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Using orientation statistics to investigate variations in human kinematics

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Summary. This paper applies orientation statistics to investigate variations in upper limb posture of human subjects drilling at six different locations on a vertical panel. Some of the drilling locations are kinematically equivalent in that the same posture could be used for these locations. Upper limb posture is measured by recording the co-ordinates of four markers attached to the subject's hand, forearm, arm and torso. A 3×3 rotation characterizes the relative orientation of one body segment with respect to another. Replicates are available since each subject drilled at the same location five times. Upper limb postures for the six drilling locations are compared by one-way analysis-of-variance tests for rotations. These tests rely on tangent space approximations at the estimated modal rotation of the sample. A parameterization of rotations in terms of unit quaternions simplifies the computations. The analysis detects significant differences in posture between all pairs of drilling locations. The smallest changes, less than 10° at all joints, are obtained for the kinematically equivalent pairs of locations. A short discussion of the biomechanical interpretation of these findings is presented.

Keywords: Directional data; Drilling task; Joint kinematics; Multivariate statistics; Quaternions; Rotations

1. Introduction

The analysis of human movement has been the subject of several studies in the past. An interesting review of the various domains of research on human kinematics is presented in Woltring (1989). Several companies have marketed camera systems to record automatically the spatial motion of markers, fixed to the body, using different technologies, e.g. Optotrack, Selspot, Peak, Vicon, Elite, Polhemus and MacReflex. The data from these high performance recording systems permit the calculation of time-varying human kinematics such as position, velocity and acceleration. For some systems, the markers consist of one landmark whose time-varying co-ordinates are recorded in the system reference frame (see Olshen *et al.* (1989)). The one-landmark markers do not permit a complete characterization of the orientation of the body segment to which they are attached; at least three landmarks on each body segment must be used to characterize orientation. For instance, a four-landmark marker, as shown in Fig. 1, can be used to characterize the orientation of any of the upper limb body segments. Knowledge of the relative orientations of adjacent limb segments provides a means of characterizing the limb geometric configuration, i.e. limb posture.

Characterization of limb posture is helpful for conducting motor control studies in various biomedical research fields, e.g. neurophysiology, rehabilitation and performance in sport. For instance, by analysing limb motion, we may be able to infer the structure of the neural

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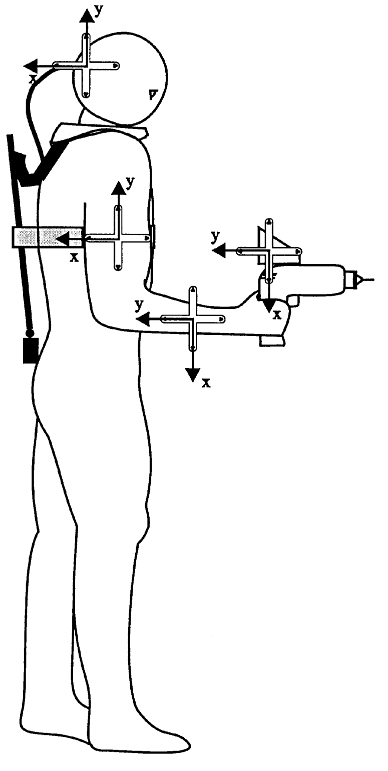

Fig. 1. Location and orientation of markers in the drilling task investigated (for all the markers, the *z*-axis points backwards into the plane of the paper)

system that supervises the control of the limb (for example see Sabes and Jordan (1997)). We may also want to use upper limb kinematic data to investigate different optimization schemes underlying the control of limb motion, e.g. minimization of the torque change criterion for a given limb movement (Kawato *et al.*, 1990). It is thought that such optimization is necessary to determine the particular posture that a human subject would choose to perform a task. As the upper limb is a redundant manipulator, there is an infinite number of allowable limb postures to perform hand-interactive tasks. The upper limb alone has at least 7 degrees of freedom: three at the shoulder, one at the elbow and three at the wrist (McCarthy (1990), chapter 5). This additional degree of freedom allows a human subject to modify its upper limb posture while keeping the same hand position and orientation relative to the environment.

The experiment considered in this paper was conducted to investigate whether such an optimization scheme exists in the context of a drilling operation. This particular task was chosen because of the interests of one author in investigating human motor control in tool usage. The repeated choice of a particular posture over the infinite number of possibilities may indicate that limb posture could be chosen to fulfil some specific criterion. For instance, subjects could choose a more extended limb to maximize the force that can be produced at the hand in the axial direction of the drill. Several other criteria could be suggested but currently no-one knows whether any such criterion exists. The drilling study attempted to investigate the existence of such a criterion by analysing the consistency in upper limb posture of human subjects in a given task.

The data set for this experiment was recorded by using markers represented in Fig. 1. Each marker consists of four non-collinear infra-red emitting diodes whose time-varying co-ordinates are recorded by a camera system. At each time point the co-ordinates of the four landmarks of a marker can be used to calculate a 3×3 rotation matrix characterizing its orientation. The relative orientation of two human body segments is obtained by computing the relative orientation of two rigid markers, assuming that each marker is rigidly fixed on one of the segments.

Despite recent advances in computing human kinematics, one aspect is still poorly covered, that of providing the end-user with adequate statistical tools to analyse experimental data. Translations of markers in space can be analysed with standard statistical methods since these are linear variables. The situation is quite different when we are interested in statistical methods for investigating the relative rotations between two adjacent body segments. This problem has not really been addressed in the biomechanical literature yet. Veldpaus *et al.* (1988), De Lange *et al.* (1990) and Woltring (1994) have suggested evaluating the stability of an estimated orientation by sensitivity analysis; however, statistical tests and inference procedures were not presented in any of those references.

This paper introduces tests for comparing the relative orientation of markers by using techniques of orientation statistics. Downs (1972), Khatri and Mardia (1977) and Prentice (1986) have contributed to this subject; see also Jupp and Mardia (1989). Commonly, in human kinematics studies, the sample size is small and rotations are clustered together. Thus the so-called tangent space techniques, introduced by Downs (1972), are suited to human kinematics. Substantial computational savings are obtained by working with the quaternion representations of rotations, as proposed by Prentice (1986).

Section 2 of the paper builds on Downs (1972) to construct one-way analysis-of-variance tests for clustered samples of rotations. It uses a residual vector, characterizing the discrepancy between a rotation in a sample and the sample mean rotation, to construct statistical tests. Standard multivariate normal procedures are shown to apply to the tangent space model for rotations presented in Section 2. Section 3 suggests a computationally efficient method, based on quaternions, to calculate the statistics of Section 2. Section 4 applies this methodology to the drilling data and suggests techniques to interpret differences between several samples of rotations. It shows that there does not appear to be a criterion for selecting drilling postures since most experimental subjects had statistically different postures for the drilling locations that were kinematically equivalent. The data that are analysed in this paper can be obtained from

`http://www.blackwellpublishers.co.uk/rss/`

2. One-way analysis of variance for rotations

2.1. Random rotations

A rotation is an orthogonal matrix whose determinant is equal to 1. The set of rotations for $\mathfrak{R}^3$-vectors is denoted SO(3). The matrix Fisher–von Mises distribution introduced by Downs (1972) (see also Khatri and Mardia (1977)) is a general exponential family model for describing the scatter of random orthogonal matrices around their modal value. Prentice (1986) showed how to reparameterize this model for data points that are rotations. In postural kinematics studies, the random rotations are closely clustered around their modal value. This means that the experimental errors are small and that tangent space techniques apply. Downs's (1972) model specifies that the errors, in the tangent space, have a trivariate normal distribution.

If M is an SO(3) rotation, the tangent space to SO(3) at M is the three-dimensional vector space of the 3×3 skew symmetric matrices. This means that

$$M\{I + \Phi(a)\}$$

describes the rotations around M when a varies in a neighbourhood of $\Re^3$'s origin, where the function $\Phi(\cdot)$ maps an $\Re^3$-vector a into the skew symmetric matrix

$$\Phi(a) = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}. \tag{2.1}$$

It is clear that $\Phi(\cdot)$ is an invertible mapping. If V is a 3×3 skew symmetric matrix then $a = \Phi^{-1}(V) = (V_{32},\ V_{13},\ V_{21})^{\mathrm{T}}$. Unfortunately, $M\{I + \Phi(a)\}$ is not a rotation. The exponential map provides a more satisfactory description of neighbourhoods of M. One can show that

$$\begin{aligned} \exp\{\Phi(a)\} &= I + \Phi(a) + \frac{1}{2!}\Phi(a)^2 + \frac{1}{3!}\Phi(a)^3 + \ldots \\ &= \cos(\theta)I + \frac{\sin(\theta)}{\theta}\,\Phi(a) + \frac{1-\cos(\theta)}{\theta^2}\,aa^{\mathrm{T}} \end{aligned} \tag{2.2}$$

is a rotation of angle $\theta = (a^{\mathrm{T}}a)^{1/2}$ around axis $a/(a^{\mathrm{T}}a)^{1/2}$. This means that the formula

$$M\ \exp\{\Phi(a)\}$$

describes, when a is close to $\Re^3$'s origin, rotations that are close to M. Chapter 2 of McCarthy (1990) provides an elementary presentation of the tangent space and exponential map for SO(3) rotations.

Let M denote a fixed 3×3 rotation representing the true orientation of a marker. The rotation X recorded for this marker is not usually equal to M. A simple model for describing the scatter of a sample of experimental rotations around M is given next.

2.2. Estimation of the modal rotation

Downs's (1972) statistical model for a sample of n rotations clustered around their modal value can be expressed as

$$X_i = M\ \exp\{\Phi(\epsilon_i)\}, \qquad i = 1, \ldots, n, \tag{2.3}$$

where the ϵ_i are independent copies of a trivariate normal random vector ϵ whose components are small or $O_p(\kappa^{-1/2})$. Here κ indexes the magnitude of the error: large κs give small errors. The tangent space inference relevant to kinematics studies is obtained by letting κ go to ∞. This section reviews the sampling properties of Downs's (1972) estimator for M and suggests a formula for estimating residuals in equation (2.3).

Implicit in Downs's (1972) model is the idea that the maximum likelihood estimator for M under a Fisher–von Mises matrix distribution is the rotation matrix M that is closest to $\bar{X} = \Sigma\, X_i/n$ in the least squares sense, i.e. such that $\mathrm{tr}(\bar{X} - M)^{\mathrm{T}}(\bar{X} - M)$ is a minimum. Thus, the maximum likelihood estimator of M maximizes $\mathrm{tr}(M^{\mathrm{T}}\bar{X})$. This maximization problem occurs in spherical regression (Stephens, 1979; Chang, 1993) and in Procrustes analysis (Sibson, 1978). Its solution is based on the singular value decomposition of $\bar{X}$,

$$\bar{X} = \hat{P}\,\mathrm{diag}(\hat{\gamma}_1,\ \hat{\gamma}_2,\ \hat{\gamma}_3)\hat{Q}^{\mathrm{T}} \tag{2.4}$$

where $\hat{P}$ and $\hat{Q}$ are rotations and $\hat{\gamma}_1 > \hat{\gamma}_2 > |\hat{\gamma}_3|$ are the singular values of $\bar{X}$. We have that $\hat{M} = \hat{P}\hat{Q}^{\mathrm{T}}$; $\hat{M}$ is called the sample mean rotation, or the mean rotation, throughout this paper. Inference procedures for M are constructed using residual vectors as building-blocks; see Jupp (1988) for a discussion of residuals for directional data. The residual for rotation X_i in model (2.3) can be defined by

$$r_i = \Phi^{-1}\left(\frac{\hat{M}^{\mathrm{T}}X_i - X_i^{\mathrm{T}}\hat{M}}{2}\right) \tag{2.5}$$

for $i = 1, \ldots, n$. Note that the three elements of r_i are less than 1 in absolute value. This vector characterizes the discrepancy between X_i and $\hat{M}$; $r_i = 0$ when $X_i = \hat{M}$. We also have that

$$\bar{r} = \Phi^{-1}\{(\hat{M}^{\mathrm{T}}\bar{X} - \bar{X}^{\mathrm{T}}\hat{M})/2\} = 0,$$

since, from the construction of $\hat{M}$ using equation (2.4), $\hat{M}^{\mathrm{T}}\bar{X}$ is a symmetric matrix.

The following proposition, whose proof is given in Appendix A, describes the first-order propagation of the sampling errors ϵ_i to the $\hat{M}$ and r_i.

Proposition 1. When in model (2.3) the ϵ_i have components which are $O_p(\kappa^{-1/2})$, we have

(a) $\hat{M} = M \exp\{\Phi(\bar{\epsilon})\} + O_p(\kappa^{-1})$ and
(b) $r_i = \epsilon_i - \bar{\epsilon} + O_p(\kappa^{-1})$.

From the proof of proposition 1 we have the representation

$$\mathrm{tr}(\hat{M}^{\mathrm{T}}\bar{X}) = 3 - \sum r_i^{\mathrm{T}} r_i/n + o_p(\kappa^{-1}).$$

Proposition 1 shows that, in concentrated samples, estimating the mean rotation is analogous to estimating the mean in a trivariate sample. When the experimental errors are normally distributed, a $100(1-\alpha)\%$ confidence region for the unknown modal rotation M is obtained from Hotelling's distribution. It is given by

$$\left\{\hat{M} \exp\{\Phi(a)\}: \frac{n-3}{(n-1)3} a^{\mathrm{T}} S^{-1} a < F_{3,n-3,\alpha}\right\}, \tag{2.6}$$

where $S = \Sigma\, r_i r_i^{\mathrm{T}}/(n-1)$ is the residual variance–covariance matrix. Confidence regions for the rotation axis and for the rotation angle can be derived from Rivest (1989).

2.3. *Statistical methods for comparing the modal rotations of several samples*

This section reviews standard methods of multivariate statistics that, in view of proposition 1, apply to the comparison of samples of rotations. Let $\{X_{ij}: j = i, \ldots, n_i, i = 1, \ldots, \mathcal{I}\}$ represent $\mathcal{I}$ samples of rotations of sizes $\{n_1, n_2, \ldots, n_{\mathcal{I}}\}$. The statistical model for this data set is

$$X_{ij} = M_i \exp\{\Phi(\epsilon_{ij})\}, \qquad j = i, \ldots, n_i, \quad i = 1, \ldots, \mathcal{I}, \tag{2.7}$$

where M_i is the modal rotation for the ith sample and the ϵ_{ij} are independent random vectors distributed according to an $N_3(0, \Sigma)$ distribution, and $\mathrm{tr}(\Sigma)$ is small or $O(\kappa^{-1})$. This section suggests test statistics for the hypothesis H_0: $M_1 = M_2 = \ldots = M_{\mathcal{I}}$ *versus* the alternative H_a: $M_i \neq M_j$ for at least one pair (i, j).

For each sample, we calculate the mean rotation $\hat{M}_i$ and the residuals r_{ij} defined in Section 2.2. In view of part (b) of proposition 1, an estimator for the common variance–covariance matrix Σ is given by

$$S = \sum r_{ij} r_{ij}^{\mathrm{T}} / \left(\sum n_i - \mathcal{I} \right).$$

Furthermore, the multivariate normality of the errors ϵ_{ij} can be assessed with Mardia's multivariate skewness and kurtosis coefficients (Rencher, 1995) calculated on the residuals $\hat{r}_{ij}$.

Testing H_0, the hypothesis of equality of the mean rotations, can be regarded as a repeated measures problem where each rotation contributes its three-dimensional vector of errors ϵ. The test statistic for H_0 depends on the variance–covariance matrix of ϵ_{ij}. If it is spherical, i.e. $\Sigma = cI$ for some positive c, then a simple test statistic for H_0 is given by

$$F_{\mathrm{obs}} = \frac{\left\{ \sum_i n_i \operatorname{tr}(\hat{M}_i^{\mathrm{T}} \bar{X}_{i.}) - \sum_i n_i \operatorname{tr}(\hat{M}^{\mathrm{T}} \bar{X}_{..}) \right\} / 3(\mathcal{I} - 1)}{\left\{ 3 \sum_i n_i - \sum_i n_i \operatorname{tr}(\hat{M}_i^{\mathrm{T}} \bar{X}_{i.}) \right\} \Big/ \left(3 \sum_i n_i - 3\mathcal{I} \right)}, \tag{2.8}$$

where $\hat{M}$ denotes the mean rotation calculated on the pooled sample of size $\Sigma\, n_i$. Its null distribution is F with $3(\mathcal{I} - 1)$ and $3\, \Sigma_i\, n_i - 3\mathcal{I}$ degrees of freedom. This is a generalization of the Watson and Williams (1956) statistic for the homogeneity of the mean directions.

It is well known that expression (2.8) is liberal when the sphericity assumption is violated (Crowder and Hand (1990), chapter 3). Mauchly's sphericity test rejects this hypothesis when

$$\chi^2_{\mathrm{obs}} = -\left(\sum_i n_i - \mathcal{I} - \frac{23}{18} \right) \log \left[\frac{\det(S)}{\{\operatorname{tr}(S)/3\}^3} \right]$$

is large, where $\det(A)$ denotes the determinant of A and S is the pooled variance–covariance matrix. Its null distribution is, in large samples, the χ^2-distribution on 5 degrees of freedom. Critical values for small samples are available (Kres, 1983). Using the sphericity test in directional statistics has previously been advocated by Rivest (1986).

When the sphericity hypothesis is rejected, hypothesis H_0 can be tested with Wilks's Λ-statistic. It is written in terms of $\mathbf{E}$, the dispersion matrix for the errors,

$$\mathbf{E} = \sum r_{ij} r_{ij}^{\mathrm{T}},$$

and $\mathbf{E} + \mathbf{H}$, the dispersion matrix of the residuals $\{r_{ij}^0\}$ around a common mean rotation $\hat{M}$ for all samples combined. We have

$$\mathbf{E} + \mathbf{H} = \sum r_{ij}^0 r_{ij}^{0\mathrm{T}}.$$

Wilks's test statistic rejects the null hypothesis when

$$\Lambda = \det(\mathbf{E}) / \det(\mathbf{E} + \mathbf{H}) \tag{2.9}$$

is small. We use Rao's F-approximation to the distribution of expression (2.9) to calculate p-values (see Rencher (1995), page 182). If $t = \{(9\mathcal{I}^2 - 18\mathcal{I} + 5)/(\mathcal{I}^2 - 2\mathcal{I} + 5)\}^{1/2}$, $\mathrm{df}_1 = 3(\mathcal{I} - 1)$ and $\mathrm{df}_2 = t\{\Sigma\, n_i - (\mathcal{I} + 5)/2\} - (3\mathcal{I} - 5)/2$, then

$$\frac{(1 - \Lambda^{1/t})\, \mathrm{df}_2}{\Lambda^{1/t}\, \mathrm{df}_1}$$

is approximately distributed, under the hypothesis of homogeneity, as an F-distribution with df_1 and df_2 degrees of freedom.

Test statistics should be invariant to a change from $\{X_{ij}\}$ to $\{X_{ij}^{\mathrm{T}}\}$. This holds true for expression (2.8). This is not exactly true for Mauchly's and Wilks's tests. For these two tests there are, under the null hypotheses, small, or $o_p(\kappa^{-1})$, differences between the statistics calculated on $\{X_{ij}\}$ and those obtained from $\{X_{ij}^{\mathrm{T}}\}$.

2.4. Simulation results

To investigate the quality of the tangent space approximations to the distribution of Wilks's Λ-statistics (2.9), data were simulated from model (2.7) with $\mathcal{I} = 3$, for various sample sizes. Empirical levels corresponding to a nominal level of 5% were calculated from 2000 Monte Carlo replications. The standard error associated with the estimated percentages is 0.5%. In the simulations, $\Sigma = cI$ for various values of c. The results appear in Table 1. For $c \leqslant 0.01$ the experimental p-values are equal to the nominal 5%, up to experimental errors. For $0.01 < c \leqslant 0.5$ the experimental p-values are smaller than or equal to 5%. The proposed version of Wilks's test is conservative. This suggests that the succession of approximations made in this section are reliable provided that the largest eigenvalue of the variance–covariance matrix Σ is less than 0.5.

3. A quaternion implementation of the analysis

The quaternion representation (Hamilton, 1969) of a rotation of angle θ around axis u, where u is a unit vector in $\Re^3$, is a unit vector q in $\Re^4$ defined by $q = (\cos(\theta/2),\ \sin(\theta/2)u^{\mathrm{T}})^{\mathrm{T}}$. Unit vectors q and $-q$ represent the same rotation. This is a 2:1 representation. Rooney (1977) provides a good introduction to this representation. Working with quaternions allows a substantial reduction in the computational effort that is required to calculate residuals r_i, defined in equation (2.5), and the test statistics of Section 2. Prentice (1986) introduced quaternions as a useful statistical tool for comparing samples of rotations.

Quaternions are endowed with a special product corresponding to rotation multiplication. If $p = (p_1, p_2, p_3, p_4)^{\mathrm{T}}$ and $q = (q_1, q_2, q_3, q_4)^{\mathrm{T}}$ are quaternion representations of rotations X_1 and X_2 respectively, then the dot product between p and q is given by

$$p{\cdot}q = \begin{pmatrix} p_1 & -p_2 & -p_3 & -p_4 \\ p_2 & p_1 & -p_4 & p_3 \\ p_3 & p_4 & p_1 & -p_2 \\ p_4 & -p_3 & p_2 & p_1 \end{pmatrix} q = Pq. \tag{3.1}$$

In other words, $p{\cdot}q$ is the matrix product of P by q. Since P is a rotation, Pq is a unit vector. It can be shown that this unit vector is the quaternion representation of the matrix product of

Table 1. Empirical levels for Wilks's test with a nominal level of 5% when the critical value is obtained with Rao's F-approximation

Sample sizes	*Empirical levels (%) for the following individual variances c:*								
	10^{-4}	10^{-2}	0.1	0.5	1.0	1.5	2	3	4
5, 5, 5	4.3	5	3.4	1.9	3.5	3.8	2.5	3.6	2.5
5, 5, 10	5	4	3.4	1.5	4.5	4.2	3.7	4	3.7
5, 10, 15	4.5	5.5	3	1.9	5	6.2	4	5.3	5.4
20, 20, 20	4.8	4.7	2.9	2	7.2	11.5	6.7	6.8	5.7
30, 30, 30	5.3	4.5	3.1	2.5	6.4	13	10.2	8.4	6.7

X_1 multiplied by X_2. Also, $(p_1, -p_2, -p_3, -p_4)^{\mathrm{T}}$ is the quaternion for the inverse of X_1, X_1^{T}, and $P^{\mathrm{T}}q$ is the quaternion representation of $X_1^{\mathrm{T}}X_2$. Prentice (1986) gives further properties of quaternions.

Let $\{q_i\colon i = 1, \ldots, n\}$ be a quaternion representation of the sample $\{X_i\}$. If p is the quaternion representation of rotation M, then

$$\mathrm{tr}(M^{\mathrm{T}}\bar{X}) = 4p^{\mathrm{T}}\left(\frac{1}{n}\sum q_i q_i^{\mathrm{T}}\right)p - 1.$$

This identity can be deduced from formula (2.2) of Prentice (1986). For completeness a short proof is given in Appendix A. Thus the quaternion representation of the mean rotation $\hat{M}$ is the eigenvector corresponding to the largest eigenvalue of $\Sigma\, q_i q_i^{\mathrm{T}}$.

This suggests the following strategy for analysing a sample of rotations. Let Q denote the $n \times 4$ matrix of the q_i^{T}. Take $\hat{p}$, the quaternion for the sample mean rotation, equal to the eigenvector corresponding to the largest eigenvalue of $Q^{\mathrm{T}}Q$. Then rows of $R = Q\hat{P}$, where $\hat{P}$ is constructed from $\hat{p}$ as in equation (3.1), are the quaternions of the residual rotations $\hat{M}^{\mathrm{T}}X_i$. As shown in Appendix A, the residuals r_i defined in equation (2.5) can be derived easily from R,

$$r_i = 2R_{i1}(R_{i2}, R_{i3}, R_{i4})^{\mathrm{T}}.$$

Quaternions allow substantial computational savings. All the calculations in Section 4 are based on quaternions.

Prentice (1986) derived large sample tests and inference procedures for $\hat{M}$ and the eigenvalues of $Q^{\mathrm{T}}Q$. In terms of the notation of this section, the $100(1-\alpha)\%$ large sample confidence region for the sample mean rotation given by his formula (5.1) contains rotations $\hat{M}\exp\{\Phi(a)\}$ such that

$$na^{\mathrm{T}}\hat{A}S^{-1}\hat{A}a < \chi^2_{3,\alpha},$$

where $\hat{A} = ([R^{\mathrm{T}}R]_{11}I - [R^{\mathrm{T}}R]_{22})/n$, and $[R^{\mathrm{T}}R]_{11}$ is the (1, 1) element of $R^{\mathrm{T}}R$ and $[R^{\mathrm{T}}R]_{22}$ is the 3×3 matrix obtained by deleting the first row and the first column of $R^{\mathrm{T}}R$. In clustered samples, as $\kappa \to \infty$, $\hat{A} = I + o_p(\kappa^{-1})$. Comparing this with the tangent space confidence region derived in Section 2 suggests a $100(1-\alpha)\%$ confidence region that is valid when the sample is either large or clustered. This region contains rotations $\hat{M}\exp\{\Phi(a)\}$ such that

$$\frac{n-3}{3(n-1)}a^{\mathrm{T}}\hat{A}S^{-1}\hat{A}a < F_{3,n-3,\alpha}.$$

4. Analysis of upper limb posture

This section considers the analysis of upper limb posture variations in human subjects performing a drilling task. This study was performed in the Newman Laboratory for Biomechanics and Rehabilitation at the Massachusetts Institute of Technology. A detailed description of the study can be found in Rancourt (1995). Limb postures were recorded by Selspot infra-red cameras along with the TRACK software created in the Newman Laboratory (Antonsson and Mann, 1989). Briefly, one objective of the study was to determine whether the upper limb posture chosen by human subjects in a drilling task was constant for apparently similar experimental situations. The interest in the drilling task was to investigate how drill operators deal with the redundancy issue of limb postural control. A characterization and statistical analysis of limb postures was therefore highly critical for the study. Statistical methods presented in the previous sections are well suited to this application.

4.1. Experimental design and data collection

Eight different subjects participated in the study. Each subject was instructed to drill at six different predetermined locations, on a vertically oriented metal plate, shown in Fig. 2. The subjects were told to use only one hand to operate the drill, at any of the six locations for a period of 2–3 s. During that time, the drilling upper limb posture was measured by a Selspot infra-red camera system. The limb posture was defined via four infra-red emitting diode markers fixed at different locations (see Fig. 1): the first was rigidly connected to the subject's back at the level of the seventh cervical vertebra, the second to the drilling arm, the third to the drilling forearm and the fourth to the drill itself. While the subject was drilling, the orientation and location of each of the four markers was recorded with respect to the laboratory reference frame. Each marker orientation was characterized by a 3×3 rotation matrix.

The upper limb posture was finally defined by computing the relative orientation of adjacent markers, thereby computing the relative orientation of two adjacent body segments. For instance, if X_{b} and X_{a} are rotations representing the orientations of the back and the arm respectively, then $X_{\mathrm{b}}^{\mathrm{T}} X_{\mathrm{a}}$ represents the relative orientation of the arm with respect to the back. Because the markers were not rigidly fixed to the bones, the relative orientation is not exactly equal to the rotations that occurred at the upper limb joints. The data set includes motion of the skin that perturbs sample rotations. Nevertheless, the data can still be used to investigate repeatability of limb posture if it is assumed that skin motion characteristics were stable all through the experiment. Special care was taken to ensure stability of the markers on the limb segments and thus the assumption is tenable. Furthermore, data on two different subjects are not directly comparable since markers were not placed at exactly the same orientation on every subject.

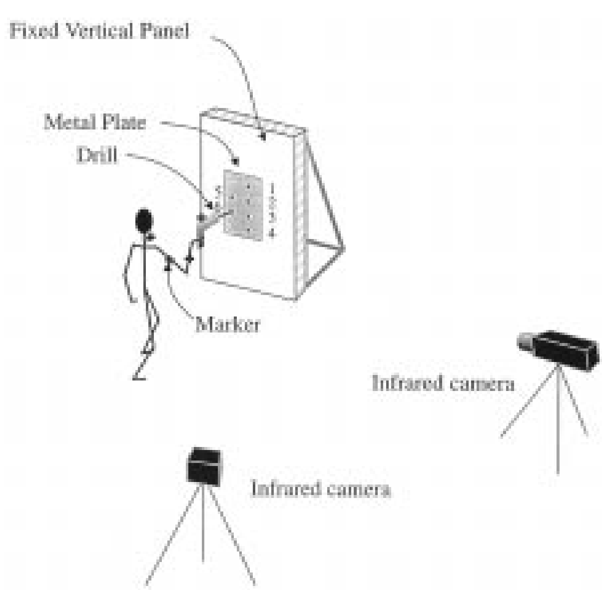

Fig. 2. The six drilling positions

Upper limb posture can be characterized by three relative rotations at the three upper limb joints: the shoulder, the elbow and the wrist. This defines the three-level factor JOINT. Each subject performed 30 trials, five at each of the six locations, in a completely randomized design. Drilling locations 1–4 were all positioned on the same vertical line. Drilling location 2 was set at the subject's shoulder height. Location 1 was set over location 2 at a distance equal to 20% of the length of the subject's upper limb. Location 3 was set below location 2 at a distance equal to 50% of the subject's arm length, whereas location 4 was set directly below location 3 at the subject's elbow height. Locations 5 and 6 were set at the same height as locations 2 and 3 respectively, but 20 cm to the left of the vertical line. Each subject contributed 90 $= 30 \times 3$ rotations to the data set. In the analysis, we considered that data set as $144 = 8$ (SUBJECT) $\times 3$ (JOINT) $\times 6$ (LOCATION) samples of size 5. There were 129 non-empty samples since, in some instances, one of the markers was out of the camera's field of view. This happened when a subject took a position that prevented some of the markers from being registered properly by the camera system. In fact, a subject could significantly rotate his whole body about the drill tip, avoiding slippage of the drill and without modifying the upper limb posture. Since limb relative orientations are investigated in this paper, we assume that the relative orientations for the empty and the non-empty samples are similar. We analysed the 24 combinations of JOINT $\times$ SUBJECT separately.

4.2. *Comparisons of postures at the six drilling locations*

The upper limb orientation analysis was performed by computing mean rotations for all non-empty experimental samples. For each sample, the trace of the variance–covariance matrix, $\Sigma\, r_i^{\mathrm{T}} r_i/4$, was calculated. For the 129 samples under investigation, the trace ranged between 0.001 and 0.12 with a mean value of 0.013. Thus replicated observations of the same subject in the same position did not exactly produce the same orientation. A trace of 0.013 means that the expected squared angle, in radians, of the rotation needed to go from the sample mean rotation to a sample rotation is 0.013. Taking the square root and converting to degrees yields 6.5° as the mean angle; this angle characterizes the variability within the average sample. This is a situation where large concentration asymptotics apply. To investigate whether in equation (2.3) the errors ϵ were normally distributed, we calculated the multivariate skewness and kurtosis coefficients on the residuals of the 129 samples. This led to two empirical distribution functions, calculated with 129 data points each, for skewness and kurtosis respectively. We compared these distributions with quantiles of the skewness and kurtosis distributions for trivariate normal samples of size 5. The corresponding *QQ*-plots are given in Fig. 3. They show that the normality assumption is tenable.

For each null hypothesis of homogeneity to be tested, the sphericity test was calculated on the pooled variance–covariance matrix. More than half of these tests were significant at the 5% level. This led us to reject the hypothesis of sphericity and to use Wilks's statistic to test the homogeneity of the rotations. For each JOINT $\times$ SUBJECT combination positions 1–4 were compared with a four-sample Wilks statistic. For each test the null hypothesis was rejected at the 10^{-3}-level. We carried out pairwise tests of homogeneity for comparing five pairs of positions: 1–2, 2–3, 3–4, 2–5 and 3–6. We would expect to observe smaller differences for pairs 2–5 and 3–6 since these tasks are kinematically equivalent. All other tests should reveal differences in upper limb posture. For position pairs 1–2, 2–3 and 3–4, 80% of the 65 tests were significant at the 5% level; this percentage dropped to 62.5% for pairs 2–5 and 3–6. More than 50% of the non-significant tests came from two subjects. For the others, most of the tests were significant.

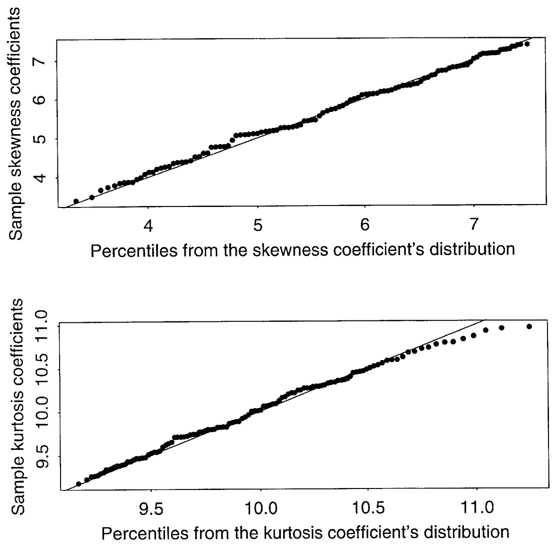

Fig. 3. *QQ*-plots for normality of the 129 samples (each represented by a point)

4.3. Understanding changes in drilling postures

To obtain a better appreciation of variations in limb posture, we can look at the orientation differences between pairs of positions. Thus, for each JOINT × SUBJECT combination, the angles of the rotation matrices for the changes in orientation for the five pairs of positions were calculated. These angles are invariant to changes in the position of the markers; thus, they characterize the movement that took place at the joint, including motion of the skin. To prove this, consider $\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_6$, the six mean rotations for say the shoulder joint of individual 1. Rotating the back marker and the arm marker by rotations of R_b and R_a respectively would yield $R_b^T \hat{M}_1 R_a$, $R_b^T \hat{M}_2 R_a, \ldots, R_b^T \hat{M}_6 R_a$ as mean rotations. The rotation for the changes in orientation from position 1 to position 2 is $R_a^T \hat{M}_1^T \hat{M}_2 R_a$. Now if $\hat{M}_1^T \hat{M}_2$ is a rotation of angle $\hat{\theta}_{12}$ around axis $\hat{u}_{12}$, then $R_a^T \hat{M}_1^T \hat{M}_2 R_a$ is a rotation of angle $\hat{\theta}_{12}$ around axis $R_a^T \hat{u}_{12}$. In other words, the angle of the rotation for the change in orientation is invariant to marker displacement, as stated above. An analysis of variance of the 106 angles (14 were missing) revealed no interaction between SUBJECT and any other factors. Least squares means for the JOINT by change of POSITION factors classification are presented in Table 2. The least significant difference at a level of 5% is about 6°.

Table 2 agrees with the hypothesis that changes in orientation at the three joints are generally smaller for the two pairs of kinematically equivalent positions, (2, 5) and (3, 6). The significance of such differences needs to be established in relation to the particular characteristics of the

Table 2. Mean angles for changes in orientation of five pairs of positions at three joints

Joint	*Mean angles (deg) for the following pairs of positions:*				
	1–2	*2–3*	*3–4*	*2–5*	*3–6*
Wrist	17.37	18.24	18.65	8.17	8.83
Elbow	12.46	7.36	12.62	9.76	7.27
Shoulder	26.88	20.02	13.75	9.64	8.23

upper limb that is investigated. For instance, variations of the order of 8–10° at each of the upper limb joints would probably not produce much difference in the mechanics of the upper limb, for the upper limb configurations that were selected.

The analysis of the upper limb posture data presented in this section raises several statistical problems that cannot be addressed here. Table 2 summarizes the angles for the changes in orientation. It would also be interesting to investigate their axes. Does the axis for the change in orientation stay constant when going from position 1 down to position 4? New techniques need to be devised to assess the variability in the estimated axes. Another issue for investigation is the relationship between the orientations at the three joints. The posture of a subject is characterized by three rotations, one at each joint. Investigating the dependences between the three joints and finding meaningful biomechanical interpretations for possible relationships is another challenging research problem that is highly relevant to the understanding of human movement. The model proposed by Prentice (1989) is relevant to this research.

5. Discussion

The statistical approach presented in this paper can provide an objective evaluation of the consistency in limb posture for the drilling task. The statistical analysis of upper limb posture suggests that the posture, in a drilling task, would not be selected on a criterion leading to a unique solution. Indeed, the variability of joint rotations within each sample of five rotations was, on average, of the order of 6.5°. In addition, comparing means of limb posture in pairs 2–5 and 3–6, two situations with kinematics and mechanics that are apparently similar, it was found that 62.5% of the tests were significant. Variables other than kinematics and mechanics might not have been properly controlled. For instance, it was found that variations in drill orientation were as much as 15° (Rancourt (1995), page 146). Clearly, this could explain the high variability in limb posture for the drilling task. Variations in posture between locations 2–5 and 3–6 could be explained by a systematic variation in drill orientation. In summary, evidence of a selection criterion for posture in a drilling task may be difficult to demonstrate because of the variability in drill operation. The results obtained in this study show, however, that variations are rather small and may not significantly influence several criteria for posture, e.g. limb isotropy (Angeles, 1992), maximizing manipulability (Yoshikawa, 1990) and shaping hand mechanical impedance (Colgate and Hogan, 1988). Experiments to test for a selection criterion for posture should thus be performed with a task precisely defined in all 6 degrees of freedom of the hand.

In addition, limited training in drill operation was given to the human subjects before the experimental data were collected. Some of the subjects had significant past experience in drilling whereas others did not. This intersubject variation was also observed in the data where consistency in limb posture for a given drilling location varied across subjects. Since an optimization criterion may only develop with usage, it would be interesting to conduct the experiment again with skilled drill operators only.

Information on relative orientation at each upper limb joint was used in this paper as means of characterizing upper limb posture. In fact, a complete description of joint motion requires information on both rotation and translation. For the wrist and the elbow, we may assume that rotation and translation are related, and hence characterizing posture by orientation statistics could be sufficient. This is probably not true in the case of the shoulder if we include the motion of the scapula in the analysis. The addition of translation changes to the analysis that has been presented is worthy of further investigations.

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Appendix A

A.1. Proof of proposition 1

Elementary calculations show that $\mathrm{tr}\{\Phi(b)\,\Phi(a)\} = -2a^{\mathrm{T}}b$ and $\Phi(b)^2 = -b^{\mathrm{T}}bI + bb^{\mathrm{T}}$, where a and b are 3×1 vectors. Thus,

$$\bar{X} = M\{I + \Phi(\bar{\epsilon}) - \sum (\epsilon_i^{\mathrm{T}}\epsilon_i I - \epsilon_i\epsilon_i^{\mathrm{T}})/2n\} + o_p(\kappa^{-1}).$$

We have that $\hat{M} = M\,\exp\{\Phi(m)\}$, where m is a 3×1 $O_p(\kappa^{-1/2})$ vector such that $\mathrm{tr}(\hat{M}^{\mathrm{T}}\bar{X})$ is a maximum. Now

$$\mathrm{tr}(\hat{M}^{\mathrm{T}}\bar{X}) = 3 - \mathrm{tr}\{\Phi(m)\Phi(\bar{\epsilon})\} - \sum \epsilon_i^{\mathrm{T}}\epsilon_i/n - m^{\mathrm{T}}m + o_p(\kappa^{-1}).$$

This is equivalent to

$$\mathrm{tr}(\hat{M}^{\mathrm{T}}\bar{X}) = 3 - \sum (\epsilon_i - m)^{\mathrm{T}}(\epsilon_i - m)/n + o_p(\kappa^{-1}).$$

Thus $m = \bar{\epsilon}$ minimizes the $O_p(\kappa^{-1})$ component of the second term of this expression. Therefore $\hat{M} = M\,\exp\{\Phi(\bar{\epsilon})\} + O_p(\kappa^{-1})$. This proves the expansion for $\hat{M}$. The expansion for the residual comes from the fact that $\hat{M}^{\mathrm{T}}X_i = I + \Phi(\epsilon_i - \bar{\epsilon}) + O_p(\kappa^{-1})$.

A.2. Quaternion implementation of the analysis

Let X_1 and X_2 be two rotation matrices with respective quaternion representations q_1 and q_2. To prove that $\mathrm{tr}(X_1^{\mathrm{T}}X_2) = 4(q_1^{\mathrm{T}}q_2)^2 - 1$, let θ be the angle of the rotation $X_1^{\mathrm{T}}X_2$. From equation (2.2), $\mathrm{tr}(X_1^{\mathrm{T}}X_2) = 2\,\cos(\theta) + 1 = 4\,\cos(\theta/2)^2 - 1$, whereas $q_1^{\mathrm{T}}q_2$ is the first element of the quaternion representation of $(X_1^{\mathrm{T}}X_2)$; thus $q_1^{\mathrm{T}}q_2 = \cos(\theta/2)$.

From equations (2.5) and (2.2), the residual r_i can be expressed as $\sin(\theta)u$, where u and θ are respectively the axis and the angle of the rotation $\hat{M}^{\mathrm{T}}X_i$. The quaternion for this rotation is $R_i = (\cos(\theta/2), \sin(\theta/2)u^{\mathrm{T}})^{\mathrm{T}}$. The formula $r_i = 2R_{i1}(R_{i2}, R_{i3}, R_{i4})^{\mathrm{T}}$ comes from applying the standard result that $\sin(\theta) = 2\,\sin(\theta/2)\,\cos(\theta/2)$.

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