ORGANIZATION OF ARM MOVEMENTS IN
THREE-DIMENSIONAL SPACE. WRIST
MOTION IS PIECEWISE PLANAR

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Abstract—It is shown that human subjects are incapable of producing with the arm, in free space, planned
or extemporaneously drawn trajectories in which the plane of wrist motion changes smoothly or
continuously. The three-dimensional nature of these movements results from the fact that the plane of
motion changes abruptly from one segment of the trajectory to the next, being confined to one plane
during each segment (i.e. piecewise planar).

In the preceding paper15 we described the kinematic
characteristics of arm motions when human subjects
were asked to draw simple and highly learned geo-
metric figures (such as 'figure 8s' and 'stars') in
three-dimensional space. We concluded that such
movements are composed of unit segments arranged
sequentially. Furthermore, we found that each of
these segments was generated according to a set of
rules which we had proposed previously12,14 to consti-
tute an algorithm by which circles and ellipses can be
drawn in an arbitrary plane of free space.

We also found that when subjects were asked to
draw a 'figure 8', the plane of motion of the hand
could change considerably and abruptly at the begin-
ing of a segment. Within a segment, instead, the
motion was close to planar. Planar wrist motion
follows directly from one ass~ption of our hypoth-
thesis, discussed in the preceding paper,15 namely that
the motion of the orientation angles13 of the arm is
sinusoidal.12,14 Given such sinusoidal angular motion,
one can demonstrate that the motion of the wrist in
three orthogonal directions will also be close to
sinusoidal. Furthermore, such a motion necessarily
describes an arc of an ellipse which lies in one plane,14
the curvature of the motion at the wrist being in-
versely proportional to the cube of the tangential
velocity, in agreement with other data.10,16

Thus, according to the algorithm, wrist motion is
confined to be piecewise planar. The question then is:
are human subjects also capable of producing, in free
space, trajectories for which the plane of wrist motion
changes smoothly and continuously? If not, then the
general validity of our hypothesis would be extended
to encompass trajectories which are intended to be
three-dimensional.

EXPERIMENTAL PROCEDURES

Motor tasks

Subjects who stood erect were asked to draw three-
dimensional figures with their right arm. Most of the
movements we investigated were visually guided in the sense
that the subjects were presented with a model of the
movement they were asked to reproduce. In some instances
the model consisted of a line traced on the surface of a
sphere; in others it was an aluminum tubing molded into a
three-dimensional curve. The subjects were given a few
minutes to practise. They were also asked to reproduce it
from memory, i.e. in the absence of the reference model.

Two other types of movements were also investigated. We
asked subjects to draw helices with the screw axis oriented
in different directions or to draw random scribbles in
three-dimensional space.

Recording system and data analysis

The system used to record arm motion and the analytical
procedures used to describe the kinematic details of these
movements are the same as those used in the previous
paper,15 with one exception. In the preceding paper we
assumed that a segment of the motion began at the max-
imum or minimum of the tangential velocity of the wrist.
For many of the movements to be described in this paper,
the extrema of the velocity and curvature were not distinct
(see Fig. 2A) and this criterion could not be utilized.
Furthermore, there was no compelling reason to assume a
priori that segments should be demarcated by the same
criteria as used in the preceding paper.

Therefore we used another procedure to determine the
number of segments and their duration for a given move-
ment. We began by assuming a given number of segments
(for example, seven segments each of 1-s duration), fitted
sinusoids to the velocities of the orientation angles over each
segment and calculated the total distortion in the angular
velocities, distortion being defined as the sum of the mean
square differences between the experimentally obtained an-
gular velocities and the fundamental component. We then
systematically changed the boundaries between segments to
minimize the distortion. Once a minimum had been ob-
tained, we increased the number of segments by one and
repeated the procedure. The process was halted if the
distortion failed to decline significantly or if the procedure
yielded a fit which was judged to be acceptable.

RESULTS

Figures 1 and 2 show typical results. In this case the
subject attempted to reproduce with his wrist the
An outline of a circle inscribed on the curved surface of a cylinder. The task was performed from memory, the subject having been previously shown a molded tube by the subject and symmetric about the sagittal and horizontal planes. Each segment of the motion is indicated by a different line symbol, in perspective and in its projection on the sagittal (X-Z) and horizontal (X-Y) planes. The segments are numbered consecutively and the numbers placed at the beginning of the segments. The dashed lines are meant to assist in visualizing the motion in three dimensions. They represent a ribbon of constant width placed perpendicularly to the plane of motion of each segment.

Figure 1 shows a perspective view (as seen by the subject) of one cycle of the wrist trajectory and its projection on the sagittal and horizontal planes. Each segment, determined according to the criterion described in Experimental Procedures, is denoted by a different line symbol. The segments have been numbered consecutively and the arrows indicate the direction of the movement of the wrist. The dotted lines adjacent to each segment are meant to assist in the visualization of the three-dimensional nature of the motion. They represent a ribbon of constant width placed perpendicularly to the plane of motion of each segment, drawn in perspective.

The plane of motion of each segment was calculated by linear regression (eq. 6 of Ref. 15) and is denoted by the dashed lines in Fig. 2B. The solid traces in this figure show the variation of the unit normal to the instantaneous plane of motion. Here $n_x$ is the component of the perpendicular in the anterior–posterior (X) direction, while $n_z$ is that for the lateral direction and $n_y$ for the vertical direction. The elevation ($\psi$) and azimuth ($\chi$) of the plane of motion were computed from these components. Figure 2A shows the variation of the tangential velocity ($V_t$) and curvature ($k$) of the wrist motion for this trial, as well as the velocities of the orientation angles of the upper arm ($\eta, \theta$) and of the forearm ($\alpha, \beta$). The segments for the cycle of the motion shown in Fig. 1 are indicated by the vertical dashed lines.

An appreciation of the extent to which each segment of wrist motion is confined to one plane can be gained from Fig. 2C. In each numbered panel of Fig. 2C, the coordinate axes have been rotated (eq. 7 of Ref. 15) so that the motion of that segment lies in the plane $x' = constant$. The upper row shows a head-on view of one cycle of the motion ($y'z'$ plane) while the lower row shows an edgewise view ($x'z'$ plane). In each panel the segment in question is indicated by the heavy solid trace; preceding and succeeding segments are denoted by lighter traces and different line symbols.

Note that the movement was performed at a much slower speed (65 cm/s) and had a much longer period (2.8 s) than for circles or ellipses drawn in any plane (typical period <1 s). Nevertheless, the trajectory of the wrist was only a poor approximation of the intended movement. Furthermore, and as predicted by the hypothesis, the plane of wrist motion did not change smoothly or continuously as it should have if the subject had been able to reproduce the model curve. This is evident in Fig. 2B. The components of the unit normal to the plane of motion ($n_x, n_y, n_z$) remain relatively constant for prolonged periods of time (400-800 ms) and then change abruptly. The times at which these changes occur agree well with the locations of the boundaries between adjacent segments. (Recall that these boundaries were determined by a different criterion, namely that the distortion from sinusoidal motion of the orientation angles be minimized.) The projections of the trajectory onto the plane of motion (Fig. 2C) reinforce this conclusion. The deviation from planar motion of each of the segments is small, as can be appreciated in the lower row of Fig. 2C, with values (computed according to eq. 8 of Ref. 15) ranging from 0.021 to 0.076. The plane of motion instead could change markedly from segment to segment, the maximum change in the planar elevation $\psi$ and azimuth $\chi$ angles exceeding 90°.

Finally, the motion of the orientation angles was close to sinusoidal within each segment, the average distortion ranging from 15 to 26%. Only in the 4th segment was the change in the yaw angle of the forearm ($\alpha$) not well approximated by a sinusoid.

Figures 3 and 4 present another example from a different subject. In this instance the subject attempted to reproduce a curve traced on the surface of a sphere. A perspective view of the wrist trajectory is shown in Fig. 3 along with a representation of the orientation of the plane of motion of each segment, while the upper and lower rows in Fig. 4 depict a frontal and edgewise view of the wrist trajectory of one cycle of the movement projected onto the plane of motion of each segment. As was the case with the
Fig. 2. (A) Variation in tangential velocity ($V_T$) and curvature ($\kappa$) of the wrist motion and the velocities of the orientation angles of the upper arm (yaw, $\eta$; elevation, $\theta$) and of the forearm ($\alpha$, $\beta$). The segments of the cycle of the motion illustrated in Fig. 1 are indicated by the vertical dashed lines, and the dashed lines superimposed on the angular velocities represent the fit of sinusoids to their modulation in each segment. Scale per division: $V_T$, 25 cm/s; $\kappa$, 0.25 cm$^{-1}$; angular velocities, 200°/s. (B) Variation in the instantaneous normal to the plane of motion and the two angles which can be used to define it ($\psi$, planar elevation; $\chi$, azimuth). The numbers denote the segments illustrated in Fig. 1, and the dashed lines indicate the best fit to planar motion for each segment. Scales per division: direction normals ($n_x$, $n_y$, and $n_z$), 1 cm; $\psi$ and $\chi$, 90°. (C) Extent to which each of the five segments in Fig. 1 deviate from planar motion. The top row shows a head-on view and the bottom row an edgewise view of the projection of one cycle of the motion onto the plane of the segment indicated by the number above the column and denoted by the solid, dark trace.

previous example, this movement was performed very slowly (period of 3.25 s) and the deviation from planar motion was small ($\epsilon$ averaging 0.076), with the largest deviation in the 4th segment ($\epsilon = 0.129$).

The two examples illustrated in Figs 1–4 are typical of the performance of the two subjects for this task. A third subject, who was asked to reproduce a circle with a cylindrical form (as in Fig. 1) and the seams on a baseball or tennis ball (whose outline also forms a figure with a continuously changing tangent plane) gave similar results. For the three subjects the average deviation from planar motion ($\epsilon$) ranged from $0.068 \pm 0.044$ ($N = 32$) to $0.101 \pm 0.047$ ($N = 35$), values which are similar to those we reported in the preceding paper for tasks in which the subjects attempted to produce planar motions. The change in the plane of motion from one segment to the next could be sizeable, the difference in the azimuth $\gamma$ or planar elevation $\psi$ between adjacent segments exceeding $20^\circ$ in 50–85% of the instances.
Aside from the fact that novel movements such as those illustrated in Figs 1 and 3 are executed at a much slower speed than the highly practised ones we considered in the previous paper, there was one other major difference in the results. In the case of ‘figure 8s’ and stars, the angular elevations of the upper arm (θ) and of the forearm (β) were generally close to 180° out of phase, with a standard deviation of about 30°. For movements such as those illustrated in Figs 1 and 3, the phase relation between these two angles was much more variable and almost random (S.D.s ranging from 70° to 103°). Accordingly, the correlation of the slant of the segment with the phase difference between forearm elevation and yaw (r > 0.97 for ‘figure 8s’) was much less (ranging from 0.676 to 0.856). The correlation between the azimuth of the plane of motion (ψ) and the phase between the two yaw angles (ν and α) was also small, ranging from 0.123 to 0.779.

We also investigated a class of movements which we expected would be easier for the subjects to perform, namely drawing a helix in three-dimensional space. Ideally, such a movement would be composed of harmonic motion in two orthogonal directions, while the speed in the third direction (the screw axis) would be constant. The tangent plane of a helix changes continuously. As expected, subjects had little difficulty in drawing such figures at speeds comparable to those at which they drew ellipses. For these helices, we used the criterion that segments begin and end when the tangential velocity at the wrist was maximal, as in the preceding paper. During each segment, the wrist deviated little from planar motion, with average values of ε of 0.053 ± 0.018 and 0.064 ± 0.026 for two subjects. For these movements the change in the plane of motion was restricted primarily to its azimuth χ, the difference in this parameter averaging 14° and 21° for the two subjects. Thus, the performance during this task also is consistent with our hypothesis. Unfortunately, the range of motion permitted by the recording system was limited, and in general the pitch of the helices drawn was shallow. When we performed an analysis of a simulated ideal helix (i.e. one whose tangent plane changed continuously) with a similar pitch, we obtained deviations from planar motion ε which were comparable to those found experimentally. Thus, the results of our investigation of this task, while not inconsistent with our hypothesis, cannot be used to lend it further support.

Results obtained from one other motor task do, however, provide such additional support. We asked a subject to generate ‘scribbles’ in three-dimensional space. Two examples are illustrated in Figs 5 and 8. As can be appreciated in Fig. 6A, which shows results for the same trial as Fig. 5, these movements were much faster, with a maximum tangential velocity (VT) in excess of 90 cm/s. Both the tangential velocity of the wrist and the curvature were highly modulated, with clear minima and maxima. The vertical dashed lines denote the boundaries between segments, computed to minimize the distortion from sinusoidal motion of the orientation angles. In this instance, the boundaries are close in time to the peaks of the tangential velocity (and the minima of the curvature). This was a general finding for this task. When
maxima and minima of these parameters could be defined easily, the boundaries of segments tended to coincide with them (average difference $46 \pm 36$ ms). Furthermore, sinusoids gave a good approximation to the modulation in the orientation angles.

As was the case with the other motor tasks we investigated, the motion of the wrist within each segment deviated little from a plane (Fig. 6B), with an average value $\epsilon = 0.091 \pm 0.042$, while the orientation of the plane of motion could change substantially from one segment to the next (see Figs 7 and 9). Finally, for these spontaneously generated movements, the angular elevations of the forearm and arm were close to $180^\circ$ out of phase ($\beta$ leading $\theta$ by $24 \pm 5$ ms).
In all these respects, these movements yielded results similar to those obtained for highly learned, planned movements such as the drawing of ellipses and 'figure 8s'.

Figure 10 illustrates one final point. The assumption that the periodicity of the motion is the same for all of the orientation angles can well account for the experimental data (cf. Figs 2 and 6). However, the patterns and the periodicity of electromyographic activity of muscles responsible for generating the movement can be very different from muscle to muscle. In Fig. 10A, the subject attempted to draw a circle inscribed on a cylinder and in Fig. 10B, to reproduce the seams of a baseball. The vertical dashed lines denote the boundaries of segments for one cycle of each of these movements. In both instances the pattern of electromyographic activity in deltoid appears to be more regular and to have a longer period than does that of biceps. Thus, in Fig. 10A, deltoid activity peaks once per movement cycle while biceps shows two peaks. Furthermore, there is no evidence of any abrupt changes in torque coincident with the beginning of a segment.

**DISCUSSION**

From the data presented here one can conclude that for all arm movements investigated, irrespective of whether they were attempts to trace a threedimensional figure in free space or they were sponta-
neously generated, the following is true: complex and apparently continuous trajectories are composed of unit segments during which the motion of the wrist is confined to a plane. The three-dimensionality of the movement actually results from the fact that the plane of motion varies from segment to segment.

This finding is taken by us as strong evidence in support of the hypothesis we have proposed and it implies that, in attempting to understand problems related to the organization and control of limb movement, two questions become prominent:

(1) how does planar wrist motion result from the co-ordinated motions at the shoulder and elbow joints and

(2) how are segments of such motion joined together?

While little is known regarding the second question, a number of investigators have dealt with the first one in recent years. The consensus which appears to emerge from these studies is that the organization of limb movements can be best understood at the kinematic level, that is in terms of the linear displacement and velocity of the wrist and of the angular motion at the shoulder and elbow. For example, we have proposed that a simple set of rules can be applied to determine the angular motions of the arm which will approximate a desired trajectory of the wrist in space.\textsuperscript{12,14}

In a relaxed form, the assumption of sinusoidal motion may be restated as follows: the motion of each of the orientation angles of the arm is periodic and all have the same period. This statement would still be consistent with the data we have presented here and in previous publications.\textsuperscript{12,14} Indeed, planar wrist motion results inevitably if the displacement of the wrist is sinusoidal in three perpendicular directions. Stated another way, wrist trajectories whose tangent plane changes continuously would result from wrist motion whose components do not all have the same period (for example, an oscillation in the z-direction at twice the frequency as those in the x- and y-directions). While this question still requires detailed analysis, one can reason that such a motion would require that the frequency of oscillations of some of the orientation angles also differ. (Some preliminary simulation studies support this conclusion.) Therefore, the inability of subjects to generate non-planar wrist motion would derive from their inability to produce oscillations of the orientation angles of the arm and forearm whose frequencies differ. Note that this constraint has been found to hold for movements requiring bi-manual co-ordination\textsuperscript{8,17} and for the co-ordination of arm and leg motion\textsuperscript{2} and it should therefore not be surprising that it holds true also for the motion of the segments of a limb (upper arm and forearm) which interact dynamically. One should also note that this appears to be a purely kinematic constraint. As can be seen from Fig. 10, the apparent periodicity of the electromyographic activity of different muscles can indeed be substantially different.

In closing we should stress that the conclusions reached here are strictly valid only in the context of the minimum number of degrees of freedom of the shoulder (three) and elbow joints (one), i.e. the restricted conditions under which the experiments were performed (trunk fixed and minimal translation of the center of rotation of the shoulder joint). If these conditions are relaxed, it is conceivable that by co-ordinated movements of the trunk and/or scapula and of the arm, the inability to produce non-planar

\textsuperscript{*}However, not all periodic motions would obey the observed power law relation between curvature and velocity.\textsuperscript{14}
Fig. 10. Torque at the elbow ($T_e$) and at the shoulder ($T_s$) and rectified electromyographic activity of elbow and shoulder muscles. The subject attempted to draw the outline of a circle circumscribed on a cylinder in (A) and of the seams of a baseball in (B). The dashed lines denote the boundaries between segments for one cycle of each of the motions. The components of shoulder torque are illustrated in a frame of reference fixed to the upper arm, $T_e$ being the torque about the anterior direction when the upper arm is vertical and the arm lies in the sagittal plane, $T_s$ the torque about the mediolateral direction and $t_{st}$ about the vertical. The scale per division is: $T_e$, 10 Nm; $T_s$, 5 Nm.

wrist motion may be overcome at least partially. There also exists the possibility that non-planar wrist motion may be generated by combining a rotation of one limb segment at a constant velocity (e.g. the upper arm) and an oscillation of the other. This possibility, suggested to us by Dr G. E. Loeb, as well as the preceding one, could not be investigated because of limitations in the experimental set-up.

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REFERENCES


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