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Estimation of the axis of a screw motion from noisy data—A new method based on Plücker lines

Koon Kiat Teu[a], Wangdo Kim[b,*]

[a]Division of Engineering Mechanics, School of Mechanical & Production Engineering, Nanyang Technological University, Singapore
[b]Biomechanics Laboratory, Legacy Research Center, Portland, OR, USA

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Abstract

The problems of estimating the motion and orientation parameters of a body segment from two n point-set patterns are analyzed using the Plücker coordinates of a line (Plücker lines). The aim is to find algorithms less complex than those in conventional use, and thus facilitating more accurate computation of the unknown parameters. All conventional techniques use point transformation to calculate the screw axis. In this paper, we present a novel technique that directly estimates the axis of a screw motion as a Plücker line. The Plücker line can be transformed via the dual-number coordinate transformation matrix. This method is compared with Schwartz and Rozumalski [2005. A new method for estimating joint parameters from motion data. Journal of Biomechanics 38, 107–116] in simulations of random measurement errors and systematic skin movements. Simulation results indicate that the methods based on Plücker lines (Plücker line method) are superior in terms of extremely good results in the determination of the screw axis direction and position as well as a concise derivation of mathematical statements. This investigation yielded practical results, which can be used to locate the axis of a screw motion in a noisy environment. Developing the dual transformation matrix (DTM) from noisy data and determining the screw axis from a given DTM is done in a manner analogous to that for handling simple rotations. A more robust approach to solve for the dual vector associated with DTM is also addressed by using the eigenvector and the singular value decomposition.

Keywords: Screw axis; Dual vector; Dual Euler angle; Dual transformation matrix; FHA (finite helical axis); Plücker lines; Random noise; Skin movement artifacts

1. Introduction

The problem of estimating the motion and orientation parameters of a body segment from two n point-set patterns is of significant importance in many areas. In most instances we have available two sets of noisy observations of n points of a rigid body, and determination of the orientation and position are based on these points data. However, this paper addresses the case where there are two sets of *lines*, which are derived from the point data.

Chasles's theorem states that a rigid body motion can be specified as a *screw*. It is intuitive that the geometry of the screw axis can be fully and easily studied using Plücker's coordinates (McCarthy, 2000). The Plücker line (dual vector) can be transformed via the dual transformation matrix (DTM) and may be manipulated in accordance with rules of motor algebra (Fischer, 1999). The robotic community has explored the large body of line transformation work established over a century ago, see e.g. Denavit and Hartenberg (1955); however, the biomechanics and photogrammetric communities have not done so. Only recently, Ying and Kim (2002) applied Plücker lines to quantify three-dimensional human joint motions.

*Corresponding author. Tel.: +503 413 5438; fax: +503 413 5216.
E-mail address: mwdkim@gmail.com (W. Kim).

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The DTM has been shown to be an effective means of three-dimensional *line* transformation in displacement analysis (Ying and Kim, 2002). Dual Euler angles were applied to investigate the ankle joint complex (Ying et al., 2004) because it facilitates the description of movement at the joints (Wong et al., 2005). The repeatability of the DTM methodology was also studied (Ying and Kim, 2005), and it was applied to the analysis of a kinematic chain in a golf swing (Teu et al., 2005). This paper demonstrates another of the dual-number transformation's applications.

Screw axis has been employed to describe joint kinematics mainly in clinical assessments. The position and orientation of the screw axis will generally change throughout the motion (Woltring et al., 1985). Woltring et al. (1994) used instantaneous screw axes estimated from low-pass-smoothed video data, and a more thorough description of these parameters has been offered in other investigations (Woltring et al., 1985; Bottlang et al., 2000; Gamage and Lasenby, 2002; Schwartz and Rozumalski, 2005). Previous reports have investigated screw axis accuracy combining the motion analysis technique and the smoothing procedure (Bottlang et al., 2000; Stokdijk et al., 1999). Bottlang et al. (2000) have also examined changes in the location of a screw axis due to an applied valgu-varus stress in the intact elbow. New methods have been proposed, and some of them do not assume a rigid body motion (Cheze et al., 1998; Halvorsen et al., 1999; Sahan and Joan, 2002). Accuracy describing relative movements between adjacent bodies with an electromagnetic tracking system was also assessed (Duck et al., 2004).

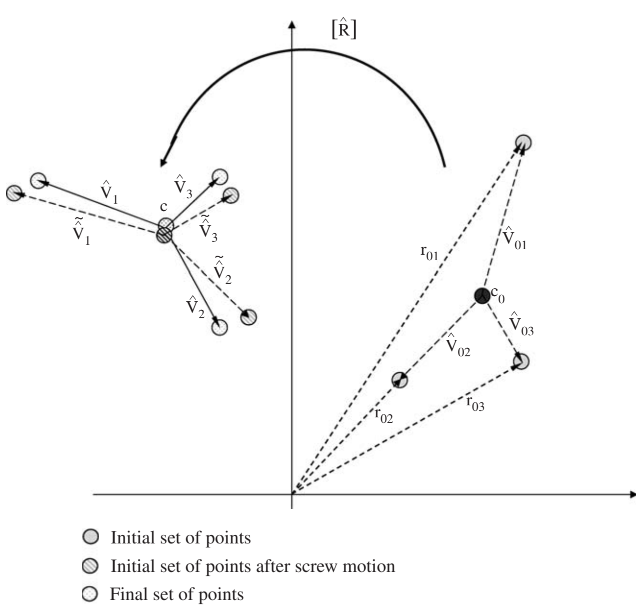

Fig. 1. Conventional algorithms typically involve mapping a set of rotated data points (initial set of points after screw motion) to the corresponding set of final data points (final set of points) using least-square methods. The new algorithm involves mapping a set of rotated vectors ($\hat{\tilde{V}}_i$) to the corresponding set of final vectors ($\hat{V}_i$) using least-square methods.

Screw axes are highly sensitive to data noise. They are also undefined with the normal set of equations when rotation angles equal zero, i.e. pure translation. Another set of equations is needed for this special case (Fischer, 1999). This caused difficulty in determining the screw axis under small rotations. Conventional algorithms generally used a set of point patterns with least-square methods to solve the noise problem. Finding a rotation matrix usually involves mapping a set of data points to the corresponding data points after movement. The new algorithm proposed takes advantage of a set of line patterns, which are derived from point data using a dual-number relationship. The method involves mapping a set of vectors (Fig. 1) to the corresponding vectors after movement, made possible by the use of DTM.

2. Methods

The background on obtaining DTM is given in Appendix A. With the DTM $[\hat{R}]$ obtained, the screw axis positions and directions can then be determined. Following the derivation by McCarthy (2000) and incorporating dual vector algebra, we show that the Plücker coordinates in dual vector representation, $\hat{\boldsymbol{V}} = \boldsymbol{V} + \varepsilon\boldsymbol{W}$ of this screw axis satisfy the condition that it is an invariant of the 3×3 DTM $[\hat{R}]$.

$$\hat{\boldsymbol{V}} = [\hat{R}]\,\hat{\boldsymbol{V}}, \tag{1}$$

where

$$\begin{aligned}\boldsymbol{V} + \varepsilon\boldsymbol{W} &= ([R] + \varepsilon[S])\,(\boldsymbol{V} + \varepsilon\boldsymbol{W})\\ &= [R]\,\boldsymbol{V} + \varepsilon([S]\,\boldsymbol{V} + [R]\,\boldsymbol{W}).\end{aligned} \tag{2}$$

We rewrite Eq. (1) as

$$[I - \hat{R}]\,\hat{\boldsymbol{V}} = 0, \tag{3}$$

and seek a solution other than $\hat{\boldsymbol{V}} = 0$. This is easily done if we separate it into the pair of vector equations compromising the primary and dual component, respectively.

$$\begin{aligned}&[I - R]\,\boldsymbol{V} = 0,\\ &[I - R]\,\boldsymbol{W} = [D]\,\boldsymbol{V},\end{aligned} \tag{4}$$

where $[D]$ is the skew-symmetric matrix defined by $[D]\,\boldsymbol{V} = \boldsymbol{d} \times \boldsymbol{V}$ for any translation, and we used the property of $[R]\boldsymbol{V} = \boldsymbol{V}$.

Now $[D]\boldsymbol{V} = \boldsymbol{d} \times \boldsymbol{V}$ must be orthogonal to $\boldsymbol{V}$. Therefore, it does not have a component in the direction of the null space of $[I - R]$, which is $\boldsymbol{V}$. This means that we can solve this equation for $\boldsymbol{V}$. Its algebraic derivation here is straightforward, because it is a direct consequence of applying the eigen-value problem to three-dimensional lines. It is probable that specifying the axis of a screw motion as a Plücker line is the most suitable linear representation because it minimizes the influence of non-linear transformations, frequently used as closed form solutions in conventional methods.

Two different methods of calculating the screw axis from DTM were addressed in Fischer (1998) and McCarthy (2000). In this paper, a different approach was stated by using the eigenvector and the singular value decomposition (SVD) (Press et al., 1992) to solve for the dual vector, especially the dual component $\boldsymbol{W}$ in Eq. (4). It is adopted on the grounds that it is more straightforward in implementation and performs better under very small rotation.

The first equation in Eq. (4) means that the $\boldsymbol{V}$, also the direction of the screw axis, is simply the eigenvector of the primary component, R, of the DTM. Looking at the dual component $\boldsymbol{W}$in Eq (4), the matrix $[I-R]$ turned out to be ill-conditioned and this can be efficiently evaluated using the SVD (Press et al., 1992):

$$(R-I)^{-1} = \boldsymbol{V}\cdot[\mathrm{diag}(1/w_j)]\cdot U^{\mathrm{T}}, \tag{5}$$

where $\boldsymbol{V}$ and U are orthogonal matrices and w is a square diagonal matrix which satisfies the following equation:

$$(R-I) = U\cdot\begin{pmatrix} w_1 & & & \\ & w_2 & & \\ & & \cdots & \\ & & & w_j \end{pmatrix}\cdot\boldsymbol{V}^{\mathrm{T}}. \tag{6}$$

Therefore the dual part $\boldsymbol{W}$ in Eq. (4) can be found as follows:

$$\boldsymbol{W} = \boldsymbol{V}[\mathrm{diag}(1/w_j)]U^{\mathrm{T}}(-[D]\boldsymbol{V}). \tag{7}$$

Because $\boldsymbol{V}\cdot\boldsymbol{W}=0$, we see that $\hat{\boldsymbol{V}} = \boldsymbol{V}+\varepsilon\boldsymbol{W}$ are the Plücker coordinates of a line, which represent the screw axis. The reference point $\boldsymbol{C}$ for the screw axis $\hat{\boldsymbol{V}}$ is determined by $\boldsymbol{C}=\boldsymbol{V}\times\boldsymbol{W}$.

It should be noted that although the case $n=3$ represents the minimum number of non-collinear points necessary to estimate the DTM of the rigid body, it is better to estimate the coordinates of $n>3$ points to reduce errors (Pittman et al., 1991).

3. Simulations and application

The proposed method was tested on simulated data using the simulation that closely follows Halvorsen et al. (1999). Four markers at coordinates (0, 0, −5), (0, 0, −30), (0, −5, −15) and (0, 5, −15) were rotated 1° per frame for a total of 60° around an axis parallel to the y-axis and going through the point (0, 0, 5). The unit is in cm.

Two simulations were performed. In the first simulation, random sequences corresponding to measurement errors were added to the dataset. For each sequence, the signal-to-noise ratio (SNR), the ratio of the energy of the signal sequence to that of the random sequence, was calculated. The signal was defined by subtracting the mean from each of the marker paths (Halvorsen et al., 1999). The energy, E_x, of a sequence, $x(n)$, is given by

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2. \tag{8}$$

In the second simulation, skin movement position artifacts were added with reference to Cappozzo et al. (1996)'s result. The authors had intended to follow Halvorsen's paper but were unsure of how the position artifacts were added. Hence, the authors implemented the following: The artifact movement at 10 instances during the swing phase of a walking stride was estimated from Cappozzo et al. (1996)'s paper for four markers (HF, LM, m6 and m7) and expressed in the coordinate system used in this paper. The skin movements were then added to the simulated data. Simulations were run with each set of errors varied at 50%, 100%, 150% and 200%.

4. Results and discussion

Schwartz and Rozumalski (2005)'s method (called as Schwartz method) were used as a reference for the proposed algorithm (called as "Plücker line method"). The simulation results of both methods were presented in Figs. 2 and 3. The general trend of the error follows that of those reference methods. Fig. 2 shows the results for the mean error in direction and position of the axis as a function of SNR in the random error simulation. Polynomial lines of an order of 2 were each fitted to the simulated results using least squares fitting. The results show that when there is little noise, both methods are comparable. However, as noise increases, the disparities in both methods become obvious. The error in direction for the Plücker line method was less when compared to the reference methods. The error in position was kept much less than that of the reference method. Fig. 3 shows the results for the mean error in direction and position of the axis as a function of skin movement in the simulation. Polynomial lines of an order of 2 were also fitted to the simulated results using least squares fitting. Again, the proposed method outperforms the reference method in terms of both direction and position estimation, especially when position artefact errors were large. Table 1 shows the mean and standard deviation of the results for various simulated situations. The pairs of means that are significantly different, $p<0.01$ are marked *. The proposed method clearly outperforms

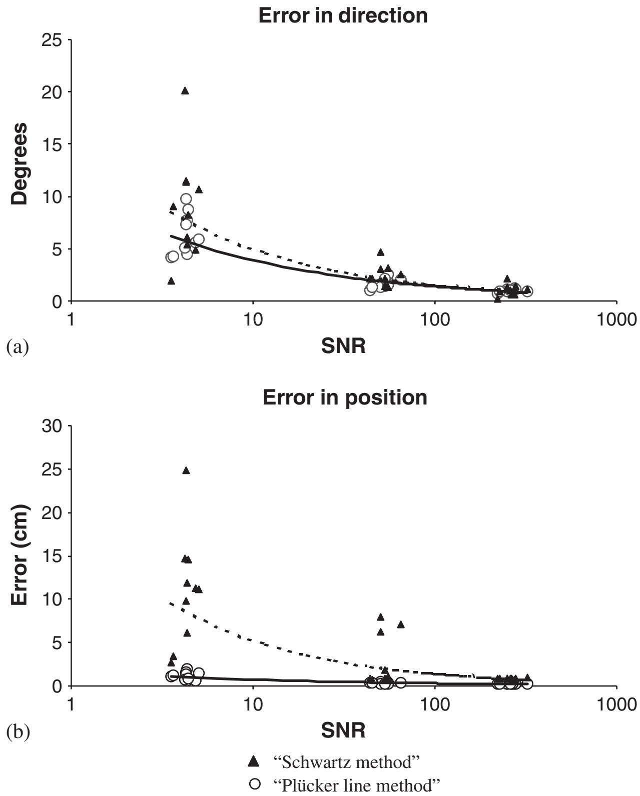

Fig. 2. Mean error in the (a) estimated direction and (b) estimated position of the axis plotted against the SNR. The solid lines correspond to the "Plücker line method" and the dotted lines to the "Schwartz method".

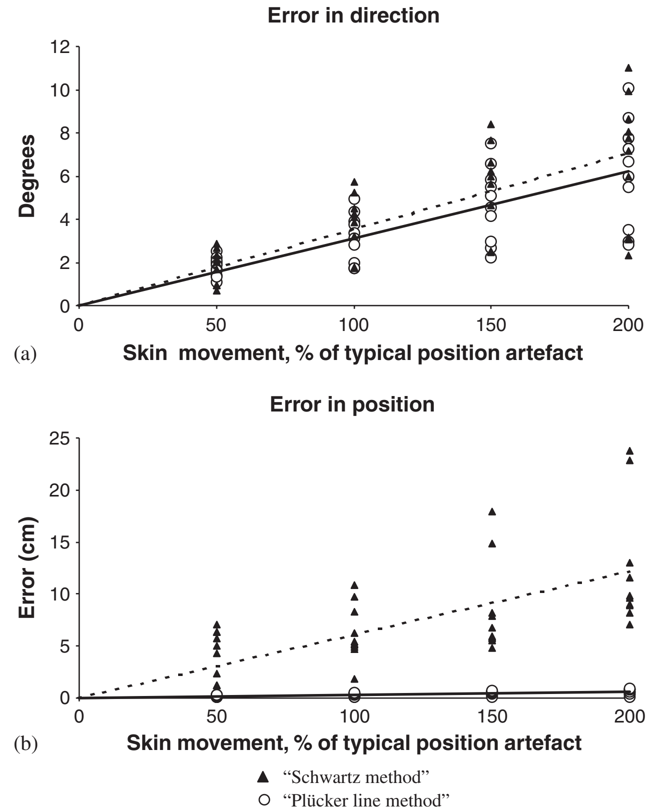

Fig. 3. Mean error in the (a) estimated direction and (b) estimated position of the axis plotted against the magnitude of the skin position artifact. The solid lines correspond to the "Plücker line method" and the dotted lines to the "Schwartz method".

Table 1
Regression mean and standard deviation of the results for (a) random error simulation and (b) skin movement simulation

SNR	Error in direction (°)				Error in position (cm)			
	Plücker line method		Schwartz method		Plücker line method		Schwartz method	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
(a) SNR								
4	6.29*	1.98	8.93*	5.02	1.17*	0.46	11.06*	6.43
50	1.71	0.46	2.40	1.04	0.36*	0.11	2.84*	2.98
250	0.96	0.14	0.99	0.53	0.24	0.04	0.85	0.06
(b) %								
50	1.78	0.44	1.85	0.76	0.18*	0.09	4.07*	2.38
100	3.16	1.11	3.81	1.38	0.31*	0.15	6.23*	2.67
150	4.69	1.73	5.56	2.06	0.45*	0.20	8.36*	4.42
200	6.11	2.47	6.71	3.01	0.59*	0.25	12.39*	5.99

The pairs of means that are significantly different are marked as "*".

the reference method, implying that it can give a better estimate for the screw axis.

The proposed method worked well for its applications, especially when noise was present, due to the fundamental difference between the proposed algorithm and the conventional methods. The mapping of line vectors gives a better estimate of the rotation than mapping of points data and hence better results could be derived from it. The results are highly significant as they demonstrate an advantage of using lines instead of points. Together with the smoothing, the presented method is highly robust and provides reliable estimates of the screw axis, even under high noise situations. We have chosen one conventional method (noted as

"Schwartz method") as a reference, but even if the general behavior should be similar, the differences observed can vary depending on the reference method selected.

5. Conclusion

In our study, the estimation of screw axes from a set of Plücker lines clearly outperforms conventional methods using point data. The presented method also showed that the screw axis could be easily determined from the DTM simply because of the fact that the axis is invariant during the dual transformation. This opens the range of new practical applications of Plücker lines in bio-kinematics situations. The eigenvector and SVD method proposed also worked well and gives a simple alternative to the dual vector calculation. The method provides an alternative analysis tool for estimation of screw axes and can be applied to any actions such as gait analysis in clinical settings to estimate the screw axis.

Appendix A. Calculating dual-number transformation matrix based on the three-dimensional coordinates of the points on the rigid body

A.1. Description of a vector constrained on a line with dual vectors

In a coordinate system, a vector constrained to lie upon a definite line in space, as shown in Fig. A1, can be expressed in the dual vector form as

$$\hat{\boldsymbol{V}} = \boldsymbol{V} + \varepsilon \boldsymbol{W}. \tag{A1}$$

In the above equation, the primary part $\boldsymbol{V}$, called the resultant vector, comprises the magnitude and direction of the vector. It is independent of the location of the coordinate system origin (reference point). The dual part $\boldsymbol{W}$ called moment vector is defined by $\boldsymbol{W} = \boldsymbol{C} \times \boldsymbol{V}$, where $\boldsymbol{C}$ connects the origin to any point on the line of the vector. $\boldsymbol{W}$ is invariant for the choice of point on the line, but it does vary with the choice of the coordinate system origin.

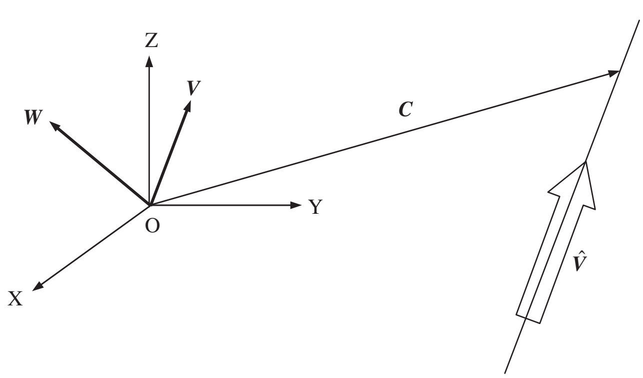

Fig. A1. Vector constrained on a line and its dual vector form description.

A.2. Algorithm for computing dual-number transformation matrix from point coordinates

The method in this paper involves obtaining the DTM from coordinated points. The obtained DTM represents a screw motion through a given axis in space. A three-dimensional line is typically specified with six parameters (Plücker's coordinates), three for position and three for direction, and the coordinates can be written as a dual vector. For applications of its use in robotics and kinematics, see McCarthy (2000). Because most measurement systems used in joint kinematics studies acquire the coordinates of points (markers) on a rigid body with respect to a pre-defined fixed coordinate system, the algorithm for calculating the DTM $[\hat{R}]$ based on the three-dimensional coordinates of the points on the rigid body is developed below. This method has been mentioned in Ying and Kim (2002), and Fig. 1 is added to aid understanding.

It is well known that the coordinates of n ($n \geqslant 3$) non-collinear points on a rigid body can determine the position and orientation of the rigid body in space. Suppose that, with respect to the pre-defined global coordinate system, the coordinates of n ($n \geqslant 3$) points on the rigid body at the initial position and final position are measured as $\boldsymbol{r}_{0i}$ and $\boldsymbol{r}_i$ ($i = 1,2,3,\ldots,n$), respectively. Thus, the centroids of the points at the initial and final positions are given by $\boldsymbol{c}_0 = 1/n\sum_{i=1}^{n}\boldsymbol{r}_{0i}$ and $\boldsymbol{c} = 1/n\sum_{i=1}^{n}\boldsymbol{r}_i$, respectively. According to the dual transformation relationship, at the final position, the vector connecting the centroid to the ith point can be estimated by

$$\tilde{\hat{\boldsymbol{V}}}_i = \tilde{\boldsymbol{V}}_i + \varepsilon \tilde{\boldsymbol{W}}_i = [\hat{R}]\,\hat{\boldsymbol{V}}_{0i} \tag{A2}$$

where $\hat{\boldsymbol{V}}_{0i} = (\boldsymbol{r}_{0i} - \boldsymbol{c}_0) + \varepsilon \boldsymbol{c}_0 \times (\boldsymbol{r}_{0i} - \boldsymbol{c}_0)$ represents the vector connecting the centroid to the ith point when the rigid body is at the initial position. On the other hand, at the final position, the same vector can also be calculated from the measured data as

$$\hat{\boldsymbol{V}}_i = \boldsymbol{V}_i + \varepsilon \boldsymbol{W}_i = (\boldsymbol{r}_i - \boldsymbol{c}) + \varepsilon \boldsymbol{c} \times (\boldsymbol{r}_i - \boldsymbol{c}). \tag{A3}$$

Fig. 1 shows the graphical representations of the points and vectors. Because of noise, there is a difference between $\tilde{\hat{\boldsymbol{V}}}_i$ and $\hat{\boldsymbol{V}}_i$. In the least-square error sense, the dual-number transformation matrix $[\hat{R}]$ should minimize the following function:

$$J = \frac{1}{n}\sum_{i=1}^{n}\left(\left\|\boldsymbol{V}_i - \tilde{\boldsymbol{V}}_i\right\|^2 + \left\|\boldsymbol{W}_i - \tilde{\boldsymbol{W}}_i\right\|^2\right) \tag{A4}$$

where $\|\|$ represents the norm of a three-dimensional vector. This optimization problem is subject to the orthogonal constraint $[\hat{R}][\hat{R}]^{T} = [I]$. The dual form constraint can be expanded into the ordinary matrix form as

$$\begin{aligned}[R][R]^{T} &= [I],\\ [R][S]^{T} + [S][R]^{T} &= [0],\end{aligned} \tag{A5}$$

where $[R]$ and $[S]$ are the primary and dual components of the dual transformation matrix, respectively. From Eq. (4), 12 ordinary constraint functions are obtained. The 18 elements in the dual-number transformation matrix were determined by solving the constrained optimization problem using sequential quadratic programming (SQP) methods (Fletcher, 1980). The optimization toolbox in MATLAB (The Math Works Inc., Natick, MA, USA) was used as the computational tool. The solution provides the optimal estimate of the dual-number transformation matrix $[\hat{R}]$.

References

Bottlang, M., Madey, S.M., Steyers, C.M., Marsh, J.L., Brown, T.D., 2000. Assessment of elbow joint kinematics in passive motion by electromagnetic motion tracking. Journal of Orthopaedic Research 18, 195–201.

Cappozzo, A., Catani, F., Leardini, A., Benedetti, M.G., Croce, U.D., 1996. Position and Orientation in space of bones during movement: experimental artefacts. Clinical Biomechanics 11, 99–100.

Cheze, L., Fregly, B.J., Dimnet, J., 1998. Determination of joint functional axes from noisy marker data using the finite helical axis. Human Movement Science 17, 1–15.

Denavit, J., Hartenberg, R.S., 1955. A kinematic description for lower pair mechanisms based on matrices. Journal of Applied Mechanics 22, 215–221.

Duck, T.R., Ferreira, L.M., King, G.J.W., Johnson, J.A., 2004. Assessment of screw displacement axis accuracy and repeatability for joint kinematics description using an electromagnetic tracking device. Journal of Biomechanics 37, 163–167.

Fischer, I.S., 1998. The dual angle and axis of a screw motion. Mechanism and Machine Theory 33 (3), 331–340.

Fischer, I.S., 1999. Dual-number Methods in Kinematics, Statics, and Dynamics. CRC Press, Boca Raton.

Fletcher, R., 1980. Practical Methods of Optimization. Wiley, New York.

Gamage, S.S.H.U., Lasenby, J., 2002. New least squares solutions for estimating the average centre of rotation and the axis of rotation. Journal of Biomechanics 35, 87–93.

Halvorsen, K., Lesser, M., Lundberg, A., 1999. A new method for estimating the axis of rotation and the center of rotation. Journal of Biomechanics 32, 1221–1227.

McCarthy, J.M., 2000. Geometric Design of Linkages. Springer, New York.

Pittman, P., Colbaugh, R., Glass, K., Rowen, B., 1991. An approach to determining the kinematic parameters of biomechanical system with application to the wrist. In: Proceedings of the ASME Winter Annual Meeting, vol. 20, pp. 567–570.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 1992. Numerical Recipes in C. Cambridge University Press, Cambridge.

Sahan, S.H.U.G., Joan, L., 2002. New least squares solutions for estimating the average center of rotation and the axis of rotation. Journal of Biomechanics 35, 87–93.

Schwartz, M.H., Rozumalski, A., 2005. A new method for estimating joint parameters from motion data. Journal of Biomechanics 38, 107–116.

Stokdijk, M., Meskers, C.G.M., Veeger, H.E.J., de Boer, Y.A., Rozing, P.M., 1999. Determination of the optimal elbow axis for evaluation of placement of prostheses. Clinical Biomechanics 14, 177–184.

Teu, K.K., Kim, W., Tan, J., Fuss, F.K., 2005. The analysis of golf swing as a kinematic chain using dual Euler angle algorithm. Journal of Biomechanics, in press.

Woltring, H.J., Huiskes, R., De Lange, A., 1985. Finite centroid and helical axis estimation from noisy landmark measurements in the study of human joint kinematics. Journal of Biomechanics 18, 379–389.

Woltring, H.J., Long, K., Osterbauer, J., Fuhr, A.W., 1994. Instantaneous helical estimation from 3D video data in neck kinematics for whiplash diagnostics. Journal of Biomechanics 27 (12), 1415–1432.

Wong, Y., Kim, W., Ying, N., 2005. Passive motion characteristics of the talocrural and the subtalar joint by dual Euler angles. Journal of Biomechanics 38 (12), 2480–2485.

Ying, N., Kim, W., 2002. Use of dual Euler angles to quantify the three-dimensional joint motion and its application to the ankle joint. Journal of Biomechanics 35, 1647–1657.

Ying, N., Kim, W., 2005. Determining dual Euler angles of the ankle complex in vivo using 'Flock of Birds' electromagnetic tracking device. Journal of Biomechanical Engineering 127, 98–107.

Ying, N., Kim, W., Wong, Y., Kam, B.H., 2004. Analysis of passive motion characteristics of the ankle joint complex using dual Euler angle parameters. Clinical Biomechanics 19, 153–160.

Further reading

Savitzky, A., Golay, M., 1964. Smoothing and differentiation of data by simplified least square procedures. Analytical Chemistry 36, 1627.