Jet drive mechanisms in edge tones and organ pipes

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Measurements of the phases of free jet waves relative to an acoustic excitation, and of the pattern and time phase of the sound pressure produced by the same jet impinging on an edge, provide a consistent model for Stage I frequencies of edge tones and of an organ pipe with identical geometry. Both systems are explained entirely in terms of volume displacement of air by the jet. During edge-tone oscillation, 180° of phase delay occur on the jet. Peak positive acoustic pressure on a given side of the edge occurs at the instant the jet profile crosses the edge and starts into that side. For the pipe, additional phase shifts occur that depend on the driving points for the jet current, the Q of the pipe, and the frequency of oscillation. Introduction of this additional phase shift yields an accurate prediction of the frequencies of a blown pipe and the blowing pressure at which mode jumps will occur.

Subject Classification: [43]75.60.

INTRODUCTION

The phenomenon of edge tones and the maintenance of oscillation in organ pipes and flutes have long been considered to be connected. Many texts put forth an explanation of the second in terms of the first, a rather circular procedure in view of the fact that there are many gaps in the theoretical basis for both. The subject matter of this paper can be illustrated by the results plotted in Fig. 1. Here are plotted the observed oscillation frequencies versus blowing velocity for two jet-edge situations, a normal organ pipe, and a simple edge-tone generator with identical jet-edge geometry. For the pipe one observes oscillation near the frequencies of the normal modes of the pipe, the frequency for each mode rising slightly as blowing velocity is increased. The oscillations jump (with hysteresis) to the successively higher modes as the blowing velocity is increased. There is a point on each plateau at which the frequency equals the passive resonance frequency of the pipe mode. At this point the jet may be said to be driving the pipe at its natural frequency. The oscillation has already become quite strong, though it may increase slightly at higher blowing pressure. Coltman[1] has shown that the flute is normally played near this condition. Note now that the edge-tone frequency obtained for the same geometry and blowing condition is far removed. An operating line drawn through the normal organ-pipe resonance points lies, in frequency, well below the edge-tone characteristic. Thus the picture of an organ pipe as a resonator coupled to an edge-tone generator, and "amplifying" its response is certainly oversimplified. It is the intent of this paper to describe, mostly from direct observation but partly theoretically, some of the various mechanisms involved in these two tone generators, and to present a consistent picture of their behavior. In doing so we must tackle several of the many uncertainties in our current picture of the mechanism, and give evidence for resolving these uncertainties. The evidence is provided, as far as possible, by direct experimental measurement, with as little recourse to mechanistic hypothesis as possible.

I. OSCILLATION MECHANISMS

The mechanisms of maintenance of oscillation in edge tones and in organ pipes have been the subject of experimental and theoretical treatment by a number of authors in the last two decades.[2–8] The process has generally been to examine a feedback loop which comprises: (1) the initiation, near the point of issuance, of a wave on the jet, (2) the growth and propagation of this growing wave down the jet, and (3) the impingement of the oscillating jet on the edge or wedge and the subsequent generation near this point of the acoustic disturbance which initiated the process. Oscillation takes place when the loop gain is unity and the sum of the phase shifts around the loop is an integral multiple of 2π.

Unfortunately, none of the processes named above are well determined, either experimentally or theoretically. Karamcheti[8] and his students have characterized quite completely the jet flow field when operating as an edge-tone generator. Their results show that the velocity of propagation of the wave on the jet is not uniform in space, and that it is not independent of frequency. We also know[9] that the behavior of the jet depends on whether it is laminar, and on the distribution of jet velocities across the thickness of the jet. Phase relationships between the disturbing acoustic oscillation and the starting of a jet wave have nowhere been reported. Powell[3] discusses this uncertainty and its implications with respect to determining edge-tone frequencies. Finally, the manner in which the jet pro-

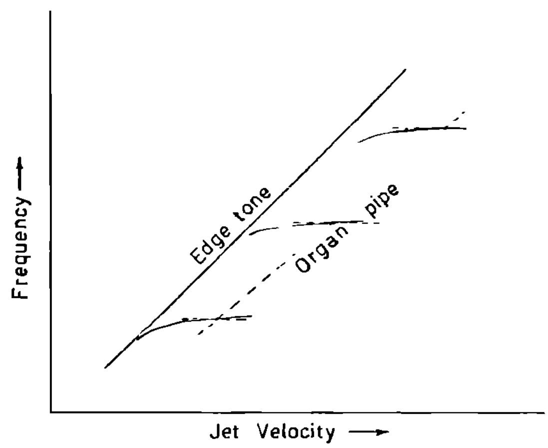

FIG. 1. Frequency behavior of jet-edge oscillators.

duces the acoustic disturbances is arguable—Powell[2] offers two possibilities for edge-tone generators—the annihilation by the wedge of transverse jet momentum, or the creation of a dipole current source by the jet. In the case of organ pipes, Cremer and Ising[5] ascribe it to the jet current inserted behind the impedance represented by the mouth end correction, while Coltman[4] took the view that the driving force was due to momentum transfer from the jet to the acoustic flow. Depending on which of these views is taken, one is also faced with uncertainties as to where the interaction takes place, and its duration. In trying to pin down some of these questions, one is forced to choose a rather limited variation of parameters so as not to further complicate an already quite complex situation. In this work then, we have fixed the dimensions (width and thickness) of the jet, made the jet passage long to assure laminar flow with a parabolic velocity distribution, and confined the treatment to the case of first stage oscillations, i.e., those in which the jet wave executes less than one complete oscillation between lip and wedge. We also maintained symmetry in the geometry where possible, and avoided situations where harmonics were strongly generated. The picture is therefore far from exhaustive—we can only hope that it clarifies some of the basic processes in the simpler cases.

II. PHASE DELAYS ON THE JET

An important portion of the entire delay of 2π occurs as a result of propagation times of jet waves. Determination of these is not as simple as it might seem. "Jet" is, in fact, not well defined. The particles of air issuing from the slit almost immediately begin to impart some of their motion to the surrounding air. To photograph patterns of smoke-filled jets, as has been done by most of those who have studied the question, tells us only what space, at a given instant, is occupied by air that came from the jet. It does not tell us how the jet particles are individually moving, what paths they followed, or what other motion is taking place in air that is not marked with smoke. Karamcheti[8] and his students have greatly improved on this technique by measuring the velocity of air motion with hot wire anemometers throughout the whole flow region. Figure 2, reproduced from the thesis of William Shields[10] is illustrative of the measurements they have made. With these one can in principle decide what portions of the field he will consider to be the jet, and what characteristic (displacement of the centerline of the forward momentum profile, transverse velocity at the axis of symmetry, etc.) should be measured when characterizing propagation velocities. Fortunately, Shields'

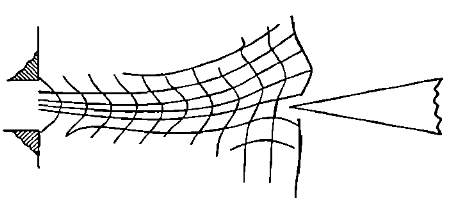

FIG. 2. Streamlines and velocity profiles during edge tone oscillation from Shields.[10]

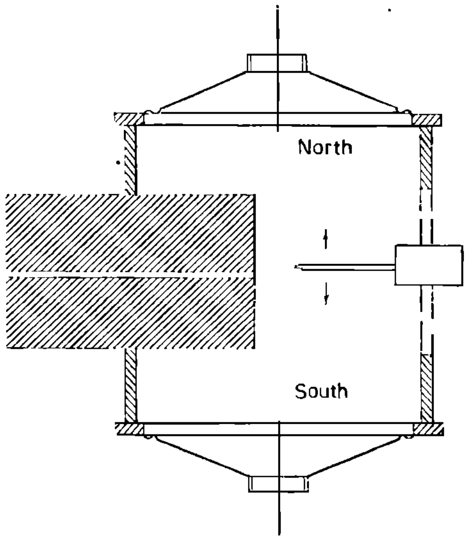

FIG. 3. Cross section of arrangement of slit blocks, loudspeaker exciter and Pitot tube microphone for measuring jet phase.

measurements show that the jet is, by and large, well behaved, and that earlier streak photographic work gives a reasonable basis for physical behavior. For the kinds of displacements shown, the jet profiles remain coherent, and the jet behavior in the region between the slit and the edge can generally be looked at as a transverse displacement of a group of largely forward-moving particles. The vortex patterns seen in streak photographs seem to play a negligible part in the oscillation mechanism. For our own purposes, we chose to observe the pressure generated by the jet as it swept over the tip of a Pitot tube microphone, giving a pressure waveform whose phase and shape could be used to describe the position of the forward momentum convection profile as a function of time. The transverse location of the maximum of this profile we call the jet position, and its variation in phase of y displacement with position along the x axis is the property whose propagation we deal with.

The experimental arrangement for measuring jet wave propagation is detailed in Fig. 3. The blowing slit was formed by the gap between two long blocks of wood, with cardboard glued above and below. The jet, whose velocity profile is parabolic after traveling through the long passageway, emerges from a wall perpendicular to the jet axis. The long passage assured laminar flow at the velocities used. The slit width (minor dimension) was 1.59 mm, the same as used by Shields, so that direct comparison with his very extensive data would be possible. The large slit dimension was 17.7 mm. The probe microphone had a sensitive element made from a piezo ceramic disk. A thin-walled Pitot tube, 0.10-cm inside diameter and 3.2 cm long, extended from a very shallow cavity in front of the disk, and was flattened at its outer end to make an entrance aperture 0.13 mm wide. The tube and cavity had a resonance frequency of about 3 kHz. A piece of thread inserted in the tube damped this resonance. Phase-shift measurements on this microphone gave an equivalent response time delay of about 2×10^{-4} sec. This correction was applied where necessary. Micrometer movements were used to position the tip at desired points in the xy plane that bisected the vertical dimension of the slit.

Positioned on either side of the slit were two high-compliance loudspeaker units, of 10-cm nominal diameter. These were joined facing each other by a plexiglass cylinder which had openings to admit the slit blocks and the probe microphone. The two speakers were driven by a two-channel stereo amplifier, into which was fed a sinusoidal audio signal of the desired frequency. Phasing was such that the two diaphragms moved simultaneously in the same direction, carrying a mass of air between them. A phase shifter in one channel, together with volume control adjustments, permitted attainment of an excellent acoustic pressure null on the central plane, so that no acoustic signal was picked up by the probe microphone when it was on axis. A signal from the driving oscillator, variable in phase and amplitude, could also be injected into the microphone circuit to cancel any acoustic signal, if desired, when the microphone was off axis.

With this arrangement, the jet profile and its position could be inferred from the wave forms of the microphone signal recorded at various x and y positions. A complete set taken at a distance of 6.7 mm from the slit, with a jet velocity (centerline) of 900 cm/sec shows a smooth velocity profile only about 10% wider than the slit, moving bodily in the y direction at the driving frequency of 256 Hz through an excursion about equal to its half-width, with essentially no change in shape. While complete sets were not analyzed for other conditions, no abnormal behavior was observed within reasonable distances, even when excursions were large.

With the microphone on the centerline, the time of crossing of the jet could be recorded, and by temporarily displacing the microphone slightly, a distinction could be made between a northbound and a southbound crossing. (The diagram of Fig. 3 has the regions above and below the centerline labeled North and South for reference purposes.) The microphone could also be used, when the jet was turned off, to explore the acoustic field produced by the loudspeakers. A y traverse near the jet wall gave a linear variation of pressure with distance—this gradient sufficed to determine the acoustic particle velocity exciting the jet. At the same time, the phase of the acoustic pressure (with respect to the driving oscillator) could be determined, and the jet crossing time correlated with this. As a reference, we chose zero time to be the time of occurrence of the peak positive (above atmospheric) acoustic pressure in the northern half. This is the instant at which the air particles in the vicinity of the jet have zero acoustic velocity, and are about to begin a southward movement.

Our eventual purpose is to describe the phase delays in the feedback loop which will necessarily add up, under oscillating conditions, to an integral multiple of 2π. We first must deal with the question of our reference zero time, and its implication with respect to any initial phase shifts that may be present.

At frequencies approaching zero, the jet will cross

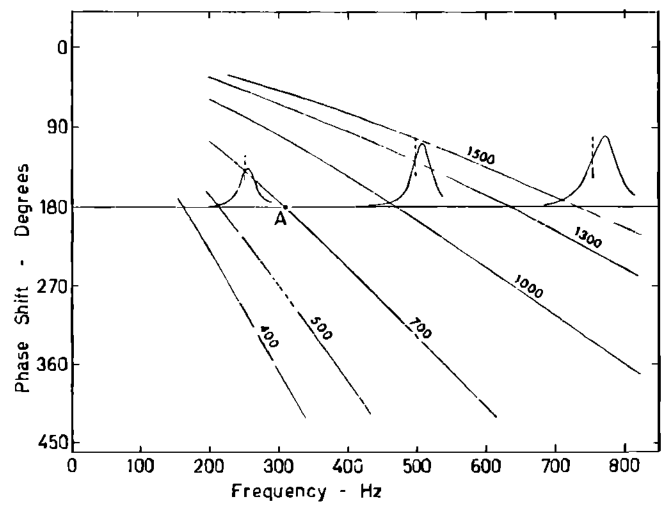

FIG. 4. Phase shifts on the jet (long lines labeled with centerline velocities in cm/sec) and in the organ pipe (humped curves). The passive resonance frequencies of the three pipe modes are at the vertical dashed lines. Slit to edge distance 7 mm.

the space from slit to edge before the exciting velocity has appreciably changed. Thus for our chosen zero time, there is no acoustic crosswind anywhere in the region, and the jet, emerging into a still region, should travel undisturbed down the centerline to impinge symmetrically on the edge. A little later, there will be a southward acoustic velocity of the surrounding air, and the jet will be deflected into the region south of the centerline. Thus in the limit of zero frequency, the jet deflection is in phase with the acoustic volume velocity. Any departure from this condition, that is, failure of the jet to make a southbound crossing of the edge at zero time, we ascribe to a delay occasioned by finite travel time of a wave on the jet. This delay we report as an angular phase shift at the frequency used, and we expect such phase shifts to extrapolate to zero for zero frequency.

Measurements of the instants of crossing were made for a variety of slit to edge spacings, acoustic driving frequencies, and jet velocities. No appreciable dependence on amplitude of the acoustic excitation was found, so this was set at a value that gave a jet swing, at 1-cm distance, of several jet widths. The data can be displayed in several ways. For purposes of frequency prediction, it is convenient to plot, for a fixed distance of the detector from the slit, the phase shift as a function of frequency. Figure 4 gives curves for each of several jet centerline velocities (controlled by the blowing pressure) for the case of a 7-mm distance. The curves can be plausibly extrapolated back to zero at frequency, though they are not straight lines, as would be obtained if the jet wave propagated with uniform velocity. Similar data were taken at 3-, 5-, and 10-mm distances.

From these data one can also plot the delay time (time from our zero reference to the time of southbound crossing of the jet) as a function of distance. This is done in Fig. 5 for a jet centerline velocity of 1000 cm/sec and two frequencies. Remarkably enough, we find the curves for the two frequencies do not super-

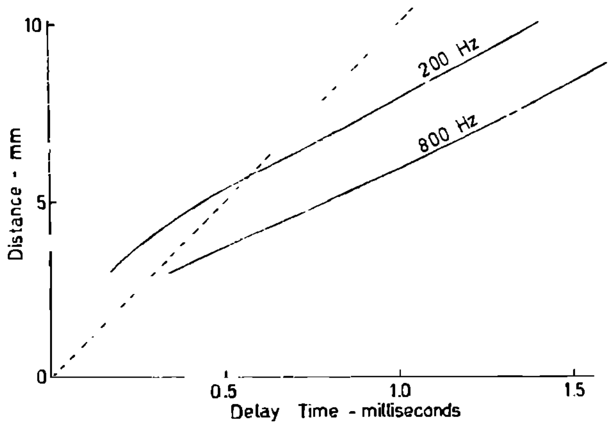

FIG. 5. Propagation of centerline crossing for two different exciting frequencies. Jet centerline initial velocity 1000 cm/sec. Dashed line uniform propagation at 1000 cm/sec.

impose, though their slopes on the straight portions are nearly the same, and correspond to a propagation velocity, once 3–5 mm away from the slit, of 500 cm/sec. This is just half the centerline velocity of the jet itself, which is represented by the dashed line. Note also that the propagation velocity at the beginning seems to be faster than that of the jet particles themselves. If we think of the acoustic motion as carrying the jet bodily from side to side, then when the jet wave itself is small in amplitude the apparent crossing time is independent of distance, i.e., it appears to propagate with infinite velocity. This may account for the initial displacements of these curves. The nonuniform propagation velocity and the dependence on frequency contribute to the difficulties encountered by many authors in attempting to arrive at simple theories to fit edge tone frequencies, especially since we anticipate that jets with different velocity profiles will exhibit different wave velocity characteristics. It must be pointed out that what is being measured here—the position in time and space of the forward velocity profile—is a somewhat artificial quantity. The bodily motion of the acoustic field carries the whole jet pattern with it, except near the slit where the requirement for infinite transverse acceleration of the newly emerging jet particles makes such an occurrence impossible. This can give rise to apparent phase shifts which may depend on the choice of the quantity being measured. Thus no attempt is made here to theoretically explain the measured shifts, but we merely exhibit the various delays which take place in the feed-back loop.

III. ACOUSTIC DISTURBANCES GENERATED BY THE JET

As mentioned in the introduction, there is no general agreement as to how the energy carried by the jet is converted into acoustic oscillations. To investigate this in more detail, the slit system described in Sec. II was fitted with an enclosure that could be converted from a jet-edge system to an organ-pipe system without changing the slit-edge geometry. At the same time, 12.7-cm top and bottom plates of the enclosure served to make the acoustic field near the jet two dimensional rather than three dimensional, i.e., the acoustic and jet motions were independent of the z dimension. The arrangement is sketched in Fig. 6. The plates were made from $\frac{1}{8}$-in.-thick plexiglass. The top plate was pierced with 1.4-mm diameter holes in a square array – 0.5 cm on centers in the neighborhood of the slit and wedge, and 1 cm on centers farther away. These holes, a few of which are indicated in Fig. 6, were used to sample the acoustic pressure at the corresponding points by using a small probe microphone fitted with a soft rubber face to make a seal around the hole. Holes not being sampled were closed with small squares of plastic electrical tape. The wedge, of 10° total angle and 2.5 cm long, was cemented between the two plates, and had bored in it a canal leading through the upper plate. Scotch tape was placed on each side of the wedge after the canal was bored, so that a very narrow aperture was formed at the leading edge, whose width was about $\frac{1}{4}$ mm. The sampling microphone, placed on the canal, could observe the impulse as the jet swept over the edge. A small correction for acoustic transit time in the canal was used.

This arrangement produced edge tones very clearly over a wide range of slit-edge distances and blowing pressures. The apparatus differed from those which other workers have used in the study of edge tones by virtue of its two-dimensional rather than three-dimensional acoustic field. To see if this change was important, experiments were made comparing the frequencies generated here with those produced using a free-standing, long (vertically) wedge. The results differed by only a few percent, and there seems to be no reason to believe that the mechanisms observed here are different than those of the usual edge-tone apparatus.

A square pipe, also made of $\frac{1}{8}$-in. plexiglass, 1.9-cm inside diameter and some 60 cm long, was shaped at one end as shown in Fig. 6 so that it could be fitted into the wedge enclosure to form an organ pipe. After sliding it in, modeling clay was used to seal the joints. Probe holes were also provided all along the upper surface of the pipe. The response of this pipe, whose first mode was at a frequency about 250 Hz, was very good. It differed from the usual organ pipe, in that the jet had a parabolic profile and no offset, that is, the slit and wedge center planes were coplanar, and by the existence of the large confining plates needed to make the flow patterns two dimensional. These differences are believed inconsequential to the behavior characteristics measured here.

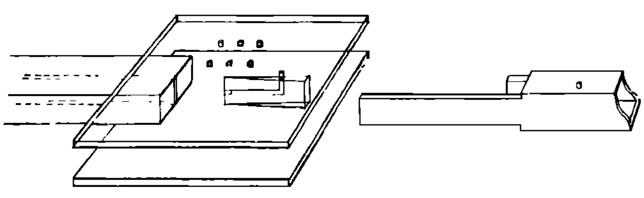

FIG. 6. Sketch of enclosed jet edge system, and the removable pipe that could be fitted in to make an organ pipe.

IV. EDGE-TONE ACOUSTIC FIELD

A microphone placed at the edge of the enclosure was used to synchronize the oscilloscope on which both the pressure waveforms and jet impulses were observed, so that their phases could be related. When oscillating as an edge-tone generator with a jet centerline velocity of 900 cm/sec and a slit-edge distance of 7 mm, a frequency of 419 Hz was produced. The acoustic pressure observed at the sample holes was more nearly triangular in waveform than sinusoidal, and had very nearly the same time phase all over the region probed. Since this extent (about 2 cm around the wedge) was small compared to a wavelength, this is not surprising. From readings of the acoustic pressure at each sampling hole, one can construct the contours of constant pressure given in Fig. 7. The limited array of sampling points makes the contour positions near the wedge uncertain. However, the dipole nature of the field, inferred by Powell[2] from farfield measurements, is very clear. Moreover, the apparent source of the pressure must lie quite close to the tip of the wedge.

A very significant finding concerns the phase of this pressure with respect to the jet-crossing time. For a wide variety of blowing pressures and slit-to-edge spacings this phase was, within a few degrees, invariant, and followed this rule: *The peak positive pressure in the north half occurs simultaneously with the northbound crossing of the jet at the edge.* This finding reinforces the often noted observation that the location of the tip of the edge is the all important factor in determining edge-tone frequencies, even though the jet forces may in some fashion be exerted farther away and at a later time. It is exactly consistent with one of the two suggestions put forth by Powell[2] (and used by Cremer and Ising[5] in organ-pipe theory) that the jet, blowing alternately above and below the edge, supplies the current for a hypothetical dipole acoustic source located at or near the edge. We can resort to a symmetry argument to establish the phase. The dimensions in the region being considered are small compared to the wavelength, and we may treat the air as imcompressible. Any excess jet air introduced into the northern half will displace the surrounding air, which may go south through the gap or northeast to escape from the region. Likewise, when the jet is blowing into the southern region, air will move north through the gap. At the instant the jet centerline crosses the edge, air is being introduced equally into the two regions. At this instant, the readjustment (acoustic) flow must be zero. Since the air movement is controlled by its inertia, the pressure is 90° out of phase, that is the pressure is at a maximum at this instant, and of such a polarity as to begin to force air out of the region into which the jet is just entering. This is in accordance with the observations. The role of the wedge is seen then, not as an obstruction on which the jet plays, but merely a partitioning agent which determines the region into which air from the jet flows. The crucial part played by the position of its leading edge becomes obvious.

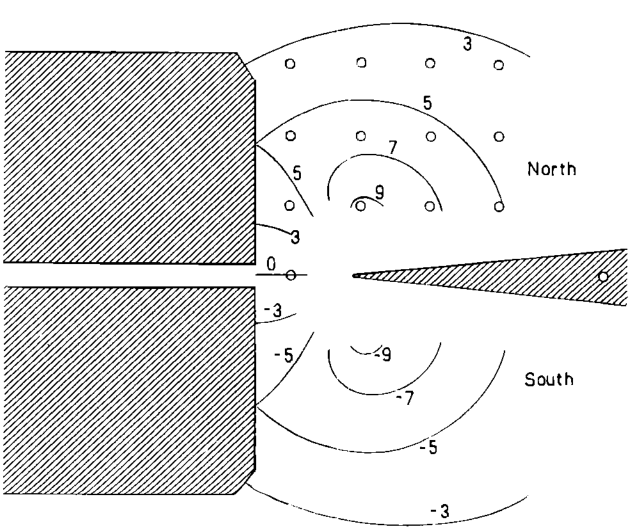

FIG. 7. Contours of acoustic pressure (relative peak values) during edge tone generation. Frequency 419 Hz, slit-to-edge distance 7 mm.

This picture is further reinforced by quantitative measurements. The probe microphone could be calibrated directly by observing the impulse generated by sweeping it through the stationary jet. That stagnation pressure could in turn be related to the pressure observed in the acoustic field with the same microphone. A sketch of the flow lines orthogonal to the contours of Fig. 7, and spaced so as to form squares, permitted an estimate of the total acoustic flow represented by the field. For the geometry used, about $\frac{6}{10}$ of the flow returns through the gap, and $\frac{4}{10}$ goes out into the space. The saddle point on contour 5 represents the division. The total acoustic flow was found to be about 80% of the jet flow, a reasonable number in view of the fact that the jet may not have been swinging completely beyond the cutting edge.

When the blowing pressure is increased, it is found that the sound pressure in the acoustic field rises. The relative acoustic current is determined by dividing the acoustic pressure by the frequency, which also varies. A plot of acoustic flow versus jet flow is very nearly linear, except for a dropping off at low jet velocities, which again may be ascribed to a jet swing less than the jet width. Thus we have a consistent picture that the acoustic air movement may be described, both in phase and amplitude, as a redistribution of the surrounding, incompressible air when jet air is forced alternately into the spaces above and below the edge. It will be noted that effects of entrainment, which Cremer and Ising called upon to explain a square-law rise of acoustic pressure with jet velocity in an organ pipe, are not apparent. Entrainment will enter into this picture only to the extent that air particles not belonging to the jet are transferred from the lower half to the upper half (and vice versa)—entrainment along each edge, staying within the original half, does not contribute. Nor does the momentum transfer which Coltman used to explain similar observations on the flute, explicitly appear. The lack of any "momentum transfer" may be inferred both from Karamcheti's streamline and profile measurements, and from the directions of acoustic current flow in the diagram of Fig. 7. Here the jet flow and acoustic flow are in the same direction only in the region east of the apparent source—nearer the edge they are in opposition, so that the momentum transfer effects could cancel out.

Powell's alternative explanation, that the force on the wedge is due to the annihilation of transverse momentum, is not upheld by the observations reported here. His view that the dipole acoustic pressure is the reaction force of an aerodynamic (circulation) lift force on the wedge is still valid—we see that at the instant the jet crosses the centerline, net current flow from one half to the other is zero, but one side of the wedge has a marked excess of moving jet air flowing along it from the previous half cycle. The excess is maximum at this instant, corresponding to the maximum pressure we inferred from the current flow picture. The role of the edge position is much more clearly displayed, however, by the simple current redistribution picture.

V. EDGE-TONE FREQUENCIES

Armed with the phase relation between jet-crossing time and acoustic pressure and with the empirical information in Fig. 4 about the phase delay between acoustic excitation and crossing of the jet at the centerline, we are in a position to predict the edge-tone frequencies which such a jet will produce. Our time zero is that of peak positive pressure in the northern half. Our rule in Sec. IV says this is also the instant at which the jet makes its northbound crossing of the edge. This is 180° out of phase with the southerly deflection that the north half pressure is just beginning to induce in the jet as it exists from the slit. To get the total loop delay of 360° required for Stage I oscillation, we need another 180° of delay due to jet wave propagation time. Thus the condition for oscillation is given by the intersections of the curves of Fig. 4 with a line drawn at the 180° phase-shift ordinate. The point marked A is such an intersection. These intersections are used to predict the edge-tone frequency as a function of blowing velocity for a slit-to-edge distance of 7.0 mm.

The predictions are listed in Table I, together with experimental values of actual observed edge tone frequencies. The fit to within a few percent is probably all that could be expected, in view of the possible disturbance of the jet flow by the wedge, and the fact that the edge tone waveform is not sinusoidal, as was the exciting acoustic excitation used for measuring jet phase delays.

TABLE I. Edge-tone frequencies, 7-mm gap.

Jet Velocity cm/sec	Frequency (in Hz) Predicted	Measured
400	162	175
500	215	220
700	310	315
1000	468	470
1300	637	620
1500	735	720

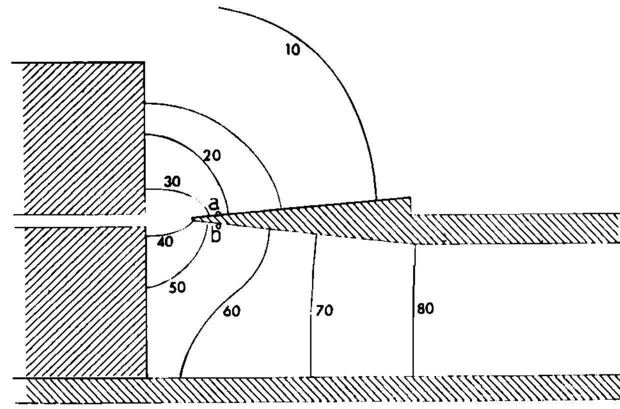

FIG. 8. Contours of acoustic pressure (relative peak values) for the organ pipe sounding in its first mode at 250 Hz.

An examination of Shields' streamline diagrams[10] also shows direct correspondence with the picture presented here. One finds that the direction of particle trajectories as they leave the slit gives an initial deflection of the jet centerline just 180° out of phase with the jet centerline position at the wedge. Thus we may say with some confidence that a 180° phase shift between jet centerline position at the edge and the acoustic velocity at the slit face, combined with a 180° phase delay between this quantity and jet centerline position at the wedge, make up the necessary 360° of phase shift for the Stage I edge tone. Note that these amounts depend on the choice of the quantities used; had we expressed the picture in terms of acoustic particle displacement, we would find a phase delay of 90° on the jet, and 270° between jet displacement at the wedge and acoustic displacement at the slit face.

VI. EXCITATION OF ORGAN PIPES

The same slit system and wedge, with its upper and lower plates, could be converted into an organ-pipe mouth by the insertion of the square pipe shown at one side in Fig. 6. The acoustic flow when blown at resonance is now quite different, not only because the geometry has been changed to an asymmetric one, but also because at resonance the acoustic flow at the mouth is dominated by the organ pipe oscillation, and contributions due directly to jet flow are relatively small.

A set of acoustic pressure contours, derived from sampling the array of probe holes in the neighborhood of the mouth, is displayed in Fig. 8. We wish now to inquire into relationships between the jet crossings and the acoustic flow, and deduce from these something about the nature of the driving mechanism of the jet.

The passive resonance frequency of the first mode of the pipe was determined, and the blowing pressure set to make the pipe oscillate at this frequency. Under these conditions, the pressure wave form at the pipe center (maximum) was found to be nearly triangular, while at the ends the pressure wave form was nearly square. In between, the wave forms were trapezoidal. From these waveforms, the shape of the standing wave in the pipe could be derived. It takes very closely the form of the wave on a string plucked in the middle. At the instant of pressure maximum the pressure distribution is triangular—maximum at the center and near zero at the ends. A little later a flat spot develops at the center, with no change outside this spot. The flat spot extends while its level sinks, reaching the ends just as it goes to zero, leaving the pressure at this moment zero throughout the pipe. The process then continues in a reverse fashion as the pressure goes negative, until a fully developed negative triangle appears. The particle velocity, or current, has a similar history, but displaced one quarter wave in space and time, so its distribution when the pressure is everywhere zero is a straight line (positive at the end, zero in the middle, negative at the other). Then flats develop on the ends, decreasing in magnitude and extending in length until they meet at zero value, in the center. The time wave form of the current is therefore triangular at the ends, approaching square at the middle, while the time waveform of the pressure is square near the end and triangular at the middle. This interesting behavior appears quite clearly when the pipe is blown near its passive resonance frequency.

The canal that was pierced in the wedge could be used to determine the time of crossing of the jet. We find experimentally, with a high-Q pipe sounding close to resonance, that the jet crosses the wedge going into the pipe (southbound in our previous notation) at about the time the acoustic current is maximum going in. The jet remains inside the tube all during the half cycle in which the current diminishes to zero and reverses its direction to become maximum flowing out. During this same half cycle, the pressure in the center of the pipe has risen from zero to maximum positive, and declined again to zero.

The phase we find here is shifted approximately 90° from the situation we found in the edge tone. This represents an additional delay between appearance of the jet in the northern half and the beginning of a southbound, or inward acoustic current at the mouth. To get 360° total delay calls for less propagation delay on the jet itself and therefore a higher blowing velocity to achieve the same frequency as for the edge tone (see Fig. 1). The observation also contradicts the momentum transfer drive mechanism I proposed earlier,[4] since during the first quarter cycle the acoustic current is moving with the jet and the second quarter cycle against it. If we assume instantaneous momentum transfer, the two quarter cycles should cancel out in drive. If we assume a stopping time spread over a quarter cycle, the counter flow period dominates, i.e., most of the time the jet is blowing against the acoustic flow, which would tend to lessen the amplitude.

We look, therefore, to the more widely accepted current drive postulated by Cremer and Ising,[5] and which we found in Sec. IV to account entirely for the edge-tone drive.

Referring to the edge-tone acoustic flow pattern (Fig. 7), we can infer from the acoustic contours the existence of an effective alternating current source located a few millimeters behind the tip of the wedge. The acoustic pressures generated are quite small, and Bernoulli effects of the jet flow over the sampling holes prevent meaningful measurements close to the wedge. Nevertheless, we can make an estimate, from Fig. 7, of the location of the apparent source, and we chose to place it 4 mm behind the wedge tip. It seems plausible to choose the same location for a current source in the organ pipe geometry. In Fig. 8, the acoustic contours, which largely display the effects of the pipe oscillation rather than the jet effects, can be used to estimate the effective "end correction" against which the jet current acts. The equivalent circuit, in terms of a transmission line into which the jet current is coupled, is shown in Fig. 9.

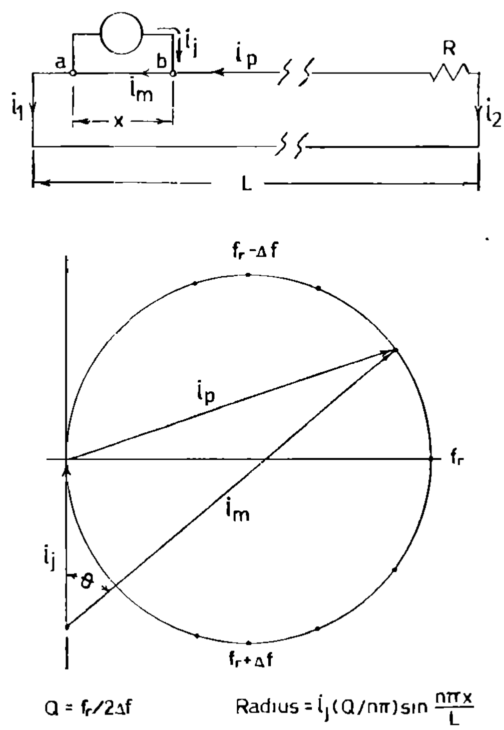

FIG. 9. Equivalent circuit for organ pipe and jet current drive, with vector diagram showing current relationships in the neighborhood of a pipe resonance.

The points a and b are the location of the effective current source on the outside and inside of the wedge, and the length of transmission line x between them represents the impedance of the mouth constriction between the respective contours. The remaining distance to the left end of the line represents the pressure field outside the point a in Fig. 8. The length L is the entire effective length of the pipe, with end corrections. All of the dissipative mechanisms have been lumped into a small resistance R, which may have a different value for different frequencies. The value of the characteristic impedance is taken as unity. The jet current, which in fact is a direct current injected alternately at a and b, can be exactly represented as far as its alternating component is concerned by an alternating current generator connected between a and b. The current i_1, at the left end is the source of acoustic radiation from the mouth, i_2 at the other end is the acoustic source at the open end of the pipe. Under resonance conditions, these two currents have identical values, in accord with the observations of sound radiation from an organ pipe.[11]

The argument in that paper concerning the absence of jet-current drive, however, was wrong. As Elder[6] has pointed out, the jet current outside the pipe, added algebracially to the mouth current, is just such as to compensate for the dissipation losses which reduce the current at the opposite end.

Manipulation of the equations for waves on a transmission line gives the following expression for i_m, the mouth current, in relation to the jet current i_j, as a function of the frequency:

$$i_m/i_j = \cos\beta x - \sin\beta x(\tan\beta L - jR)/(R^2+\tan^2\beta L)\ , \qquad (1)$$

where

$$\beta = 2\pi/\lambda = \omega/c\ .$$

In the derivation it is assumed that the equivalent distance between a and the left-hand end is small compared to a quarter wavelength, and can be lumped in with the right-hand portion of the transmission line.

The phase angle between the jet current and the mouth current in the above expression is

$$\theta = \tan^{-1}\frac{-R}{\cot\beta x(R^2-\tan^2\beta L)-\tan\beta L}\ . \qquad (2)$$

The value of R may be related to the Q of the nth mode by $R = n\pi/2Q$. The behavior of the relationship between these two currents in the neighborhood of the nth resonant mode, $\beta \cong n\pi/L$ may be more easily comprehended by the vector diagram in Fig. 9. For values of Q not too low, the pipe current is represented by the vector i_p, whose terminus lies on a resonance circle or "Q circle." The resonance frequency f_r lies on the real axis, the half-power points Δf away are on the vertical diameter. Points for other frequencies are such that the vector i_p intersects a vertical linear frequency scale, i.e., its angle with the real axis is $\tan^{-1}(-df/\Delta f)$, where df is the departure from the resonance frequency. The jet current i_j lies on the vertical axis, and the radius of the circle is given by $i_j(Q/n\pi)\sin n\pi x/L$. The sum of these two vectors is the mouth current i_m. As the frequency increases, the vector tips sweep around the circle clockwise. One notes that the maximum amplitude of mouth current will occur prior to resonance, and that a phase shift of $\theta = 90°$, if it occurs at all, happens only at frequencies above resonance. Beyond resonance the phase-shift θ will reach a maximum value and then fall off again.

With a knowledge of this phase shift between the jet current and the mouth current, we should be able to predict the frequencies for a blown pipe using the jet propagation shifts measured on the free jet. To make the examination more critical, a small amount of polyester wool was distributed along the inside of the pipe to lower the Q. The passive resonance frequencies and the Q for each of the modes were measured. The values of Q with the absorber in place were 10.7, 17.3, and 16.7 for modes $n = 1$, 2, and 3, respectively. The value of $x = 2.8$ cm was estimated from the values of the pressure contours at the points a and b in Fig. 8. The remainder of the end correction, 2.6 cm, lies outside the point a. In an actual organ pipe, the absence of the confining plates will diminish the magnitude of this additional end correction.

With these measured parameters, the phase shifts between mouth and jet currents were calculated for each mode by Eq. (2). The effective pipe length L for each mode was adjusted slightly to yield the measured resonance frequency for each mode.

Since we want the sum of the jet delay and phase delay to equal 180° (the other 180° is in our definition of mouth current direction at the slit face), these phase shifts are plotted upward from the 180° line in Fig. 4. The intersection of any blowing-velocity curve with these curves should predict the blown frequency. Examining these intersections we see for example that the 700 cm/sec blowing curve, which gave an edge-tone frequency (intersection with 180° line) of 307 Hz should sound the pipe very close to its mode-1 resonance at 250 Hz. But the 500 cm/sec blowing should give a frequency very little different than the edge tone. Just beyond 700 cm/sec, no intersection with the mode-1 phase curve is possible, and the only frequency we can expect is a jump to a 350-Hz edge tone. Further increase in blowing will raise this frequency smoothly until the mode-2 phase shifts enter around 500 Hz. Double values are permitted and observed in some regions. At 1300 cm/sec velocity, both 495 and 637 Hz are possible, but at 1500 cm/sec, mode 2 cannot sound.

These expectations are borne out by the measured behavior of the pipe. A comparison of the measured and calculated blown frequencies is made in Fig. 10. The agreement is very close, except for the beginning of mode 3, where slightly more blowing velocity is required to reach a given frequency than the model predicts. The discrepancy is in the same direction as the slight discrepancy for high frequencies in the edge-tone observations. The double-valued effects are clearly shown, and each of the first two modes terminates at its upper end quite closely as predicted. In view of the

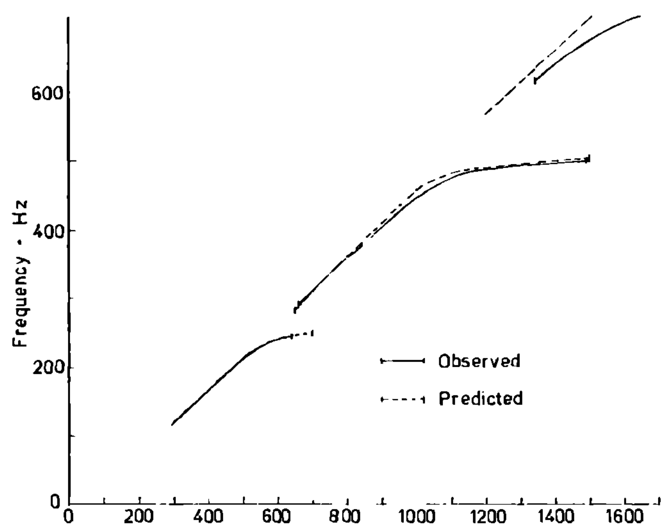

FIG. 10. Observed and predicted frequencies of oscillation for a low-Q organ pipe.

sensitivity of the phase calculations to the choice of x, and the rather rapid variation of x with the assumed geometrical position of the current sources, the correspondence is highly satisfactory.

As far as the blowing frequencies and observed phases of the acoustic wave and jet crossings are concerned, the simple concept of an alternating jet current source located a few millimeters behind the wedge tip seems to provide an adequate and consistent drive mechanism for both the edge tone and the organ pipe. For the geometry and blowing conditions used here calculations of possible maximum momentum drive effects show them to be minor in nature. Depending on just how and over what time the jet particles slow down, momentum effects may be completely negligible.

But there are some puzzles left. It would be gratifying to find that the triangular wave form described earlier resulted from an integral of the nearly square jet current wave. Unfortunately, the model calls for a derivative, not an integral. And while the amplitude of the acoustic flow in the edge tone experiment rose directly with jet flow as expected, the magnitude of the oscillating current in the pipe does not rise with jet flow in the manner one calculates from the model. Here one might fall back, as did Cremer and Ising, on entrainment, about which these observations have little to say. In spite of these questions, the observations relating the phases of the various parts of the feedback loop, and the very close predictions of edge-tone and pipe frequencies provide very strong evidence of the correctness of the jet drive mechanisms described here.

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