Practical Stability and Bounds of Heterogeneous AIMD/RED System with Time Delays

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***Abstract*—The Additive Increase and Multiplicative Decrease (AIMD) congestion control algorithm of TCP protocol deployed in the end systems and the Random Early Detection (RED) queue management scheme deployed in the intermediate systems contribute to Internet stability and integrity. Previous research based on the fluid-flow model analysis indicated that an AIMD/RED system may not be asymptotically stable when the feedback delays or the link capacity becomes large [3]. However, as long as the system operates near its desired equilibrium, small oscillations are acceptable and the network performance is still satisfactory. Deriving the bounds of these oscillations for the heterogeneous AIMD/RED system with time delays is non-trivial. In this paper, we study the practical stability of the AIMD/RED system with heterogeneous flows and feedback delays, and obtain theoretical bounds of the AIMD flow window size and the RED queue length, as functions of number of flows, link capacity, RED queue parameters, and AIMD parameters. Numerical results with Matlab and simulation results with NS-2 are given to validate the correctness of the theorems and demonstrate the tightness of the derived bounds. The analytical and simulation results provide important insights on which system parameters contribute to higher oscillations of the system and how to set system parameters to ensure system efficiency with bounded delay and loss.**

***Index Terms*—Practical stability, bounds estimate, heterogeneous AIMD/RED system, time delay system.**

I. INTRODUCTION

Internet stability depends on the Transmission Control Protocol (TCP), which is voluntarily deployed in the end system based on the Additive Increase and Multiplicative Decrease (AIMD) congestion control mechanism. On the other hand, the active queue management (AQM) algorithms, such as Random Early Detection or Random Early Discard (RED), have been developed and deployed in the intermediate systems to fairly distribute network congestion signals to all on-going flows, which further improve TCP and network performance. AIMD and RED both contribute to the overwhelming success of the Internet. With the rapid advances in optical and wireless communications, the Internet is becoming a more diverse network with higher data rate, a larger number of flows, supporting heterogeneous applications. It is important to understand whether an AIMD/RED system can be stable, scalable, and efficient for future more diversified Internet.

Different from many previous work [1], [2], [3], [4], [14] on the sufficient conditions for the asymptotic stability of AIMD/RED or other network control systems, in this paper, we study the practical stability of the AIMD/RED system, and derive its theoretical bounds. The definitions of boundedness and stability are listed below, which follow those in [17], [18].

Definition 1: Consider the dynamic system with time delays

$$\frac{dx}{dt} = f(t, x(t), x(t-\tau_1(t)), \cdots, x(t-\tau_m(t)))$$

where $x \in R^n$, $f: I \times R^n \times R^n \times \cdots \times R^n \to R^n$ is continuous. Let $\tau = \sup_{i=1,..,m} \tau_i(t)$. The solutions of the system are said to be

- *uniformly bounded* if there exists a constant c, for every $a \in (0, c)$, there is $B = B(a) > 0$, such that for any $\xi(t) \in C[[t_0-\tau, t_0], R^n]$, $\|x(t, t_0, \xi)\| \leq B$ for all $t \geq t_0$ when $\|\xi\| \leq a$.

The trivial solution of the above system is said to be

- *stable* if for every $\epsilon > 0$ and $t_0 \in \mathbb{R}_+$, there exists some $\delta = \delta(t_0, \epsilon) > 0$ such that for any $\xi(t) \in C[[t_0-\tau, t_0], R^n]$, $\|\xi\| < \delta$ implies $\|x(t, t_0, \xi)\| < \epsilon$ for all $t \geq t_0$;
- *asymptotically stable* if the system is stable and for every $t_0 \in \mathbb{R}_+$, there exists some $\eta = \eta(t_0) > 0$ such that $\lim_{t\to\infty} \|x(t, t_0, \xi)\| = 0$ whenever $\|\xi\| < \eta$;
- *practically stable* if given (λ, A) with $0 < \lambda < A$, we have, for any $\xi(t) \in C[[t_0-\tau, t_0], R^n]$, $\|\xi\| < \lambda$ implies $\|x(t, t_0, \xi)\| < A$, $t \geq t_0$ for some $t_0 \in \mathbb{R}_+$.

It has been pointed out that an AIMD/RED system may not be asymptotically stable when the delay or the link capacity becomes large [3]. However, even if the system as a whole is not asymptotically stable, as long as the end systems do not overshoot the available bandwidth too severely, the overall system efficiency can still be very high, and the packet loss rate and queuing delay can still be well bounded. In other words, if the system oscillates sufficiently close to the desired operating point, its performance is still acceptable. Therefore, it is critical to investigate that, does the AIMD/RED system always operate in the area close to the desired equilibrium state, and what are the theoretical bounds? To answer these questions, studying system practical stability and bounds is the key, which is also the focus of this paper.

With clearly defined bounds, a system is considered practically stable. The bounds can be used as a guideline to set up the AIMD/RED system parameters to enhance system performance. The boundedness issue for some TCP-like congestion control algorithms has been studied in [6], [7], [8] by applying Lyapunov-like method. Shakkottai and Srikant justified the use of the deterministic model for Internet congestion control in [9], and in [5], the upper bound on the transmission rate for two types of TCP-like traffic were given. However, to the best of our knowledge, the theoretical bounds of congestion window size and bottleneck queue length of heterogeneous AIMD/RED systems considering feedback delays have not been reported in the literature. Because of the heterogeneity of the Internet, understanding the stability properties and bounds of the AIMD/RED system with heterogeneous flows is critical for future network planning and design.

Using the fluid-flow model of the heterogeneous AIMD/RED system, instead of applying the Lyapunov-like method, we derive upper and lower bounds of congestion window size and queue length by directly studying the inherent properties of the AIMD/RED system. The derived theoretical bounds provide important insights on which system parameters contribute to high oscillations of the system and how to choose system parameters to ensure system efficiency with bounded delay and loss. The theorems given in the paper can also help to predict the system performance for the future Internet with higher capacity and more flows with different flow parameters.

The remainder of the paper is organized as follows. Sec. II introduces the fluid model of the heterogeneous AIMD/RED system. Sec. III derives the upper and lower bounds of the AIMD/RED system with feedback delays. In Sec. IV, numerical results with Matlab and simulation results using NS-2 are presented to validate the derived bounds, followed by concluding remarks in Sec. V.

II. A FLUID-FLOW MODEL OF HETEROGENEOUS AIMD/RED SYSTEM WITH TIME DELAYS

A stochastic model of TCP/RED was developed using fluid-flow and stochastic differential equations in [10]. We extend the fluid-flow model for general AIMD(α, β) congestion control: the window size is increased by α packet per round-trip time (RTT) if no packet loss occurs; otherwise, it is reduced to β times its current value. The general AIMD congestion control has been proposed to support heterogeneous applications with different tolerance on flow throughput variations [11], [12], [13], [14]. TCP is a special case of AIMD with $\alpha = 1$ and $\beta = 0.5$.

We consider the case when there are two classes of flows with parameters (α_1, β_1), (α_2, β_2), time-invariant traffic loads N_1, N_2, respectively, as depicted in Fig. 1. We assume that all the flows have the same round-trip time. The model in this section can be extended to any certain number of flows in multiple classes with heterogeneous AIMD parameters and feedback delays.

With a RED queue, the packet dropping or marking probability, p, is determined by the average queue length q_{act}:

$$p = \begin{cases} 0 & 0 \le q_{act} \le \min_{th} \\ K_p(q_{act} - \min_{th}) & \min_{th} < q_{act} \le \max_{th} \\ 1 & q_{act} > \max_{th} \end{cases}$$

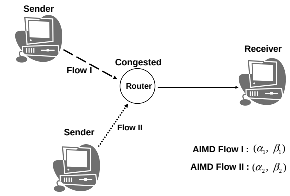

Fig. 1. Heterogeneous AIMD/RED System

where $K_p > 0$. When $q_{act} \le \min_{th}$, $\frac{dW(t)}{dt} = \frac{\alpha}{R}$, the window size of AIMD flows will keep increasing and will not converge to any value. Thus, in the following, we will discuss the stability of this model when $q_{act} > \min_{th}$. Without loss of generality, let $q(t) = q_{act}(t) - \min_{th}$. In addition, since the queue behaves the same as a Drop-Tail queue once q_{act} exceeds $\max_{th}$, we choose $\max_{th}$ to be sufficiently large such that $K_p(\max_{th} - \min_{th}) = 1$.

Taking time delays into consideration, a heterogeneous AIMD/RED system shared by two classes of flows can be modeled as

$$\frac{dW_I(t)}{dt} = \frac{\alpha_1}{R(t)} - \frac{2(1-\beta_1)}{1+\beta_1}\frac{W_I(t)W_I(t-R(t))}{R(t-R(t))}K_p q(t-R(t)),$$

$$\frac{dW_{II}(t)}{dt} = \frac{\alpha_2}{R(t)} - \frac{2(1-\beta_2)}{1+\beta_2}\frac{W_{II}(t)W_{II}(t-R(t))}{R(t-R(t))}K_p q(t-R(t)),$$

$$\frac{dq(t)}{dt} = \begin{cases} \frac{N_1 W_I(t)}{R(t)} + \frac{N_2 W_{II}(t)}{R(t)} - C, & q > 0, \\ \{\frac{N_1 W_I(t)}{R(t)} + \frac{N_2 W_{II}(t)}{R(t)} - C\}^+, & q = 0. \end{cases} \quad (1)$$

where $\{a\}^+ = \max\{a, 0\}$, $\alpha > 0$, $\beta \in (0, 1)$; W_i is the ensemble average of AIMD congestion window size (in the unit of packets) of flow of class i, $i = I, II$; q is the ensemble average of queue length; $R(t)$ is the round-trip time with $R(t) = \frac{q(t)}{C} + T_p$ (secs) where C is the link capacity (packets/sec) and T_p is the deterministic round-trip delay; $p(t) \in [0, 1]$ is the probability of a packet being marked or dropped. It should be noted that, in the fluid model, q and W are positive and bounded quantities; i.e., $W_i \in [1, W_{\max}]$ and $q \in [0, q_{\max}]$ where $q_{\max}$ and $W_{\max}$ denote buffer size and maximum window size, respectively.

With ever-increasing link capacity and appropriate congestion control mechanisms, variation of queuing delays becomes negligible compared to the round-trip delays [2], [4]. We thus ignore the effect of the delay jitter on the round-trip time and assume that the round-trip time of each flow is a constant, $R(t) = R$. It is shown in [2] that $W_i(t)W_i(t-R)$ in (1) can be approximated by $W_i^2(t)$ for $i = I, II$ when the window size is much larger than one.

For the heterogeneous system (1), the equilibrium point (W_I^*, W_{II}^*, q_0^*) is given by

$$W_I^* = \frac{GCR}{N_1 G + N_2}, W_{II}^* = \frac{CR}{N_1 G + N_2}, q_0^* = \frac{\alpha_1(1+\beta_1)}{2(1-\beta_1)W_I^{*2} K_p},$$

where $G = \sqrt{\frac{\alpha_1(1+\beta_1)(1-\beta_2)}{\alpha_2(1-\beta_1)(1+\beta_2)}}$.

Remarks: At the equilibrium, the total arrival rate equals the total link capacity, so the link bandwidth is fully utilized. If the total window size is larger than $N_1W_I^* + N_2W_{II}^*$, the queue will build up, which results in a longer queuing delay; if the total window size is less than $N_1W_I^* + N_2W_{II}^*$, the network load is smaller than its capacity, i.e., the network resources are not fully utilized. In conclusion, the equilibrium point is the most desired operating point of the system.

III. Practical Stability and Bounds of Heterogeneous AIMD/RED System with Time Delays

It was demonstrated in [3] that an AIMD/RED system becomes *asymptotically* unstable with the increase of round-trip delays of the system. Using the fluid model, sufficient conditions for the asymptotic stability of the AIMD/RED system with feedback delays were derived in [14]. In this paper, we show that even though the system may become asymptotically unstable because of the effects of time delays, its window size and queue length are still bounded, and in most cases, the upper bounds are close to their equilibria.

In this section, we study the delayed heterogeneous AIMD/RED system as defined by (1). As mentioned, variation of queuing delays becomes relatively small to propagation delays because of the ever-increasing link capacity and appropriate congestion control mechanisms. It is revealed in [16] that the variable nature of RTT due to queuing delay variation helps to stabilize the TCP/RED system. In light of this, we derive upper and lower bounds of the AIMD/RED system assuming RTT to be constant. These results will be good approximations even if RTT is slightly time-varying.

Notice that the AIMD/RED system defined by (1) are described by delayed differential equations. Its initial conditions are given by $1 \le W_i(t) \le W_i^*$ for $i = I, II$, and $0 \le q(t) \le q_0^*$ on the interval $t \in [-R, 0]$.

In (1), we take $\bar{W}(t) = N_1 \cdot W_I(t) + N_2 \cdot W_{II}(t)$, $M_1 = \frac{(1-\beta_1)}{1+\beta_1}$, $M_2 = \frac{(1-\beta_2)}{1+\beta_2}$, $r_1 = M_1/N_1$, and $r_2 = M_2/N_2$, then

$$\begin{aligned}\dot{\bar{W}} &= (N_1\alpha_1 + N_2\alpha_2)/R \\ &- 2[r_1 \cdot (N_1W_I)^2(t) + r_2 \cdot (N_2W_{II})^2(t)] \cdot K_p q(t-R)/R.\end{aligned} \tag{2}$$

Notice that $W_i(t) \ge 0$ for $i = I, II$ and take $r_{min} = \min(r_1, r_2)$, $r_{max} = \max(r_1, r_2)$, the following inequality can be obtained:

$$-2r_{max}\frac{\bar{W}^2(t)}{R} \le \frac{\dot{\bar{W}}(t) - \frac{N_!\alpha_1 + N_2\alpha_2}{R}}{K_p q(t-R)} \le -r_{min}\frac{\bar{W}^2(t)}{R}. \tag{3}$$

Also, we have

$$\dot{q}(t) = \begin{cases} \bar{W}(t)/R - C, & q > 0, \\ \{\bar{W}(t)/R - C\}^+, & q = 0. \end{cases} \tag{4}$$

Thus, with the new variable pair $(\bar{W}(t),\ q(t))$, the original heterogeneous AIMD/RED system (1) can be rewritten by (2) and (4). We will study the properties of $(\bar{W}(t),\ q(t))$ in the following to show the practical stability and derive the bounds of the system.

Remarks: Our focus in the analysis below is $\bar{W}(t)$, the total window size at t. This is because $\bar{W}(t)$ indicates the entire throughput of the heterogeneous AIMD/RED system, which is more useful than the throughput of each individual flow.

A. Upper Bound on Window Size

Theorem 1: Let $U_B > 0$ be the largest real root of

$$U_B{}^2 \cdot [U_B - R \cdot C - (N_1\alpha_1 + N_2\alpha_2)]^2 = \frac{4(N_1\alpha_1 + N_2\alpha_2)^2}{r_{min} \cdot K_p},$$

then $\bar{W}(t) \le U_B$ for $t \ge 0$.

Proof: With (2), $\dot{\bar{W}}(t) \le (N_1\alpha_1 + N_2\alpha_2)/R$ for $t \ge 0$. For $\tau > 0$, take integration on both sides from $t - \tau$ to t:

$$\bar{W}(t) - \bar{W}(t-\tau) \le (N_1\alpha_1 + N_2\alpha_2) \cdot \tau/R. \tag{5}$$

We show that $U_B > 0$ in the theorem is an upper bound of $\bar{W}(t)$ for $t \ge 0$, i.e., if $\bar{W}(t) = U_B$ for some $t = t_1 \ge 0$, then $\dot{\bar{W}}(t_1) \le 0$.

Integrating on both sides of (4) from $t_1 - a \cdot R$ to $t_1 - R$ for $a > 1$ gives

$$\int_{t_1-aR}^{t_1-R} \dot{q}(s)ds \quad \ge \quad \frac{1}{R}\int_{t_1-aR}^{t_1-R} \bar{W}(s)ds - (a-1)R \cdot C.$$

Note that (5) implies $\bar{W}(t_1 - \tau) \ge U_B - a \cdot (N_1\alpha_1 + N_2\alpha_2)$ when $\tau \in [R,\ aR]$. Thus,

$$q(t_1 - R) \ge [U_B - a \cdot (N_1\alpha_1 + N_2\alpha_2)] \cdot (a-1) - R \cdot C \cdot (a-1), \tag{6}$$

since $q(t) \ge 0$.

Taking $f(a) = (a-1) \cdot [U_B - a \cdot (N_1\alpha_1 + N_2\alpha_2) - R \cdot C]$ and computing the maximum value of $f(a)$ by letting $f'(a) = 0$ gives

$$f(a) = [U_B - R \cdot C - (N_1\alpha_1 + N_2\alpha_2)]^2/[4(N_1\alpha_1 + N_2\alpha_2)], \tag{7}$$

with $a = [U_B - R \cdot C + (N_1\alpha_1 + N_2\alpha_2)]/[2(N_1\alpha_1 + N_2\alpha_2)]$ and $f''(a) < 0$.

Therefore, it follows from (3), (6) and (7) that, $\dot{\bar{W}}(t_1) \le 0$ if U_B satisfies

$$U_B{}^2 \cdot [U_B - R \cdot C - (N_1\alpha_1 + N_2\alpha_2)]^2 = \frac{4(N_1\alpha_1 + N_2\alpha_2)^2}{r_{min} \cdot K_p}, \tag{8}$$

which implies $\bar{W}(t) \le U_B$ for $t \ge 0$. ■

By the continuity property of $U_B{}^2 \cdot [U_B - R \cdot C - (N_1\alpha_1 + N_2\alpha_2)]^2$ and the fact that the RHS of (8) is always greater than zero, we can conclude that there exists at least one real root for (8) and the largest root must be greater than $R \cdot C + (N_1\alpha_1 + N_2\alpha_2)$. Therefore, the upper bound U_B itself will increase with the increment of $R \cdot C$ and $(N_1\alpha_1 + N_2\alpha_2)$. In addition, the oscillation of the window size from its equilibrium value will increase with the increment of $N_1\alpha_1 + N_2\alpha_2$ and the decrement of K_p.

It is also noted that the upper bound derived in Theorem 1 is global for the time t, i.e., the window size $\bar{W}(t)$ will not go above U_B for any $t > t_1$. If we assume, instead, that there exists $t_1' > t_1$ and $\Delta W > 0$, such that $\bar{W}(t_1') = U_B + \Delta W$, there must be some $\tau' \in (0,\ t_1' - t_1)$ such that $\bar{W}(t_1' - \tau') = U_B$ and $\dot{\bar{W}}(t_1' - \tau') > 0$. However, similar to the proof of Theorem 1, we have $\dot{\bar{W}}(t_1' - \tau') \leq 0$, which is a contradiction. Therefore, the window size is upper bounded by U_B for all $t \geq 0$.

B. Lower Bound on Window Size and Upper Bound on Queue Length

In the previous subsection, we proved that the AIMD window size $\bar{W}(t)$ is bounded by U_B, which is defined by (8). In this subsection, we show that the window size is lower bounded while the queue length is upper bounded.

Theorem 2: Let $L_{B1} := (\frac{N_1\alpha_1 + N_2\alpha_2}{2\cdot r_{max}})^{1/2}$, then $\bar{W}(t) \geq L_{B1}$ for $t \geq 0$.

Proof: Showing that $L_{B1} > 0$ is the lower bound of $\bar{W}(t)$ for $t \geq 0$, we should prove that if $\bar{W}(t) = L_{B1}$ at time $t = t_2 \geq 0$, then $\dot{\bar{W}}(t_2) \geq 0$.

Since the dropping/marking probability $p(t) = K_p \cdot q \leq 1$ for all t, then

$$\begin{aligned}\dot{\bar{W}}(t_2) &\geq \frac{N_1\alpha_1 + N_2\alpha_2}{R} - 2\cdot r_{max}\frac{\bar{W}^2(t)}{R}K_p q(t-R) \\ &\geq \frac{N_1\alpha_1 + N_2\alpha_2}{R} - 2\cdot r_{max}\frac{\bar{W}^2(t)}{R}.\end{aligned}$$

Therefore, $\dot{\bar{W}}(t_2) \geq 0$ when $\bar{W}(t) = L_{B1}$ with L_{B1} defined in the theorem, which implies $\bar{W}(t) \geq L_{B1}$ for $t \geq 0$. ■

Notice that L_{B1} in Theorem 2 is the lower bound of $\bar{W}(t)$ for all $t \geq 0$, which is a global bound. To show this, similar analysis to the upper bound of window size U_B can be applied to check that the window size $\bar{W}(t)$ will not go below L_{B1} for any $t > t_2$. However, the value of L_{B1} is actually small because of the loose approximation of $K_p \cdot q$ and the fact that $(\alpha_i,\ \beta_i)$ pair are all small real numbers for i=$1,\ 2$. Therefore, the global lower bound does not provide much information about the system performance. Since window size oscillates around its equilibrium in the steady state, the amplitude of the oscillation is more important than the global lower bound. Next, we derive the upper bound of queue length and local lower bound of the window size after the first time it reaches the peak value at t_1. The local lower bound is more useful for understanding the performance of the AIMD/RED system.

Theorem 3: Define T_1 and U_Q as

$$T_1 := \frac{U_B - R{\cdot}C}{r_{min}\cdot RC^2\cdot K_p\cdot(q_0^*{+}\Delta q) - \dfrac{N_1\alpha_1 + N_2\alpha_2}{R}},$$

$$U_Q {:=} \inf_{\Delta q > 0}\{(q_0^* + \Delta q) + (\frac{U_B}{R} - C)\cdot(T_1 + R)\},$$

where U_B is defined in Theorem 1. Let $L_{B2} > 0$ satisfy

$$L_{B2}^2 \cdot K_p \cdot U_Q = \frac{N_1\alpha_1 + N_2\alpha_2}{2r_{max}}, \tag{9}$$

then $q(t) \leq U_Q$ for $t \geq 0$ and $\bar{W}(t) \geq L_{B2}$ for $t \geq t_1$.

Proof: We first derive the upper bound of $q(t)$ for $t \geq 0$. Suppose that $\bar{W}(t)$ reaches its peak value at moment $t = t_1$. To get a loose upper bound of $q(t)$, we introduce the comparison theorem [18]. Instead of following system (2) and (4), we consider its comparison system: $\dot{q}(t) = U_B/R - C$, and $\bar{W}(t) \equiv U_B$ for $t{\in}[t_1,\ t_1']$. Notice that the solutions of the comparison system are larger than those of the original system, so the bounds derived in the following are also the bounds for system (2) and (4).

Assume that $\bar{W}(t)$ does not decrease for some time after t_1, and thus $q(t)$ increases at the rate of $U_B/R - C$. t_1' is chosen such that $q(t_1') = q^* + \Delta q$ with $\Delta q > 0$, then $\bar{W}(t)$ decreases from t_1' while $q(t)$ keeps increasing till t_2 such that $\dot{q}(t_2) = 0$ ($\bar{W}(t_2) = RC$) with $t_2 \geq t_1' + R$. Therefore, $q(t_2)$ is the local maximum value of $q(t)$. It should be noticed that this estimate of $q(t)$ might be greater than the real maximum value of $q(t)$ since $\bar{W}(t)$ may not stay at its peak value after t_1, and $q(t)$ will still increase after t_1, but with the rate less than $U_B/R - C$.

From the above analysis, for $t \in [t_1',\ t2]$, $\dot{q}(t) \leq \dfrac{U_B}{R} - C$, which implies

$$\begin{aligned}q(t_2) &\leq q(t_1') + (\frac{U_B}{R} - C)\cdot(t_2 - t_1') \\ &= (q_0^* + \Delta q) + (\frac{U_B}{R} - C)\cdot(t_2 - t_1').\end{aligned} \tag{10}$$

To estimate the length of the interval $[t_1',\ t_2]$, for $t \in [t_1' + R,\ t_2]$, it follows from the analysis above that

$$\begin{aligned}\bar{W}(t) &\geq \bar{W}(t_2) = RC, \\ q(t-R) &\geq q(t_1') = q_0^* + \Delta q,\end{aligned}$$

for some $\Delta q > 0$.

Thus,

$$\dot{\bar{W}}(t) \leq \frac{N_1\alpha_1 + N_2\alpha_2}{R} - r_{min}\cdot\frac{(RC)^2}{R}\cdot K_p\cdot(q_0^* + \Delta q), \tag{11}$$

for $t \in [t_1' + R,\ t_2]$.

On the other hand,

$$\int_{t_1'+R}^{t_2} \dot{\bar{W}}(s)ds = \bar{W}(t_2) - \bar{W}(t_1' + R) \geq RC - U_B. \tag{12}$$

It follows from (11) and (12) that,

$$\begin{aligned}RC{-}U_B \leq\ & [(N_1\alpha_1 + N_2\alpha_2)/R{-}r_{min}{\cdot}RC^2{\cdot}K_p{\cdot}(q_0^*{+}\Delta q)] \\ & \cdot(t_2 - t_1' - R),\end{aligned}$$

i.e.,

$$t_2{-}t_1'{-}R \leq \frac{U_B - RC}{r_{min}RC^2K_p(q_0^*{+}\Delta q) - (N_1\alpha_1 + N_2\alpha_2)/R}.$$

With the definition of T_1 in Theorem 3, we have $t_2 - t_1' \leq T_1 + R$. Therefore, it follows from (10) that

$$q(t) \leq \inf_{\Delta q > 0}\{(q_0^* + \Delta q) + (\frac{U_B}{R} - C)\cdot(T_1 + R)\}, \tag{13}$$

i.e., $q(t) \leq U_Q$ for $t \geq 0$, which indicates that U_Q is the upper bound of the RED queue length. Since the packet loss in a RED queue is proportional to the queue length, the derived queue length upper bound also reflects the maximum packet loss rate.

We finally show that $L_{B2} > 0$ is a lower bound of $\bar{W}(t)$ for $t \geq t_1$, i.e., if $\bar{W}(t) = L_{B2}$ at time $t = t_3 > t_1$, then $\dot{\bar{W}}(t_3) \geq 0$.

With (3) and (13),

$$\dot{\bar{W}}(t_3) \geq \frac{N_1\alpha_1 + N_2\alpha_2}{R} - 2r_{max} \cdot \frac{L_{B2}^2}{R} \cdot K_p \cdot U_Q.$$

Thus, $\dot{\bar{W}}(t_3) \geq 0$ if L_{B2} is chosen to satisfy (9). Therefore, L_{B2} is the lower bound of $\bar{W}(t)$ for $t \geq t_1$. ■

Therefore, the heterogeneous AIMD/RED system is practically stable with the bounds derived in Theorems 1 and 3.

The approach in this section can also be extended to obtain the theoretical bounds for AIMD/RED systems when they are shared by more than two classes of flows. Details are omitted here due to space limit.

IV. Performance evaluation

In this section, numerical results with Matlab and simulation results with NS-2 are given to validate the theorems and evaluate the system performance with different parameters. It should be noted that, in the fluid model, q and W are ensemble averages with positive and bounded quantities. In ergodic systems, ensemble average equals time average. If the AIMD window size oscillates between $2\beta W/(1+\beta)$ and $2W/(1+\beta)$ in a round, the average duration of a round equals $2(1-\beta)WR/((1+\beta)\alpha)$. The ensemble average of the window size in the fluid model can be used to predict its time average over a round in a real system.

Since the Internet contains mixed traffic, we evaluate the performance of AIMD/RED systems with heterogeneous flows. Parameters are chosen as follows: C=$10,000$ packet/sec, K_p=0.005, and $R = 0.05$ sec for 5 TCP flows competing with 5 AIMD(1/5, 7/8) flows. For comparison, we also choose C=$20,000$ packet/sec, K_p=0.005, and $R = 0.05$ sec for 10 TCP flows and 10 AIMD(1/5, 7/8) flows.

For the case of 5 TCP flows competing with 5 AIMD(1/5, 7/8) flows, the upper bound of $N_1W_I+N_2W_{II}$ is 508.9 packets, the lower bound L_{B2} is 28.28 packets, and the upper bound of the queue length is 10.2 packets. For the case of 10 TCP flows competing with 10 AIMD(1/5, 7/8) flows, the upper bound of $N_1W_I + N_2W_{II}$ is 1016.1 packets, the lower bound L_{B2} is 55.80 packets, and the upper bound of queue length is 19.6 packets. In the NS-2 simulations, since the RED threshold $\min_{th}$ is set to 20 packets, the upper bounds of total window size and queue length are enlarged by 20 packets accordingly. For the simulation results, we compare the theoretical bounds with both the total window size of all flows and its time average over a round. The correctness of our theoretical bounds and the tightness of the upper bound of window size are demonstrated by the numerical and simulation results, as shown in Fig. 2. The average window sizes in the NS-2 simulation results are slightly larger than the numerical results. This is because the numerical simulations with Matlab ignore the queuing delays in RTT, which may under-estimates the window size. It is also observed from Fig. 2 that, if the number of flows and the link capacity are increased proportionally, the upper bound of per-flow window size is closer to its optimal value. With both the number of flows and the link capacity being doubled, the upper bound of the queue length is less than twice of the previous bound. Therefore, the queuing delay bound is slightly reduced because of the multiplexing gain. An interesting conclusion is that although the increase of link capacity may cause an AIMD/RED system to become asymptotically unstable [3], the system queuing delay has lower bound and the upper bound of flows window size is closer to the optimal operating point. This result demonstrates the importance of studying practical stability and bounds of the AIMD/RED system.

Fig. 3 shows the window trace and queue length when 20 TCP flows share the bottleneck with 40 AIMD($1/5$, $7/8$) flows with K_p=0.005 and K_p=0.001, respectively. For the case of K_p=0.005, the upper bound of $N_1W_I + N_2W_{II}$ is 3034.4 packets and the upper bounds of queue length is 43.1 packets; while for the case of K_p=0.001, the upper bound of $N_1W_I + N_2W_{II}$ is 3042.4 packets and the upper bounds of queue length is 60.7 packets. It can be seen that a smaller value of K_p results in a slightly larger bounds on both window size and queue length. This observation is consistent with our analysis in Sec. III. However, in the case of higher bandwidth, the impact of K_p is less. Similar results with homogeneous AIMD flows are reported in [15].

V. Conclusion

In this paper, we have studied the practical stability of the heterogeneous AIMD/RED system by deriving theoretical bounds of window size and queue length. The theorems in the paper can provide important insights and guidelines for setting up parameters for heterogeneous AIMD/RED systems in order to maintain system practical stability and to fully utilize network resources without excessive delay and loss. In contrast to the previous pessimistic opinion that an AIMD/RED system becomes asymptotically unstable when the link capacity is larger, our results show that the deviation of the AIMD/RED system from its optimal operation region is smaller with higher link capacity. Thus, AIMD/RED should perform well in future Internet with higher data rate links and heterogeneous flows.

Acknowledgment

This work has been supported in part by research grants from the Natural Science and Engineering Council of Canada.

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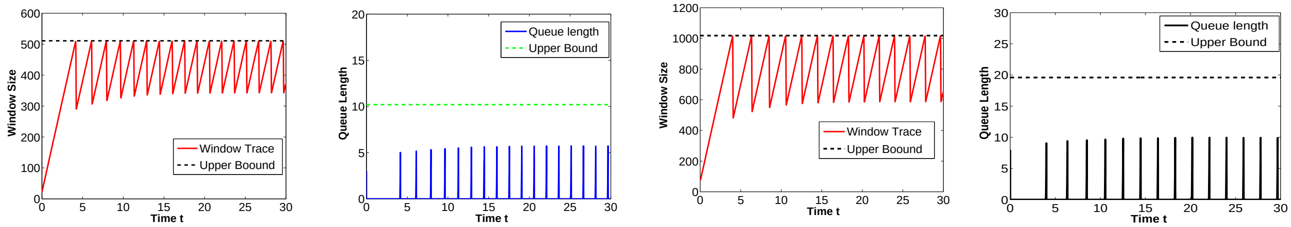
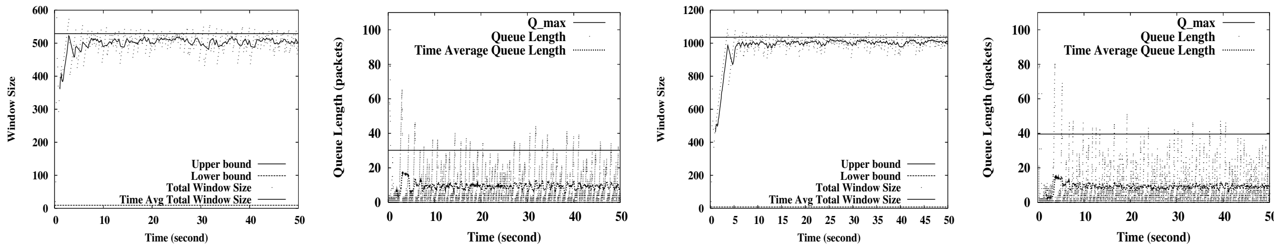
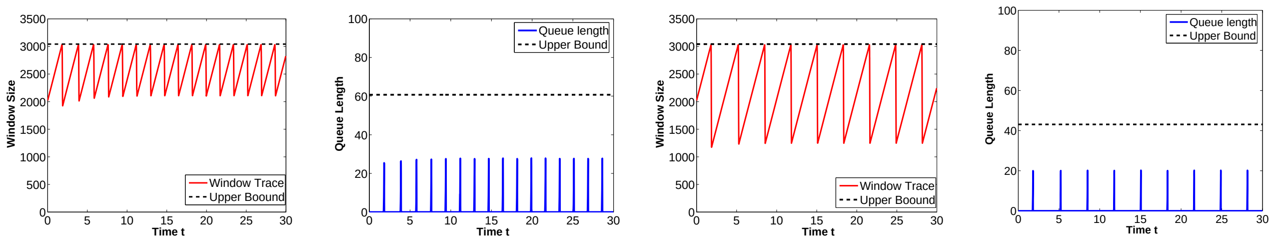


5 TCP vs. 5 AIMD flows, C=10,000 packet/sec (Matlab) 10 TCP vs. 10 AIMD flows, C=20,000 packet/sec (Matlab)

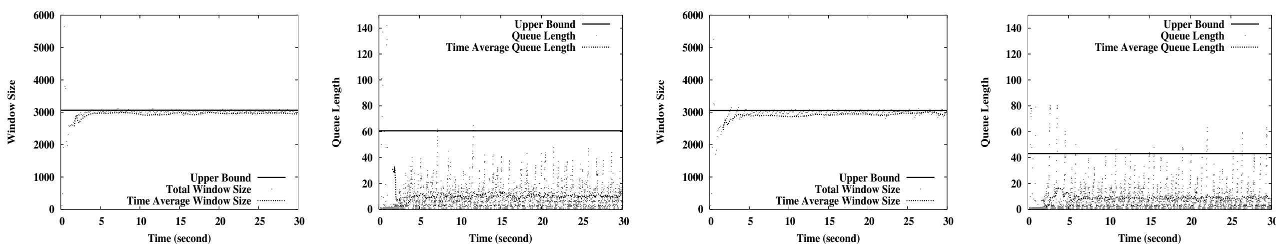


5 TCP vs. 5 AIMD flows, C=10,000 packet/sec (NS-2) 10 TCP vs. 10 AIMD flows, C=20,000 packet/sec (NS-2)

Fig. 2. Theoretical Bounds of Heterogeneous flows, K_p=0.005, and $R = 0.05$ sec



20 TCP vs. 40 AIMD flows, $K_p = 0.001$ (Matlab) 20 TCP vs. 40 AIMD flows, $K_p = 0.005$ (Matlab)



20 TCP vs. 40 AIMD flows, $K_p = 0.001$ (NS-2) 20 TCP vs. 40 AIMD flows, $K_p = 0.005$ (NS-2)

Fig. 3. Theoretical Bounds of Heterogeneous flows, C=60,000 packet/sec, and $R = 0.05$ sec

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