the starting point to the subject's nose and asked the subject to make guided movements by following the string, thus constraining the wrist to follow an approximately straight-line trajectory.

A comparison between free and guided movements by one subject is given in Figs 11 and 12. In part A of each figure are shown the kinematics (Fig. 11A) and the torque and the EMG activity (Fig. 12A) when the subject was asked to touch his nose with no constraint on wrist trajectory. Parts B of each figure show analogous data obtained when the wrist was constrained to follow a straight-line path. In the latter condition, motion at the elbow reverses direction (Fig. 11B), initially moving posteriorly (X-axis) and then forwards. In contrast, in free movements (Fig. 11A) there is a monotonic progression of the elbow. In terms of the orientation angles, angular elevation of the upper arm (θ) initially decreases and then increases in Fig. 11B; in Fig. 11A the increase is monotonic.

The constraint on the motion of the wrist also affects the evolution of the joint torques and the pattern of muscular activities underlying them. In Fig. 11A, biceps and anterior deltoid begin to increase at the same time, as do T_e and T_{sy}. The initial backward motion at the elbow (Fig. 11B) requires a pause in anterior deltoid at the same time that biceps activity is increasing. Similarly, the increase in shoulder torque T_{sy} is delayed markedly relative to that in T_e. Note that the guided movements were performed more slowly than the free movements. Had the former been performed faster, one would expect the reversal of elbow motion to require an actual decrease in T_{sy} and thus activation of the antagonists to anterior deltoid. In short, both the EMG activity and the torque present more complex patterns in this instance than they do in the case of unconstrained movements (Figs 6, 10 and 12A).

DISCUSSION

The two main findings for three-dimensional point-to-point movements reported in this paper to be discussed first are: (1) some of the orientation angles are linearly related* to each other for a given movement, and (2) changes in each of the components of torque at the shoulder and elbow and the patterns of EMG activity responsible for them have a simple appearance, similar to those found in movements restricted to one degree of freedom.

Concerning the first point, it should be noted that for the movements which were studied, there is little trial-to-trial variability in the trajectory of the wrist and elbow. Thus, as in the case of planar point-to-point movements[3,14] and the continuous drawing movements described in the preceding paper,[16] shoulder and elbow motion is tightly coupled. For movements in the sagittal plane, it had also been found that the angular elevations of the arm (θ) and of the forearm (β) were linearly related during the deceleratory phase of the movement, with a slope which was independent from the final position of the wrist.[14] Since this invariant relationship led to an elbow torque which was nearly constant, it was suggested that it could reflect a control law.[14]

However, in the more general case presented in this paper, the slope of linear relationships between some of the orientation angles did depend on final wrist position and some of the linear relationships were valid throughout the movement. The present data also differ from those obtained for the drawing movements, where β and θ were linearly related throughout. Furthermore, which of the four orientation angles were linearly related depended on the location of the end point in space (see Figs 2 and 3). Finally, inter-subject differences were found in the sense that for some target locations different pairs of angles were linearly related.

All these findings tend to argue against the interpretation that linear relations between angles can be taken to indicate a rigid control law, as suggested originally. At the present time the alternative interpretation of invariance presented in the preceding paper [16] appears more plausible. From this point of view linear relationships between angles as described in this paper would be the expression of a general constraint which facilitates the mapping between extrinsic and intrinsic coordinates. While the structure of such a mapping could be easily deduced in the preceding paper, since both angular motion and wrist motion were sinusoidal, for point-to-point movements the question cannot be resolved at this time. However, if one were to generalize the point of view put forth in the preceding paper, one would suggest an identity between the linear relations of joint angles and the expected linear wrist movement in space. (The word expected is used since the actual trajectory may in some instances deviate from a straight line without the subject's cognizance.) If so, the sensorimotor mapping would be straightforward.

As for the second point, namely the changes in torque during inwardly and outwardly directed movements, they were found to be either biphasic with an initial increase followed by a decrease leading to the arrest of the movement, or monotonic. Also,

*Note that in general the presence of a linear relationship between two variables cannot be taken *per se*, as evidence for the existence of a control process leading to such a relationship.

the minima and maxima of the components of the torque at the shoulder and elbow tended to coincide. These features may reflect the presence of other general constraints involving torque and/or patterning of EMG activity. In particular, the observation that naturally executed movements directed to the nose are highly curvilinear may be due to such constraints. In this case, straight-line movements require an initial pause in the agonist for arm elevation (anterior deltoid) and possibly activation of its antagonists at the onset of the movement (Fig. 12). To generalize this observation, one can then hypothesize that naturally executed movements can only be initiated by activation of the agonists for that movement. This sample constraint would limit the number of possible trajectories in space.

One final point can be mentioned. As already pointed out in the Results, the requirement that the orientation of the hand remain constant (such as when carrying a cup of water) had no effect on the motion of the more proximal limb segments (arm and forearm). This is in agreement with previous observations in which wrist pronation–supination[8] as well as flexion–extension[13] were shown to have no influence on shoulder and elbow motion. However, this does not imply that wrist motion is always independent of motion at the more proximal joints.

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