1 Basics of Digital Waveguide Modeling

Digital waveguide synthesis models are computational physical models for certain classes of musical instruments (string, winds, brasses, etc.) which are made up of delay lines, digital filters, and often nonlinear elements.

Digital waveguide models typically share the following characteristics:

- Sampled acoustic traveling waves
- Follow geometry and physical properties of a desired acoustic system
- Efficient for nearly lossless distributed wave media (strings, tubes, rods, membranes, plates, vocal tract, )
- Losses and dispersion are consolidated at sparse points along each waveguide

1.1 The Digital Waveguide

A lossless digital waveguide is a bidirectional delay line at some wave impedance $R$.

\[
\begin{array}{c}
\rightarrow z^{-N} \\
R \\
\leftarrow z^{-N}
\end{array}
\]

Figure 1: Sampled traveling-wave simulation for an ideal string or acoustic tube

Each delay-line element contains a sampled traveling-wave component. For example, the number in a delay cell may represent pressure in an acoustic tube or transverse displacement in a vibrating string.
1.2 Physical Outputs

Traveling waves are efficient for simulation, but they cannot be measured directly in the physical world.

Physical variables (force, pressure, velocity, ...) are obtained by summing traveling-wave components.

![Diagram of physical signal extraction from digital waveguide.]

Figure 2: Extracting a physical signal from a digital waveguide.

1.3 Signal Scattering

Signal scattering is caused by a change in wave impedance $R$ along the waveguide.

\[ k_1 = \frac{R_2 - R_1}{R_2 + R_1} \]

![Diagram of signal scattering at a junction of different wave impedances $R_1$ and $R_2$.]

Figure 3: Signal scattering at a junction of different wave impedances $R_1$ and $R_2$. The value $k_1$ is called the reflection coefficient, and it fully characterizes the scattering junction.

2 The Rigidly Terminated Ideal String

A rigid termination is the simplest case of a string termination. It imposes the constraint that the string cannot move at all at the termination. Let $y(t, x)$ denote the transverse displacement of an ideal vibrating string at time $t$, with $x$ denoting position along the length of the string. If we
terminate a length $L$ ideal string at $x = 0$ and $x = L$, we then have the “boundary conditions”

$$y(t, 0) \equiv 0 \quad y(t, L) \equiv 0$$

where “$\equiv$” means “identically equal to,” i.e., equal for all $t$.

The corresponding constraints on the sampled traveling waves are then

$$y^{+}(n) = -y^{-}(n)$$
$$y^{-}(n + N/2) = -y^{+}(n - N/2)$$

where $N$ is the time in samples to propagate from one end of the string to the other and back, or the total “string loop” delay. The loop delay is also equal to twice the number of spatial samples along the string. A digital simulation diagram for the rigidly terminated ideal string is shown in Fig. 4. A “virtual pick-up” is shown at the arbitrary location $x = \xi$.

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Note that rigid terminations reflect traveling displacement, velocity, or acceleration waves with a sign inversion. Slope or force waves reflect with no sign inversion. Since here we have displacement waves, the rigid terminations are inverting. This result may also be obtained from the reflection coefficient formula on the previous page by setting the terminating impedance to infinity in that formula.
3 Schematics for Digital Waveguide Models of Musical Instruments

Musical instruments usually also involve nonlinear and frequency-dependent (linear) scattering junctions.

3.1 Wind Instruments

Example of wind instruments include the clarinet, trumpet, flute, and organ pipe.

![Figure 5: Digital Waveguide Woodwind Instrument.]

3.2 Bowed Strings

Example of bowed-string instruments include the violin, viola, cello, and bass viol.

![Figure 6: Digital Waveguide Bowed-String Instrument.]

4 Recommended Text Books


- P.M. Morse, *Vibration and Sound*, AIP for ASA, 1976 (1st ed. 1936, 2nd ed. 1948). $18.50 from AIP:(516)349-7800x481. (Acoustics text with more musical acoustics than is typical nowadays.)


4.1 Recommended Report