Geometric Approach for Coupling Enhancement of Magnetically Coupled Coils

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***Abstract*—This paper presents a geometric approach for the enhancement of the coupling coefficient between two magnetically coupled coils. It is demonstrated that the coupling coefficient can be considerably enhanced, if the turns of the coils are not concentrated at the circumferences, but distributed across the diameters. For analysis, each of the two coils is assumed to be composed of concentric circular loops. The experimental results are in very good agreement with the theoretical results.**

I. INTRODUCTION

ONE possibility for transcutaneously providing an implanted stimulator with power and information is to transmit radio frequency (RF)-power via an inductively coupled coil system. Such a coil system consists of the primary coil which is outside the body and the secondary coil implanted with the stimulator. When facing each other, they form a transformer which allows energy transfer from the transmitter to the implant. The distance between the coils essentially determines their minimum geometric size, because with respect to the efficiency of the power transfer, the coupling coefficient between the coils has to be sufficiently high. If the coupling is too low, a higher current in the primary coil has to be used to provide the same output from the secondary coil. Thus more power is wasted in the primary coil due to I^2R-losses.

A considerable body of knowledge is available for the design of inductive links including magnetically coupled coils [1]–[7]. In medical applications an inductive link should usually fulfil two requirements. First, the RF-voltage amplitude at the secondary coil should be insensitive to variations of the relative position of the coupling coils, since this relative position is not very well defined. This property, for example, is necessary in applications where the RF-voltage at the receiver coil is used to derive an implant supply voltage which has to be kept within particular limits. Second, an optimization of the efficiency of the power transfer is often desirable. However, these two requirements cannot be met independently of each other. For example, the diameter of the secondary coil can be so chosen as to be smaller than that of the primary coil. In general, such selection reduces the coupling at a particular distance between the coils (compared to the case when the diameter of the secondary coil is equal that of the primary coil), and thereby reduces the power efficiency, but it makes the coupling coefficient more insensitive to lateral coil displacement, as long as the secondary coil remains within the diameter of the primary coil [3], [8]. This "geometric" approach aims at directly keeping the coupling coefficient constant. However, the coupling coefficient is still sensitive to variations of the coil separation.

Instead of trying to keep the coupling coefficient itself constant, the receiver RF-amplitude can be made insensitive to varying coil coupling by employing resonant circuits in the transmitter and the receiver, where the coils represent the inductances. The characteristics of these resonant circuits can be exploited to obtain a sufficiently constant receiver RF-amplitude within a defined range of coil coupling. This approach is the one which is most commonly used (e.g., [2]–[4], [6], and [7]).

Most inductive link designs found in literature employ two circular coils whose turns are concentrated at the coils' circumferences. Investigations of the coils' geometry have exclusively concentrated on calculating the coupling coefficient or the mutual inductance as a function of the ratios of the coil diameters and distance parameters [3], [5], and [8]. Attempts to enhance the coil coupling in order to reduce I^2R-losses are limited to the recommendation that ferrite backings be used (cf. [4]).

The fundamental question examined in this paper is, whether it is possible to exploit the area *within* the outer circumferences of primary and secondary coils to enhance the coupling coefficient. It turns out that the coupling coefficient can be significantly improved, if the turns of the coils are not concentrated at the outer circumferences, but distributed across the radii. This distribution can be achieved by using single concentric circular turns to construct the coils, allowing a closed-form mathematical calculation of the coupling coefficient. As shown in Fig. 1, primary and secondary coils are composed N_a and N_b circular coils, respectively. The connection between these circular coils is assumed to accomplished in such a way that:

1) the direction of the current is assumed to be equal in all the turns of a coil (and thus each turn enhances the magnitude of the mutual inductance between primary and secondary coil), and
2) the overall length of the connection paths is sufficiently shorter than the geometric dimensions of the coils, so that the influence of the current through the paths on the self and mutual inductances is negligible.

In practical applications spiral-shaped coils are more useful.

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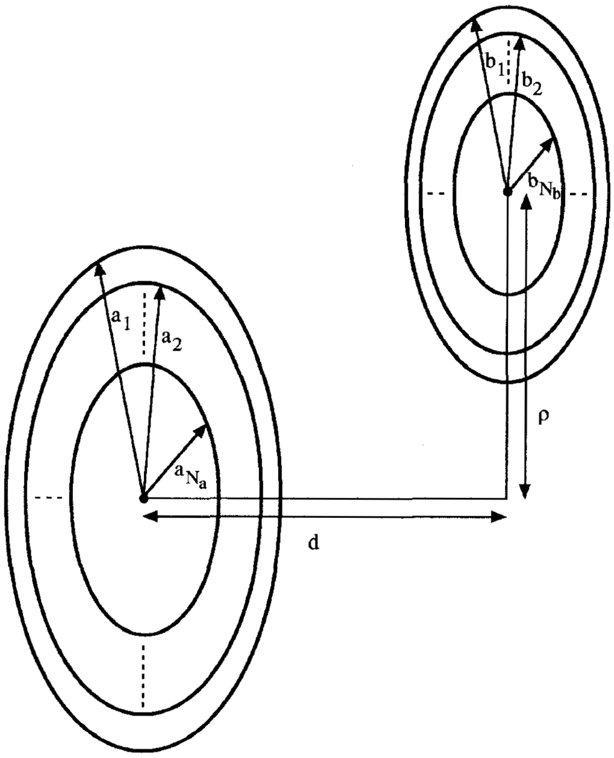


Fig. 1. Geometric arrangement and notation for primary and secondary coils composed of circular concentric loops.

However, it is much more complicated to analyze the coupling between spiral-shaped coils. It is assumed that the results derived in the following approximate the case of spiral-shaped coils sufficiently well.

The enhancement of coupling, as demonstrated in this paper, may be exploited in two ways.

1) For given (maximum) sizes of primary and secondary coils and given self-inductances, an enhancement of coupling will reduce the I^2R-losses and thus improve the power transmission efficiency of the inductive link.
2) For a given coupling coefficient, the size of primary and secondary coils can be reduced. This property enables a reduction of the overall size of an implanted device, if the size of the device is determined by the secondary coil.

II. Analysis

In general, the coupling coefficient k between two magnetically coupled coils is defined as

$$k = \frac{M_{ab}}{\sqrt{L_a L_b}} \tag{1}$$

where M_{ab} is the mutual inductance, and L_a, L_b are the self-inductances of the coils.

In the following, the relative permeability of the coil material and its surrounding medium is assumed to be $\mu_r = 1$.

Following [9], the mutual inductance of two circular air-cored loops whose axes are parallel (radii a and b, coil distance d, distance between the axes ρ) can be expressed by the single integral

$$M(a,b,\rho,d) = \pi\mu_0\sqrt{ab}\int_0^\infty J_1\left(x\sqrt{\frac{a}{b}}\right)J_1\left(x\sqrt{\frac{b}{a}}\right) \times J_0\left(x\frac{\rho}{\sqrt{ab}}\right)\exp\left(-x\frac{d}{\sqrt{ab}}\right)dx \tag{2}$$

where J_0 and J_1 are the Bessel functions of zeroth- and first-order, respectively, [9]. This expression does not contain the radius R of the coil's wire. It is assumed that the ratios $\frac{R}{a}$ and $\frac{R}{b}$ are sufficiently small (cf. [3]).

For the case of perfect alignment, i.e., $\rho = 0$, (2) leads to

$$M(a,b,\rho=0,d) = \mu_0\sqrt{ab}\left[\left(\frac{2}{\kappa} - \kappa\right)K(\kappa) - \frac{2}{\kappa}E(\kappa)\right] \tag{3}$$

where

$$\kappa \equiv \left(\frac{4ab}{(a+b)^2 + d^2}\right)^{\frac{1}{2}} \tag{4}$$

and $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kind, respectively.

Equation (3) can be used to derive a formula for the self-inductance of a single circular loop. As shown in [10], for the condition $\frac{R}{a} \ll 1$, the self-inductance of such a loop (radius a and wire-radius R) can be approximated by

$$L(a,R) = \mu_0 a\left(\ln\left(\frac{8a}{R}\right) - 2\right). \tag{5}$$

As mentioned above, primary and secondary coils employed in an inductive link usually consist of a particular number of single circular loops of approximately equal diameter. The self-inductance of such coils is approximately equal to the self-inductance of a single loop [as derived in (5)], multiplied by the square of the number of turns. For a coil composed of N_a concentric circular loops (cf. Fig. 1) with different radii, a_i $(i = 1,2,\cdots,N_a)$, and with wire-radius R, the overall self-inductance L_a becomes

$$L_a = \sum_{i=1}^{N_a} L(a_i,R) + \sum_{i=1}^{N_a}\sum_{j=1}^{N_a} M(a_i,a_j,\rho=0,d=0)(1-\delta_{i,j}) \tag{6}$$

where $\delta_{i,j} = 1$ for $i = j$, and $\delta_{i,j} = 0$ otherwise.

The mutual inductance between primary and secondary coils, M_{ab}, can be calculated using

$$M_{ab} = \sum_{i=1}^{N_a}\sum_{j=1}^{N_b} M(a_i,b_j,\rho,d). \tag{7}$$

III. Computed Results

Equations (6) and (7) allow the computation of the coupling coefficient as defined in (1). All calculations have been performed on an IBM-PC using MATLAB (The MathWorks, Inc., South Natick, MA). For convenience, the MATLAB notation is used here for the description of coil configurations. Coil "a" is described by $\mathbf{a} = [a_{\max}\colon -\Delta\colon a_{\min}]$, where the first and the

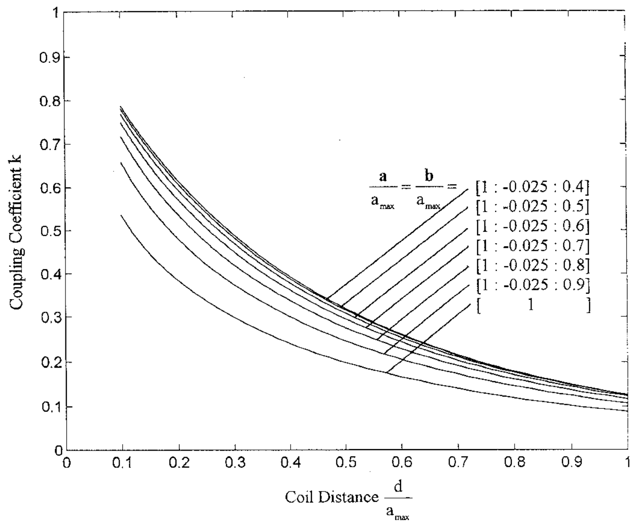


Fig. 2. Coupling coefficient k as a function of spacing for various coil configurations. Primary and secondary coils are assumed to be identical. The normalized wire radius is $\frac{R}{a_{\max}} = 12.5 * 10^{-3}$.

third number within the brackets are the radii of the maximum and minimum loops, respectively, and the second number, Δ, is the increment between the radii. With this, $a_1 = a_{\max}$ and $a_{\mathrm{Na}} = a_{\min}$. For example, coil **a** $= [1: -0.1: 0.1]$ denotes coil "a" which is composed of 10 loops with radii $1, 0.9, 0.8, \cdots, 0.2$, and 0.1.

Fig. 2 depicts the coupling coefficient k between two identical coils as a function of the normalized spacing for $\rho = 0$. Various coil configurations with different minimum radii $a_{\min} = b_{\min}$ are considered. The minimum physically possible increment between the windings $\Delta = 2R = 0.025 a_{\max}$ (with wire radius $\frac{R}{a_{\max}} = 12.5 * 10^{-3}$) is chosen here (except for the trivial case of the single turn coil configuration. Obviously, the lowest coupling coefficient is obtained for the single turn coil configuration, and the coupling coefficient is increasing for decreasing minimum coil radii. For minimum radii smaller than $0.4 a_{\max}$, the coupling coefficient remains almost unchanged.

The coupling coefficient is not very sensitive to variations of increment Δ, when radii $a_{\max}$ and $a_{\min}$ remain unchanged. For example, when equal maximum and minimum radii as in Fig. 2 are considered, and the increment chosen is $\Delta = 4R$, the relative deviation from the results shown in Fig. 2 is within $+0.3\%$ and -2.1%. For $\Delta = 8R$, the relative deviation lies between $+3.7\%$ and -0.7%. This insensitivity is a strong indication for the assumption that the coupling coefficient will also not change very much, if spiral coils (with corresponding minimum and maximum radii) are used instead of coils composed of circular turns.

Fig. 3(a), (b), and (c) depicts the coupling coefficient between two identical coils as a function of the normalized lateral displacement for three different coil distances $\frac{d}{a_{\max}} = 0.2, 0.5$, and 0.8, respectively. In each plot, four coil configurations are considered, the single-coil configuration ($\mathbf{a} = \mathbf{b} = a_{\max}$), and coils with minimum radii $\frac{a_{\min}}{a_{\max}} = \frac{b_{\min}}{a_{\max}} = 0.8, 0.6$, and 0.4. As in Fig. 2, the minimum possible increment $\Delta = 2R$ is chosen.

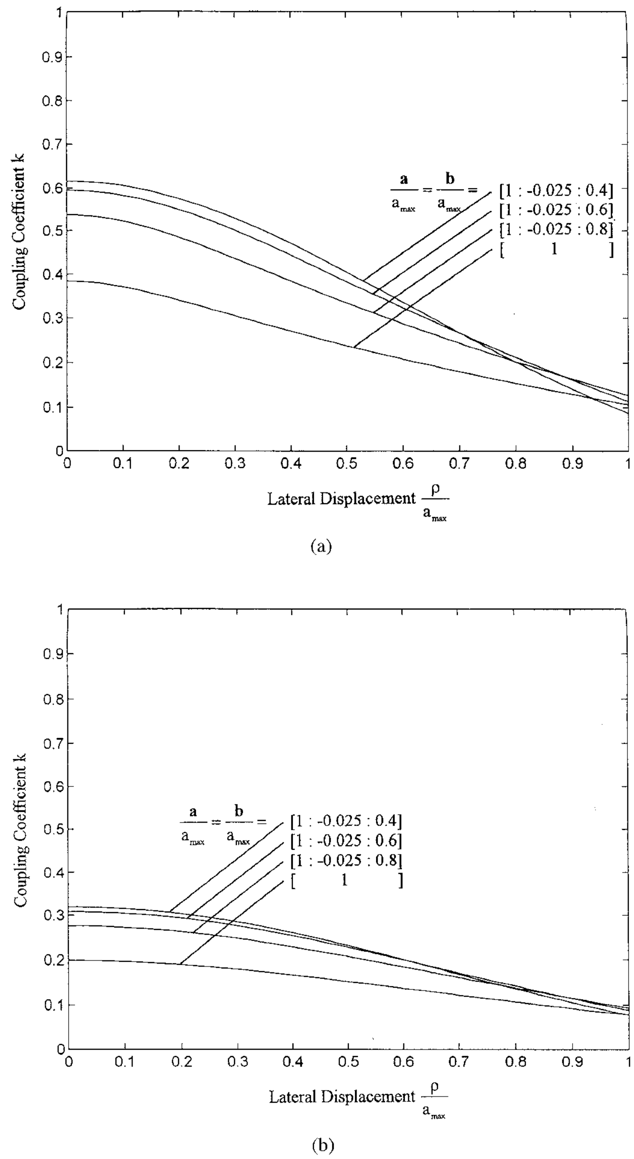
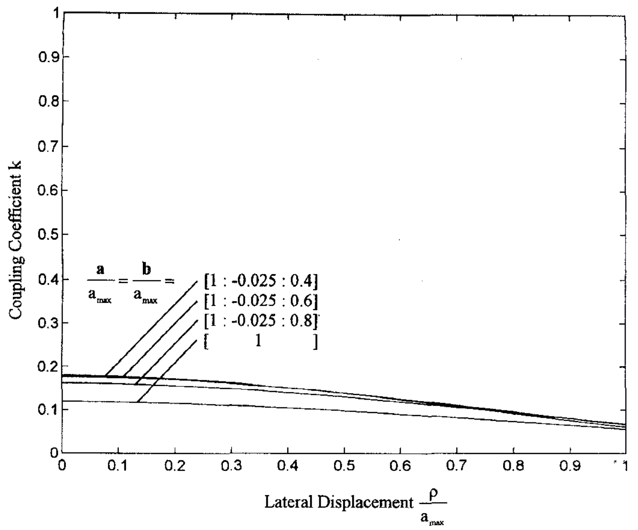


Fig. 3. Coupling coefficient as a function of the normalized lateral displacement $\frac{\rho}{a_{\max}}$ for identical primary and secondary coils. In each plot, four coil configurations $\frac{\mathbf{a}}{a_{\max}} = \frac{\mathbf{b}}{a_{\max}} = [1]$, $[1: -0.025: 0.8]$, $[1: -0.025: 0.6]$, and $[1: -0.025: 0.4]$, with increment $\Delta = 2R$ are considered. The normalized wire radius is $\frac{R}{a_{\max}} = 12.5 * 10^{-3}$. The coil distances are: (a) $\frac{d}{a_{\max}} = 0.2$, (b) $\frac{d}{a_{\max}} = 0.5$.

Fig. 3 shows that the coupling is more sensitive to lateral displacement for coils with smaller $a_{\min}$, since the coupling coefficient is considerably higher at $\rho = 0$. The coupling coefficient of all configurations is about equal at $\frac{\rho}{a_{\max}} = 1$. In many practical applications, the lateral displacement can be kept small using positioning magnets in the centre of the coils.

An intuitive explanation of the coupling enhancement of "distributed" coils is given with the help of two equal coils **a**, one primary and one secondary coil. Each of these coils shall be composed of two windings ($N_a = 2$) which are concentrated most closely to the circumferences, i.e., $\Delta = 2R$ and $\mathbf{a} = [a_{\max} \quad a_{\max} - 2R]$. Self-inductance L_0 of coils **a**



(c)

Fig. 3. (*Continued*). Coupling coefficient as a function of the normalized lateral displacement $\frac{\rho}{a_{\max}}$ for identical primary and secondary coils. In each plot, four coil configurations $\frac{\mathbf{a}}{a_{\max}} = \frac{\mathbf{b}}{a_{\max}} = [1]$, $[1{:} -0.025{:}0.8]$, $[1{:} -0.025{:}0.6]$, and $[1{:} -0.025{:}0.4]$, with increment $\Delta = 2R$ are considered. The normalized wire radius is $\frac{R}{a_{\max}} = 12.5 * 10^{-3}$. The coil distance is: (c) $\frac{d}{a_{\max}} = 0.8$.

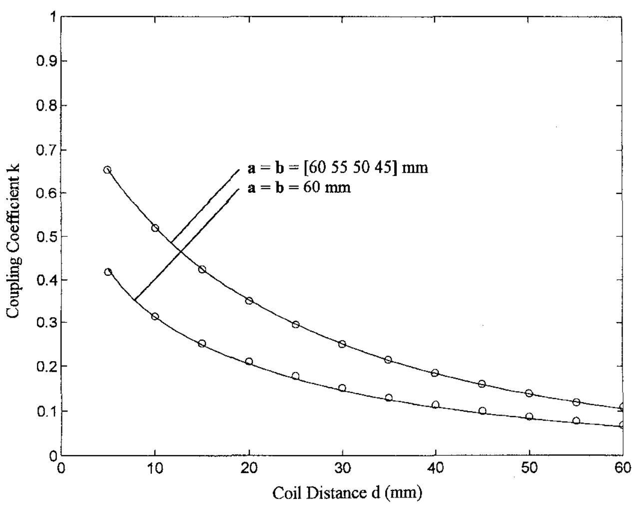
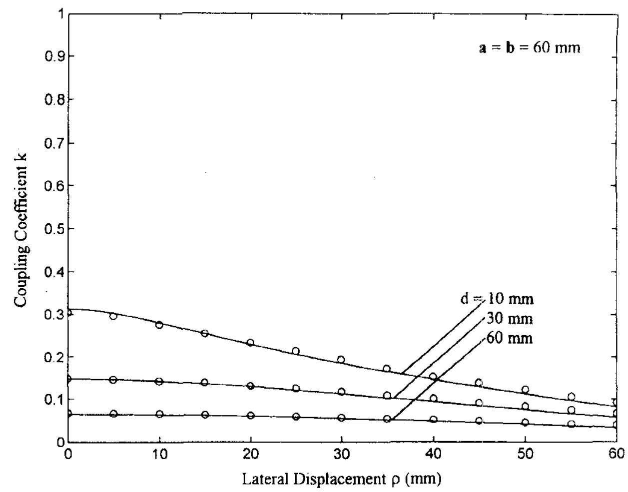


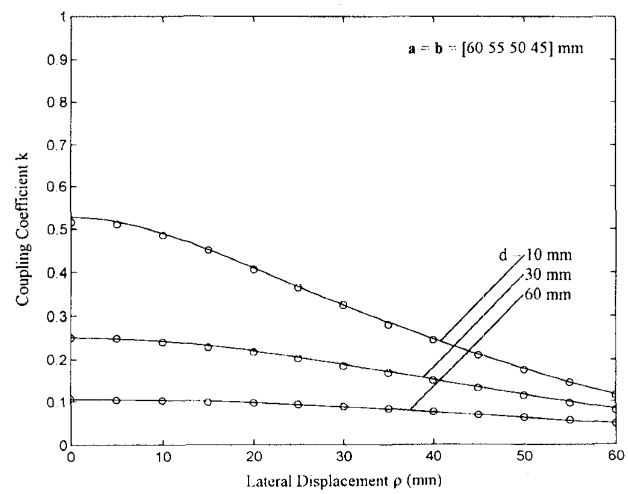
Fig. 4. Coupling coefficient as a function of the coil distance for coil configurations $\mathbf{a} = \mathbf{b} = [60 \quad 55 \quad 50 \quad 45]$ mm (upper curve) and $\mathbf{a} = \mathbf{b} = 60$ mm (lower curve). Solid curves are the computed results, circles indicate experimental data. The wire radius is $R = 0.2$ mm.

is calculated with $L_0 = L_{01} + L_{02} + 2M_{12}$, where L_{01}, L_{02} are the self-inductances of the single loops, and M_{12} is the mutual inductance between them. If the wire radius R is small compared to $a_{\max}$, then $L_{01} \approx L_{02} \approx M_{12} \approx L_{00}$, and thus $L_0 \approx 4L_{00}\,(= N_a^2 L_{00})$.

With the mutual inductance, M_0, between primary and secondary coils at a particular separation, the coupling coefficient is simply $k_0 = \frac{M_0}{L_0}$ [cf. (1)]. Now the radius of the inner turn in both coils is reduced, i.e., $\mathbf{a}' = [a_{\max} \quad a_{\max} - 2R\alpha]$, with $\alpha > 1$. This in general will reduce both, M_0 and L_0. However, the effect on L_0 will be stronger than on M_0, since especially the mutual inductance M_{12} between the single turns is rapidly decreasing, and thus the coupling factor k_0, as defined above, is increased.



(a)



(b)

Fig. 5. Coupling coefficient as a function of the lateral displacement for coil separations $d = 10, 30$, and 60 mm. Solid curves are the theoretical results, circles indicate experimental data. The wire radius is $R = 0.2$ mm: (a) Single-turn coils $\mathbf{a} = \mathbf{b} = 60$ mm and (b) "Distributed" coils $\mathbf{a} = \mathbf{b} = [60 \quad 55 \quad 50 \quad 45]$ mm.

IV. Experimental Results

The theoretical considerations presented above were verified by experiment. Measurements were performed with copper coils attached to a construction made of plexiglass. The wire radius was $R = 0.2$ mm. For measurement, a sinusoidal voltage with an amplitude of $u_0 = 1V_p$ and a frequency of $f_0 = 2$ MHz was applied to the primary coil, and the voltage at the unloaded secondary coil was detected. Equal coil configurations for primary and secondary coils were investigated. In this case, the coupling coefficient is simply

TABLE I
DISTRIBUTED COIL CONFIGURATIONS WITH $a_{\max} = 12$ mm TO IMPLEMENT COILS WITH SELF-INDUCTANCE $L_A = 0.88$ μH ($R = 0.125$ mm)

Number of Turns N_a	Minimum Radius a_{min}(mm)	Wire Length l_a(mm)	Relative Unloaded Quality Reduction $\Delta Q_L(\%)$
4	11.25	294	0
5	9.11	337	−12.8
6	6.79	365	−19.5
7	4.79	384	−23.4
8	3.09	397	−25.9
9	1.66	407	−27.8

equal to the amplitude ratio of secondary and primary voltage (this can be easily verified by using the T-equivalent circuit of two coupled coils).

Fig. 4 depicts the coupling coefficient as a function of the coil distance for $\rho = 0$. Two coil configurations $\mathbf{a} = \mathbf{b} = [60 \quad 55 \quad 50 \quad 45]$ mm and $\mathbf{a} = \mathbf{b} = 60$ mm are investigated. As expected, a considerably enhanced coupling coefficient is obtained for the "distributed" coils. For example, at $d = 40$ mm, the coupling for the "nondistributed" and "distributed" coils are $k_{\text{nondist}} = 0.109$ and $k_{\text{dist}} = 0.184$, respectively (theoretical values), which means an enhancement by 68.8%. The experimental data are in very good agreement with the theoretical results.

Fig. 5 shows the coupling coefficient as a function of the lateral displacement for various coil distances $d = 10, 30$, and 60 mm. Again, the values measured are very close to the calculated results.

V. DISCUSSION

In the previous sections of this paper it has been shown that distributing the turns of coils across the radii considerably enhances the coupling coefficient. The question arises, how this improvement can be exploited in practical applications, either to enhance the power transfer efficiency, or to reduce the geometric size of the coils.

Let us consider a typical inductive link for transcutaneous power and data transfer as shown in Fig. 6(a). This link consists of a parallel tuned transmitter and a parallel tuned receiver circuit, which are magnetically coupled. Resistor R_1 represents the output resistance of the RF-amplifier driving the link, and resistor R_2 is the load. Resistors R_{s1} and R_{s2} are the series resistances of transmitter and receiver coils, respectively. For convenience, circuit Fig. 6(a) is rearranged as shown in Fig. 6(b), where the coil losses are represented by the parallel resistors R_{p1} and R_{p2}. Series-and parallel coil resistances are related via

$$R_{pi} = \frac{(\omega L_i)^2}{R_{si}} \tag{8}$$

with angular frequency ω, and index $i = 1, 2$.

The center frequency $\omega_0 = \omega_{01} = \omega_{02}$ of transmitter and receiver resonant circuits is defined as

$$\omega_0 = \frac{1}{\sqrt{L_i C_i}}. \tag{9}$$

The overall loaded qualities, Q_i, are defined via the relation

$$\frac{1}{Q_i} = \frac{1}{R_i}\sqrt{\frac{L_i}{C_i}} + \frac{1}{R_{pi}}\sqrt{\frac{L_i}{C_i}} = \frac{1}{Q_{Ri}} + \frac{1}{Q_{Li}} \tag{10}$$

where the qualities Q_{Ri} and Q_{Li} represent the qualities of the resonant circuits either exclusively due to resistances R_i, or exclusively due to resistances R_{pi}, respectively. Qualities Q_{Li} are often called the unloaded qualities of the coils.

Following [3], efficiency η_{12} as the ratio between the power delivered to the receiver circuit and the overall power consumption is given by

$$\eta_{12} = \frac{1}{1 + \frac{1}{k^2 Q_1 Q_2}} \tag{11}$$

where k is the coupling coefficient. For this equation it is assumed that the system is operated at the center frequency, i.e., $\omega = \omega_0$.

Within the receiver circuit, the power is split up between the load, R_2, and the parallel coil resistor, R_{p2}. This is described by efficiency η_{22} defined as

$$\eta_{22} = \frac{1}{1 + \frac{R_2}{R_{p2}}} = \frac{1}{1 + \frac{Q_{R2}}{Q_{L2}}} \tag{12}$$

with Q_{R2} and Q_{L2} from (10). Thus, the overall efficiency, η, as the ratio between the power in load R_2 and the overall power consumption is given by

$$\begin{aligned}\eta &= \eta_{12}\eta_{22} \\ &= \frac{1}{\left(1 + \frac{1}{k^2}\left(\frac{1}{Q_{L1}} + \frac{1}{Q_{R1}}\right)\left(\frac{1}{Q_{L2}} + \frac{1}{Q_{R2}}\right)\right)} \frac{1}{\left(1 + \frac{Q_{R2}}{Q_{L2}}\right)}.\end{aligned} \tag{13}$$

Obviously, efficiency η keeps increasing with increasing coupling coefficient k, and therefore any coupling enhancement—at a particular coil separation—results in an improved efficiency. It can also be seen that for a high efficiency the unloaded qualities Q_{Li} should be as high as possible. As suggested in this paper, a coupling enhancement is achieved by using "distributed" coils. However, the distribution of the coil windings across the radii results in a particular reduction of the unloaded qualities. This is because the wire lengths necessary to achieve a particular inductance for the "distributed" coils—and with it the series resistances—are somewhat greater than for coils, where the windings are concentrated at the circumferences. Looking at (8) and (10) it can be seen, that the series resistances are inversely proportional to the unloaded qualities. The wire length l_a of coil $\mathbf{a}$ composed of N_a turns can be calculated by

$$l_a = 2\pi \sum_{i=1}^{N_a} a_i + 2(a_{\max} - a_{\min}) \cdots \tag{14}$$

where the term $2(a_{\max} - a_{\min})$ represents the overall wire length necessary to connect the circular turns of the coil with each other. A typical example is demonstrated with the help of Table I. Coil $\mathbf{a} = [12{:} -0.25{:}\, 11.25]$ mm with $R = 0.125$ mm is composed of $N_a = 4$ turns which are concentrated most closely to the circumference (outer radius: $a_{\max} = 12$

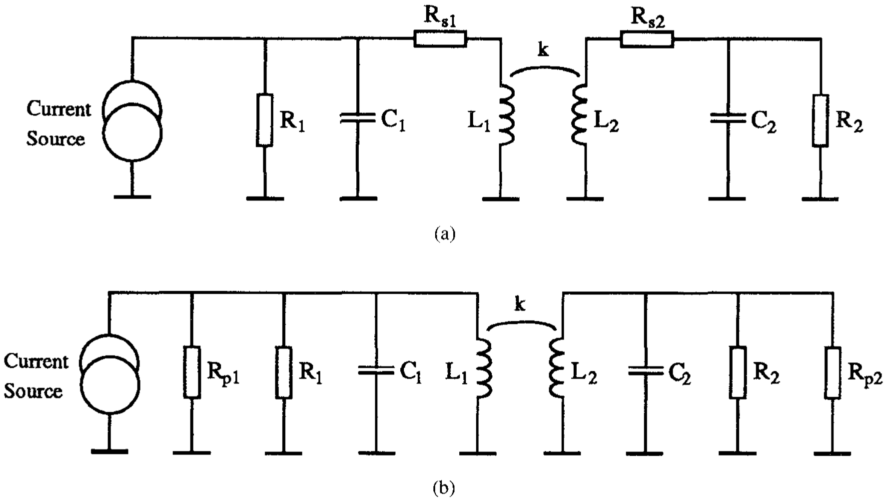


Fig. 6. (a) Circuit diagram of an inductive RF-link and (b) approximated circuit.

mm, inner radius: $a_{\min} = 11.25$ mm). The resulting self-inductance is $L_a = 0.88$ μH. The same self-inductance can be achieved with $N_a = 5, 6, 7, 8$, or 9 turns (first column), where the distances between the turns are increasing and the minimum radii $a_{\min}$ are decreasing (second column). Columns 3 and 4 depict the resulting wire lengths [according to (14)] and the relative reduction of the unloaded quality ΔQ_L in percent referred to the coil composed of four turns.

To summarize, "distributed" coils yield, on the one hand, an increased coupling between transmitter and receiver coils, but reduce the unloaded qualities on the other. Nevertheless, a net improvement of the overall efficiency can be obtained, as demonstrated with the help of Fig. 7. Here, the overall efficiency η is computed for three different implementations of transmitter and receiver coils (for each case, transmitter and receiver coils are identical ($\mathbf{a} = \mathbf{b}$) with inductances $L_a = L_b = 0.88$ μH (wire radius $R = 0.125$ mm). The qualities are chosen $Q_{R1} = 20$ and $Q_{R2} = 5$.

Case 1: Coils with $a_{\max} = 12$ mm, $a_{\min} = 11.25$ mm composed of $N_a = 4$ turns are assumed (cf. first row in Table I), and the unloaded qualities are assumed to be $Q_L = Q_{L1} = Q_{L2} = 80$.

Case 2: Coils with the same inductances as in Case 1, but with $N_a = 6$ turns (third row in Table I) are selected, resulting in increased wire-lengths of the coils and thus in reduced unloaded qualities $Q_L = Q_{L1} = Q_{L2} = 80 * 0.805 = 64.4$. Nevertheless, the overall efficiency is considerably higher than in Case 1. For example, at $d = 10$ mm, $\eta = 0.52$ in Case 1, and $\eta = 0.61$ in Case 2, which means a relative improvement of 17.3%.

Case 3: Here the calculation of η is based on a coupling coefficient k derived from single turn coils with radii $\mathbf{a} = \mathbf{b} = 12$ mm. It is assumed that for both coils $N = 3.55$ turns are concentrated *at the same radius* to achieve equal inductances as in Cases 1 and 2. This assumption might be of theoretical interest, since it represents the most extreme (but nonrealizable) "concentration" of turns and shows—at least for the present example—the greatest possible improvement of "distributed" coils compared to "concentrated" coils. In this case, the wire length is $N * 12 * 2 * \pi = 267$ mm, which means a length reduction of $\Delta l_a = -8.8\%$ and thus a slight enhancement of the unloaded qualities $Q_L = Q_{L1} = Q_{L2} = \frac{80}{0.91} = 87.9$. The overall efficiency is clearly below the efficiencies of Cases 1 and 2. At $d = 10$ mm, $\eta = 0.45$ in Case 3, which means an efficiency reduction of -13.5% referred to the corresponding efficiency in Case 1.

So far, only the overall efficiency of the power transfer of the inductive link Fig. 6 has been considered. The ideas presented in this paper may also be exploited to reduce the overall size of transmitter and receiver coils in applications, where the voltage at the receiver coil has to be insensitive to coupling variations. Following [3], a maximum of the receiver voltage as a function of coupling coefficient k occurs at "critical coupling," i.e., when $k\sqrt{Q_1 Q_2}$ equals one. At critical coupling, efficiency $\eta_{12} = 0.5$ (note, that η_{12} is the efficiency of the power transfer from the transmitter to the receiver circuit). Assuming the same circuit parameters as in Case 1 of above (i.e., $L_a = L_b = 0.88$ μH, $R = 0.125$ mm, $Q_{R1} = 20$, $Q_{R2} = 5$, and coils with $a_{\max} = b_{\max} = 12$ mm, $a_{\min} = b_{\min} = 11.25$ mm, $N_a = 4$, $Q_L = 80$), critical coupling occurs at a distance $d_{\text{crit}} = 10.8$ mm. The same distance for critical coupling and thus for efficiency $\eta_{12} = 0.5$ is achieved for coils $a_{\max} = b_{\max} = 10.3$ mm, $a_{\min} = b_{\min} = 6.8$ mm, $N_a = 6$. This means that the outer radii can be reduced from 12 mm to 10.3 mm, which corresponds to a size reduction of the coils by -14%.

VI. Conclusion

This paper presents a geometric approach for enhancing the coupling between two magnetically coupled coils. This enhancement is achieved by distributing the turns of the coils across the radii instead of concentrating them at the outer

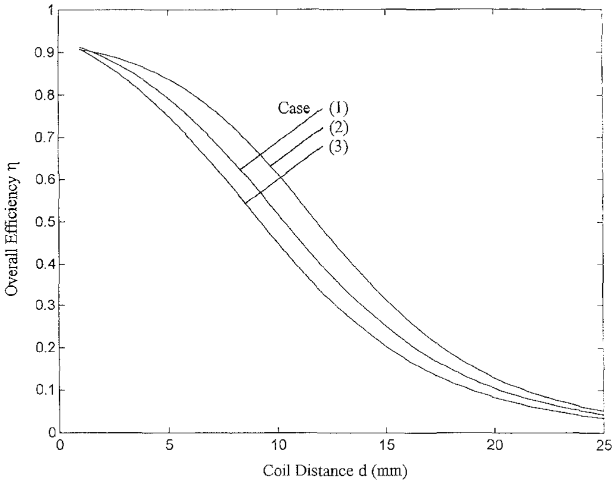


Fig. 7. Overall efficiency η as a function of coil separation d. For all cases, transmitter and receiver coils are identical, and $L_a = L_b = 0.88\ \mu$H (wire radius $R = 0.125$ mm), and qualities $Q_{R1} = 20$ and $Q_{R2} = 5$. Case 1: $a_{\max} = b_{\max} = 12$ mm, $a_{\min} = b_{\min} = 11.25$ mm, $N_a = 4$, $Q_L = 80.0$; Case 2: $a_{\max} = b_{\max} = 12$ mm, $a_{\min} = b_{\min} = 6.79$ mm, $N_a = 6$, $Q_L = 64.4$; Case 3: $a_{\max} = b_{\max} = a_{\min} = b_{\min} = 12$ mm, $N_a = 3.55$, $Q_L = 87.9$.

circumferences. As shown in Section III for identical primary and secondary coils, for given maximum and minimum coil radii, the coupling of distributed coils is insensitive to the spacing between the windings of the coils. Thus, in practical applications, the self-inductances can be chosen according to the requirements of the link, relatively independent from the coupling coefficient.

The price paid for such a coupling enhancement are moderate decreases of the unloaded qualities of the coils due to increased wire lengths (assuming equal inductances and equal outer radii of concentrated and distributed coils). Typically, the unloaded qualities are reduced by about 20%. However, in most applications, the coils are parts of transmitter and receiver resonant circuits. If the unloaded qualities are sufficiently high compared to loaded qualities, the effect of coupling enhancement on the overall efficiency η predominates over the effect of unloaded quality reduction and thus a net improvement of overall efficiency is obtained.

The results presented here are calculated for identical primary and secondary coil. However, the formulas derived can be applied for arbitrary coil configurations.

Some experiments have been performed to verify the theoretical results. The experimental data are in very good agreement with the predicted values.

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