Boundedness of AIMD/RED System with Time Delays

Lijun Wang[1], Lin Cai[2], Xinzhi Liu[1] and Xuemin (Sherman) Shen[3]
Department of Applied Mathematics[1], Department of Electrical and Computer Engineering[3]
University of Waterloo, Waterloo, ON, Canada, N2L 3G1
Department of Electrical and Computer Engineering[2]
University of Victoria, Victoria, BC, Canada, V8W 3P6

***Abstract*—Internet performance depends on the Additive Increase and Multiplicative Decrease (AIMD) congestion control algorithm deployed in the end systems and the Random Early Detection (RED) queue management scheme deployed in the intermediate systems. Previous research based on the fluid-flow model indicated that the TCP/RED system may not be asymptotically stable when the feedback delay becomes large or when the link capacity becomes large. However, so long as the system operates near its desired equilibrium, the network performance (in terms of efficiency, loss rate, and delay) is still appreciable. In this report, using the fluid-flow model for a generalized AIMD/RED system, we derive theoretical bounds of the AIMD flow window size and the RED queue length, as functions of number of flows, link capacity, RED queue parameters, and AIMD parameters. Numerical results with Matlab and simulation results with NS-2 are given to validate the correctness and demonstrate the tightness of the derived bounds. Our main findings are: 1) larger values of round-trip delay and link capacity will actually *reduce* the oscillating amplitude of window size and queue length from their equilibria in steady state; 2) if we proportionally increase the link capacity and the number of AIMD flows, the queueing delay will be slightly reduced, so the multiplexing gain slightly increases; and 3) although AIMD flows can adapt their sending rates according to available bandwidth, larger number of flows leads to longer queueing delay in the AIMD/RED system. Thus, we should limit the number of AIMD connections in a link or promote to use more conservative AIMD parameters to bound the queueing delay and loss. Our results can also help to predict and control the system performance for future Internet with higher data rate links multiplexed with more flows with different parameters.**

***Index Terms*—Boundedness, AIMD/RED system, time delay system.**

I. Introduction

The first congestion collapse in the Internet was observed in 1980's, although the Internet was in its infant stage at that time. To solve the problem, Van Jacobson proposed the Transmission Control Protocol (TCP) congestion control algorithm based on the Additive Increase and Multiplicative Decrease (AIMD) mechanism in 1988. Since then, the TCP congestion control algorithm has been widely deployed in the end systems to respond to network congestion signals and avoid network congestion collapses. On the other hand, the active queue management algorithms, Random early detection or Random early discard (RED), have been developed and deployed in the intermediate systems to fairly distribute network congestion signals to all on-going flows. AIMD and RED are considered key factors to the overwhelming success of the Internet, which has experienced explosive growth over the past two decades, with network speeds almost doubled every year and new applications emerge quickly. Future Internet will become an even more diversified system. It will contain heterogeneous wireless and wired links with speeds varying from tens of Kbps to tens of Gbps, with flow round-trip delays varying from ms to seconds. It will also support various multimedia applications with different throughput, delay, and jitter requirements. An immediate question is whether the AIMD/RED system can be stable, scalable, and efficient for the next generation Internet?

Internet stability has been an active research topic since its first congestion collapse was observed. With a fluid-flow model of the system, it has been proved that, without feedback delay, the AIMD congestion control mechanism, coupled with the RED queue management, can ensure asymptotic stability [15]. However, with a non-negligible feedback delay, the AIMD/RED system may not be asymptotically stable when the delay becomes large and/or when the link capacity becomes large [3].

On the other hand, the Internet is a very dynamic system, and can tolerate some transient congestion events. In fact, TCP controlled flows aggressively probe for available bandwidth in the network, and create transient congestions. Even the system is not asymptotically stable, so long as the end systems do not overshoot the available bandwidth too severely, the overall system efficiency can still be very high, and the packet loss rate and queueing delay can still be well bounded. Therefore, it is critical to investigate, if the network may operate at states away from the desired equilibrium state, what the theoretical bounds of the system are.

Different from the previous work [1]–[4], [15], [16] to find the conditions to ensure system asymptotic stability, this report derives the theoretical bounds of the system, i.e., flows' congestion window size and intermediate systems' queue length, given the number of flows sharing the link, their AIMD parameter pairs and round-trip times (RTTs), link capacity, and RED queue parameters. Using the fluid-flow model of a generalized AIMD/RED system, instead of applying the Lyapunov-like method, we derive a tight upper bound of congestion window size and queue length by directly studying the inherent properties of AIMD/RED systems. With clearly defined bounds, the system is considered marginally stable. The definitions of stability are listed below, which follow those in [19], [20].

Definition 1: Consider dynamic systems with time delay of the following form:

$$\frac{dx}{dt} = f(t, x(t), x(t-\tau_1(t)), \cdots, x(t-\tau_m(t)))$$

where $x \in R^n$, $f : I \times R^n \times R^n \times \cdots \times R^n \rightarrow R^n$ is continuous. Let $\tau = \sup_{i=1,..,m} \tau_i(t)$. The trivial solution of the system is said to be

- *uniformly bounded* if there exists a constant c, for every $a \in (0, c)$, there is $B = B(a) > 0$, such that for any $\xi(t) \in C[[t_0 - \tau, t_0], R^n]$, $\|x(t, t_0, \xi)\| \leq B$ for all $t \geq t_0$ when $\|\xi\| \leq a$;
- *stable* if for every $\epsilon > 0$ and $t_0 \in \mathbb{R}_+$, there exists some $\delta = \delta(t_0, \epsilon) > 0$ such that for any $\xi(t) \in C[[t_0 - \tau, t_0], R^n]$, $\|\xi\| < \delta$ implies $\|x(t, t_0, \xi)\| < \epsilon$ for all $t \geq t_0$;
- *asymptotically stable* if the system is stable and for every $t_0 \in \mathbb{R}_+$, there exists some $\eta = \eta(t_0) > 0$ such that $\lim_{t\rightarrow\infty} \|x(t, t_0, \xi)\| = 0$ whenever $\|\xi\| < \eta$;
- *marginally stable* if the system is stable but not asymptotically stable.

Although asymptotic stability of the AIMD/RED system has been extensively investigated, to the best of our knowledge, this report is the first one to present performance bounds of the system. The derived theoretical bounds provide important insights on which system parameters contribute to high oscillations of the system and how to choose system parameters to ensure system efficiency with bounded delay and loss. The theorems given in the report can also help to predict the system performance for the future Internet with higher data rate and more flows with different flow parameters.

The remainder of the report is organized as follows. Sec. II introduces the model of the generalized AIMD/RED system. Sec. III studies the boundedness of AIMD/RED systems with feedback delay. In Sec. IV, numerical results with Matlab and simulation results using NS-2 are presented to validate the derived bounds, and the impacts of different system parameters on the system performance are discussed. Related work is discussed in Sec. V, followed by concluding remarks in Sec. VI.

II. A FLUID-FLOW MODEL OF AIMD/RED SYSTEM

A stochastic model of TCP behaviors is developed using fluid-flow and stochastic differential equation analysis in [10]. Simulation results have demonstrated that this model accurately captures the dynamics of TCP. We extend the fluid-flow model for general AIMD(α, β) congestion control: the window size is increased by α packets per RTT if no packet loss occurs; otherwise, it is reduced to β times its current value. The general AIMD congestion control has been proposed to support heterogeneous applications with different tolerance on flow throughput variations [12]–[14].

For all AIMD-controlled flows with the same (α, β) parameter pair and round-trip delay, the AIMD fluid model relates to the *ensemble averages* of key network variables [4], [10], and is described by the following coupled, nonlinear differential equations:

$$\frac{dW(t)}{dt} = \frac{\alpha}{R(t)} - \frac{2(1-\beta)}{1+\beta} W(t) \frac{W(t-R(t))}{R(t-R(t))} p(t-R(t))$$

$$\frac{dq(t)}{dt} = \begin{cases} \frac{N(t)\cdot W(t)}{R(t)} - C, & q > 0 \\ \{\frac{N(t)\cdot W(t)}{R(t)} - C\}^+, & q = 0 \end{cases} \tag{1}$$

where $\{a\}^+ = \max\{a, 0\}$, $\alpha > 0$, $\beta \in [0, 1]$; $W \geq 1$ is the AIMD window size (packets), and $q \in [0, q_{\max}]$ is the queue length (packets) at time t. W and q in the fluid-flow model can approximate the ensemble averages of flow's congestion window size and queue length respectively in the real system. $R(t)$ is the round-trip time with $R(t) = \frac{q(t)}{C} + T_p$ (s), where C is the link capacity (packets/s) and T_p is the deterministic round-trip delay. $N(t)$ is the number of AIMD flows, and $p(t)$ is the probability of a packet being dropped or marked by an intermediate system.

The first differential equation of system (1) describes the AIMD(α, β) window control dynamic. α/R represents the window's additive increase, whereas $2(1-\beta)W/(1+\beta)$ represents the window's multiplicative decrease in response to packet dropping or marking probability p. Since the AIMD flow's window size in a practical system oscillates between $\beta W_{\max}$ and $W_{\max}$, its average window size W over a round[1] is $(1+\beta)W_{\max}/2$. Each time, the window size is decreased by $(1-\beta)W_{\max} = 2(1-\beta)W/(1+\beta)$. The second equation models the bottleneck queue length as simply an accumulated difference between packet arrival rate NW/R and link capacity C. $\{\cdot\}^+$ in the model guarantees that queue length is non-negative.

It should be noted that, in the fluid-flow model, q and W are positive and bounded quantities which approximate the ensemble averages of queue length and window size in practical systems. In ergodic systems, ensemble average equals time average. The values of q and W in the fluid-flow model can be used to predict its time average over a round in a practical system. Given the AIMD window size oscillating between $\beta W_{\max}$ and $W_{\max}$ in a round, the average duration of a round equals $2(1-\beta)WR/[(1+\beta)\alpha]$.

We consider the popular Active Queue Management (AQM) scheme, RED, in the system (1). With RED, the packet dropping or marking probability, p, is determined by the average queue length q_{act}:

$$p = \begin{cases} 0, & 0 \leq q_{act} \leq \min_{th} \\ K_p(q_{act} - \min_{th}), & \min_{th} < q_{act} \leq \max_{th} \\ 1, & q_{act} > \max_{th} \end{cases} \tag{2}$$

[1]A round is defined as the interval between two time instants that the flow reduces its congestion window size consecutively.

where $K_p > 0$. When $q_{act} \leq \min_{th}$, $\frac{dW(t)}{dt} = \frac{\alpha}{R}$, the window size of AIMD flows will keep increasing and will not converge to any value. Thus, in the following, we will discuss the stability of this model when $q_{act} > \min_{th}$. Without loss of generality, let $q(t) = q_{act}(t) - \min_{th}$. In addition, since the queue behaves in the same way as a Drop-Tail queue once q_{act} exceeds $\max_{th}$, we choose $\max_{th}$ to be sufficiently large such that $K_p(\max_{th} - \min_{th}) = 1$.

Eq. (1) is a generalized TCP/RED congestion control model, which includes the models studied in [2], [4], [10]. If we choose $\alpha = 1$, $\beta = 0.5$, (1) is equivalent to the traditional TCP/RED model in [10].

The equilibrium point $(W^*,\ q^*)$ for (1) and (2) is given by

$$W^* = \frac{R{\cdot}C}{N}; \qquad q^* = \frac{\alpha(1+\beta)N^2}{2(1-\beta)R^2C^2K_p}.$$

Remarks: At the equilibrium, the total arrival rate equals the total link capacity, so the link bandwidth can be fully utilized. If the window size is larger than W_0^*, the queue will build up which results in a longer queueing delay; if the window size is less than W^*, the network load is smaller than its capacity, so the network resources are not fully utilized. In conclusion, the equilibrium point is also the most desired operating point of the system.

III. Boundedness of Homogeneous-flow AIMD/RED system with Time Delay

It has been demonstrated in [3] that an AIMD/RED system becomes (asymptotically) unstable with the increase of round trip delays of the system. Using the fluid-flow model, sufficient conditions for the asymptotic stability of AIMD/RED systems with feedback delays have been derived in [16]. In this section, we show that even though the system may become (asymptotically) unstable because of the effects of time delay, its window size and queue length are still bounded, and in most cases, the upper bounds are close to their equilibria.

First, we study the delayed homogeneous AIMD system defined by (1) with RED defined by (2). We set $\min_{th} = 0$ in RED and assume that the traffic load (i.e., the number of AIMD flows) is time-invariant, i.e., $N(t) = N$. With ever-increasing link capacity and appropriate congestion control mechanism, variation of queuing delays becomes relatively small to propagation delays. In [18], it is revealed that the variable nature of RTT due to queueing delay variation helps to stabilize the TCP/RED system. In light of this, we derive upper and lower bounds of AIMD/RED systems assuming RTT to be constant. These results will be a good approximation if RTT is slightly time-varying. We thus ignore the effect of the delay jitter on the round-trip time and assume that the round-trip time of each flow is a constant, $R(t) = R$.

Notice that the AIMD/RED system defined by (1) and (2) are described by delayed differential equations, its initial conditions are given by $1 \leq W(t) \leq W^*$ and $0 \leq q(t) \leq q^*$ on the interval $t \in [-R,\ 0]$. According to (1), it is also reasonable that we let $\dot{W}(t) \leq \frac{\alpha}{R}$ for $t \in [-R,\ 0]$.

A. Upper Bound on Window Size

Theorem 1: Let $UB > 0$ be the largest real root of

$$UB \cdot (UB - \alpha) \cdot (UB - \frac{R{\cdot}C}{N} - \alpha)^2 = \frac{\alpha^2(1+\beta)}{(1-\beta)NK_p},$$

then $W(t) \leq UB$ for $t \geq 0$.

Proof: With (1) and (2), we note that $\dot{W} \leq \dfrac{\alpha}{R}$ for $t \geq 0$, since $W(t) \geq 1$ and $q(t) \geq 0$. For $\tau > 0$, taking integration on both sides from $t - \tau$ to t gives

$$W(t) - W(t-\tau) \leq \frac{\alpha}{R} \cdot \tau \quad \text{for} \quad t \geq 0. \tag{3}$$

We show that the UB (> 0) in the theorem is an upper bound of $W(t)$ for $t \geq 0$, i.e., if $W(t) = UB$ for some $t = t_1 \geq 0$, then $\dot{W}(t_1) \leq 0$.

With (3) and $W(t_1) = UB$, and taking $\tau = R$ and $t = t_1$, we have

$$W(t_1 - R) \geq UB - \alpha. \tag{4}$$

Notice that $W(t_1 - \tau) \geq UB - a{\cdot}\alpha$ when $\tau \in [R,\ aR]$ for any real number $a > 1$.

Consider

$$\dot{q}(t) = \begin{cases} \dfrac{N \cdot W(t)}{R} - C, & q > 0 \\ \{\dfrac{N \cdot W(t)}{R} - C\}^+, & q = 0 \end{cases}$$

Taking integration on both sides from $t_1 - aR$ to $t_1 - R$, we have

$$\begin{aligned} \int_{t_1-aR}^{t_1-R} \dot{q}(s)ds \quad &\geq \quad \frac{N}{R}\int_{t_1-aR}^{t_1-R} W(s)ds - (a-1)R{\cdot}C \\ &\geq \quad N \cdot (a-1) \cdot (UB - a{\cdot}\alpha) - (a-1)RC \end{aligned}$$

which implies

$$q(t_1 - R) \geq [N \cdot (UB - a{\cdot}\alpha) - R{\cdot}C] \cdot (a-1) \tag{5}$$

since $q(t) \geq 0$.

Taking $f(a) = (a-1){\cdot}[N{\cdot}(UB - a{\cdot}\alpha) - R{\cdot}C]$ and computing the maximum value of $f(a)$ by letting $f'(a) = 0$ gives $a = (N{\cdot}UB + R{\cdot}C + N{\cdot}\alpha)/(2\alpha N)$ and

$$f(a) = N(UB - R{\cdot}C/N - \alpha)^2/(2\alpha). \tag{6}$$

Therefore, it follows from (4), (5) and (6) that, $\dot{W}(t_1) \leq 0$ since UB satisfies

$$\frac{N \cdot UB \cdot (UB-\alpha) \cdot (UB - R{\cdot}C/N - \alpha)^2}{2\alpha} = \frac{\alpha(1+\beta)}{2(1-\beta)K_p}, \tag{7}$$

which implies $W(t) \leq UB$ for $t \geq 0$. ■

If all AIMD flows are TCP-friendly, i.e., the average throughput of non-TCP-transported flows over a large time scale does not exceed that of any conformant TCP-transported ones under the same circumstance [11], the (α, β) pair should satisfies the TCP-friendly condition $\alpha = 3(1-\beta)/(1+\beta)$ derived in [14], [15]. Thus, the above equality (7) becomes

$$UB \cdot (UB - \alpha) \cdot (UB - R{\cdot}C/N - \alpha)^2 = \frac{3\alpha}{NK_p}. \tag{8}$$

By the continuity property of $UB \cdot (UB-\alpha) \cdot (UB - R{\cdot}C/N-\alpha)^2$ and the fact that the RHS of (7) is always greater than zero, we can conclude that the largest root of (7) must be greater than $R{\cdot}C/N+\alpha$, where $R{\cdot}C/N$ is the equilibrium value of the window size for AIMD/RED system. Therefore, the oscillation of the window size from its equilibrium value will increase with the increment of α and the decrement of K_p. In addition, the upper bound UB itself will increase with the increment of $R{\cdot}C$, α and the decrement of N, K_p.

It is also noted that the upper bound derived in Theorem 1 is a global one for the time t, i.e., the window size $W(t)$ will not go above UB for any $t > t_1$. If we assume, instead, that there exists $t_1' > t_1$ and $\Delta W > 0$, such that $W(t_1') = UB + \Delta W$, then there must be some $\tau' \in (0,\ t_1' - t_1)$ such that $W(t_1' - \tau') = UB$ and $\dot{W}(t_1' - \tau') > 0$. However, similar to the proof of Theorem 1, we have $\dot{W}(t_1' - \tau') \leq 0$, which is a contradiction. Therefore, the window size is upper bounded by UB for any $t \geq 0$.

B. Lower Bound on Window Size and Upper Bound on Queue Length

In the previous subsection, we proved that the AIMD window size $W(t)$ is bounded from above, and an upper bound, UB, is defined by (7). In this subsection, we show that the window size is also bounded from below while the queue length is upper bounded.

Theorem 2: Define $A := \dfrac{\alpha}{R} - \dfrac{2(1-\beta)}{1+\beta}\dfrac{UB^2}{R}$ and let $LB_1 > 0$ be the root of

$$LB_1 \cdot (LB_1 - AR) = \frac{\alpha(1+\beta)}{2(1-\beta)},$$

then $W(t) \geq LB_1$ for $t \geq 0$.

Proof: From Theorem 1, $W(t) \leq UB$ for $t \geq 0$, which implies

$$\dot{W}(t) \geq \frac{\alpha}{R} - \frac{2(1-\beta)}{1+\beta}\frac{UB^2}{R} =: A$$

It can be seen from the definition of UB that $A < 0$. We show that $LB_1 > 0$ is the lower bound of $W(t)$ for $t \geq 0$, i.e., if $W(t) = LB_1$ at time $t = t_2 \geq 0$, then $\dot{W}(t_2) \geq 0$.

Taking integration on both sides from $t_2 - R$ to t_2 gives $W(t_2 - R) \leq W(t_2) - AR = LB_1 - AR$.

Since dropping/marking probability $p(t) = K_p \cdot q(t) \leq 1$ for all t, then $\dot{W}(t_2) \geq \dfrac{\alpha}{R} - \dfrac{2(1-\beta)}{1+\beta}\dfrac{LB_1 \cdot (LB_1 - AR)}{R}$. Therefore, $\dot{W}(t_2) \geq 0$ since LB_1 satisfies

$$LB_1 \cdot (LB_1 - AR) = \frac{\alpha(1+\beta)}{2(1-\beta)}, \tag{9}$$

which implies $W(t) \geq LB_1$ for $t \geq 0$. ■

Notice that LB_1 in Theorem 2 is the lower bound of $W(t)$ for all $t \geq 0$, which is a global one. By similar analysis to the upper bound of window size UB, it is easy to check that the window size $W(t)$ will not go below LB_1 for any $t > t_2$. However, the value of LB_1 is actually very small since $\alpha(1+\beta)/(2(1-\beta))$ is fairly small compared to $-AR$. Therefore, the global lower bound does not provide much information about the performance of AIMD/RED systems.

Since window size oscillates around its equilibrium in the steady state, the amplitude of the oscillation is more important than the global lower bound. Next, We will show the local lower bound of the window size after the first time it reaches the peak value at moment t_1. This local lower bound is more useful for understanding the performance of AIMD/RED systems.

Theorem 3: Define T_1 and UQ as

$$T_1 = \frac{UB - \dfrac{R{\cdot}C}{N}}{\dfrac{2(1-\beta)}{1+\beta} \cdot \dfrac{C{\cdot}K_p}{N} \cdot [\dfrac{R{\cdot}C}{N}\Delta q + \Delta W(q_0^* + \Delta q)]}$$

$$UQ = \inf_{\substack{\Delta q > 0,\\ \Delta W \in [0,\ UB - \frac{R \cdot C}{N}]}} \{(q_0^* + \Delta q) + (\frac{N}{R} \cdot UB - C) \cdot (T_1 + R)\},$$

where UB is defined in Theorem 1. Let $LB_2 > 0$ satisfy

$$LB_2 {\cdot} (LB_2 + \frac{2(1-\beta)}{1+\beta} {\cdot} UB^2 {\cdot} K_p {\cdot} UQ - \alpha) {\cdot} K_p {\cdot} UQ = \frac{\alpha(1+\beta)}{2(1-\beta)},$$

then $q(t) \leq UQ$ for $t \geq 0$ and $W(t) \geq LB_2$ for $t \geq t_1$.

Proof: We first derive the upper bound of $q(t)$ for $t \geq 0$. At moment $t = t_1$, $W(t)$ reaches its peak value. To get a loose upper bound of $q(t)$, we assume that $W(t)$ does not decrease for some time after t_1, and thus $q(t)$ increases at the rate $\dfrac{N}{R}UB - C$. Moment t_1' is chosen such that $q(t_1') = q^* + \Delta q$ with $\Delta q > 0$, then $W(t)$ decreases from t_1' while $q(t)$ keeps increasing till moment t_2 such that $\dot{q}(t_2) = 0$ (i.e., $W(t_2) = R{\cdot}C/N$). Therefore, $q(t_2)$ is the local maximum value of $q(t)$. It should be noticed that this estimate of $q(t)$ might be greater than the real maximum value of $q(t)$ since $W(t)$ may not stay at its peak value after t_1, and $q(t)$ will still increase after t_1, but with the rate less than $\dfrac{N}{R}UB - C$.

From above analysis, for $t \in [t_1',\ t2]$, $\dot{q}(t) \leq \dfrac{N}{R} \cdot UB - C$. Thus,

$$\int_{t_1'}^{t_2} \dot{q}(s)ds \leq (\frac{N}{R} \cdot UB - C) \cdot (t_2 - t_1')$$

which implies

$$\begin{aligned} q(t_2) &\leq q(t_1') + (\frac{N}{R} \cdot UB - C) \cdot (t_2 - t_1') \\ &= (q_0^* + \Delta q) + (\frac{N}{R} \cdot UB - C) \cdot (t_2 - t_1') \end{aligned} \tag{10}$$

To estimate the length of the interval $[t_1',\ t_2]$, for $t \in [t_1' + R,\ t_2]$, it follows from the analysis above that

$$\begin{aligned} W(t) &\geq W(t_2) = \frac{R{\cdot}C}{N}, \\ q(t-R) &\geq q(t_1') = q_0^* + \Delta q, \\ W(t-R) &\geq W(t_2 - R) = \frac{R{\cdot}C}{N} + \Delta W, \end{aligned}$$

for some $\Delta q > 0$ and $\Delta W \in (0,\ UB - \frac{R \cdot C}{N})$.

Thus,

$$\dot{W}(t) \quad \leq \quad -\frac{2(1-\beta)}{1+\beta} \cdot \frac{C \cdot K_p}{N} \cdot [\Delta W(q_0^* + \Delta q) + \frac{R \cdot C}{N} \Delta q] \tag{11}$$

for $t \in [t_1' + R,\ t_2]$.

On the other hand,

$$\int_{t_1'+R}^{t_2} \dot{W}(s) ds = W(t_2) - W(t_1' + R) \geq \frac{R \cdot C}{N} - UB. \tag{12}$$

It follows from (11) and (12) that,

$$\begin{aligned} \frac{R \cdot C}{N} - UB \leq \quad - \quad & \frac{2(1-\beta)}{1+\beta} \cdot \frac{C \cdot K_p}{N} \cdot (t_2 - t_1' - R) \\ & \cdot [\Delta W(q_0^* + \Delta q) + \frac{R \cdot C}{N} \Delta q] \end{aligned}$$

i.e.,

$$t_2 - t_1' - R \leq \frac{UB - \frac{R \cdot C}{N}}{\frac{2(1-\beta)}{1+\beta} \cdot \frac{C \cdot K_p}{N} \cdot [\frac{R \cdot C}{N} \Delta q + \Delta W(q_0^* + \Delta q)]}$$

With the definition of T_1 in the theorem, we have $t_2 - t_1' \leq T_1 + R$. Therefore, it follows from (10) that

$$q(t) \leq \inf_{\substack{\Delta q > 0, \\ \Delta W \in [0,\, UB - \frac{R \cdot C}{N}]}} \{(q_0^* + \Delta q) + (\frac{N}{R} \cdot UB - C) \cdot (T_1 + R)\}, \tag{13}$$

i.e., $q(t) \leq UQ$ for $t \geq 0$, which indicates that UQ is the upper bound of the RED queue length. Since the packet loss in a RED queue is proportional to the queue length, the derived queue length upper bound also reflects the upper bound of packet loss rate.

We finally show that $LB_2 > 0$ is a lower bound of $W(t)$ for $t \geq t_1$, i.e., if $W(t) = LB_2$ at time $t = t_3 > t_1$, then $\dot{W}(t_3) \geq 0$.

With (7) and (13),

$$\dot{W}(t) \geq \frac{\alpha}{R} - \frac{2(1-\beta)}{1+\beta} \cdot \frac{UB^2}{R} \cdot K_p \cdot UQ \tag{14}$$

for $t \geq 0$, we have

$$\int_{t_3 - R}^{t_3} \dot{W}(s) ds \geq \alpha - \frac{2(1-\beta)}{1+\beta} \cdot UB^2 \cdot K_p \cdot UQ$$

i.e.,

$$W(t_3 - R) \leq LB_2 + \frac{2(1-\beta)}{1+\beta} \cdot UB^2 \cdot K_p \cdot UQ - \alpha \tag{15}$$

It follows from (13) and (15) that,

$$\dot{W}(t_3) \geq \frac{\alpha}{R} - \frac{2(1-\beta)}{1+\beta} \cdot \frac{LB_2 \cdot UW}{R} \cdot K_p \cdot UQ$$

with $UW := LB_2 + \frac{2(1-\beta)}{1+\beta} \cdot UB^2 \cdot K_p \cdot UQ - \alpha$.

Therefore, $\dot{W}(t_3) \geq 0$ if LB_2 is chosen to satisfy

$$LB_2 \cdot UW \cdot K_p \cdot UQ = \frac{\alpha(1+\beta)}{2(1-\beta)}, \tag{16}$$

and thus LB_2 is the lower bound of $W(t)$ for $t \geq t_1$. ■

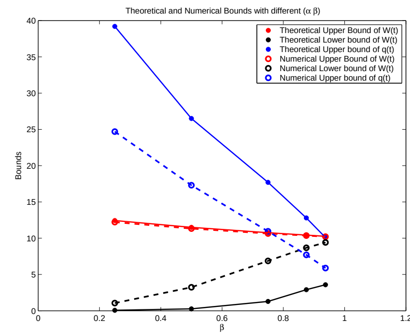


Fig. 1. Theoretical and Numerical Bounds of window size and queue length with different $(\alpha,\ \beta)$

IV. Performance Evaluation

In this section, numerical results with Matlab and simulation results with NS-2 are given to validate the theorems and evaluate the system performance with different parameters.

A. AIMD parameter pairs

First, we investigate how the AIMD parameter pair $(\alpha,\ \beta)$ affects the bounds of window size and queue length. Let N, R, C and K_p be constants: $N = 10, R = 0.1$ sec, $C = 1000$ packet/sec and $K_p = 0.01$. The AIMD $(\alpha,\ \beta)$ pairs are chosen to be TCP-friendly, varying from $(9/5, 1/4)$ to $(3/31, 15/16)$, and the results are given in Fig. 1. It can be seen that for the upper and lower bounds of the window size and the upper bound of the queue length, the numerical results are all within the bounds given by Theorem 1 and Theorem 3, which verifies the correctness of the Theorems. In addition, the upper bound of the window size given by the Theorem is very tight. The one for queue length is a loose bound as mentioned in the proof of Theorem 3. The theoretical lower bound of window size is not tight because the approximation of $\dot{W}(t)$ in (14) is not very accurate. How to find a tight lower bound for window size will be a future research issue.

Another observation is that the differences between numerical and theoretical results is getting smaller as $(\alpha,\ \beta)$ pair varies from $(9/5, 1/4)$ to $(3/31, 15/16)$, which shows that the theoretical results become tighter when the value of β gets larger.

In ideal cases, the window size should converge to $R \cdot C/N$, which is 10 packets per RTT in the above cases. The results in Fig. 1 show that with a smaller value of α and a larger value of β, the AIMD flows have less oscillation amplitude around the optimal operation point, so they can utilize network resources more efficiently with less delay and loss in steady state. This is because, with a smaller value of α, the AIMD flows overshoot the available bandwidth in a slower pace; with a larger value of β, the AIMD flows will not decrease drastically for any single packet loss. Also, as shown in Fig. 1, the upper bound of the queue length becomes smaller w.r.t. β; thus, the average queueing delay (and thus loss rate) becomes smaller in steady state.

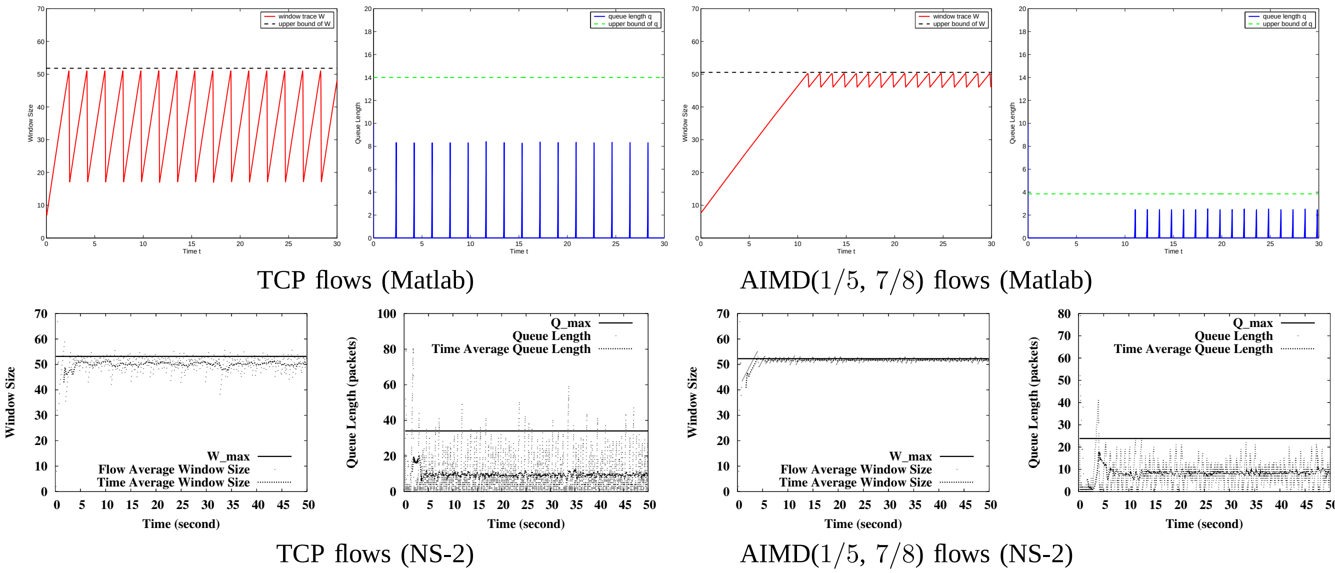


Fig. 2. Traces of window size and queue length, N=10, C=10000 packet/sec, R=0.05 sec and K_p=0.005

Fig. 2 shows the traces of TCP flows with AIMD parameter pair of $(1, 1/2)$ and those of AIMD$(1/5, 7/8)$ flows. Here, N=10, C=10000 packet/sec, R=0.05 sec and K_p=0.005. For NS-2 simulations, we set $Q_{\min}$ of the RED queue to be 20 packets. Therefore, the upper bound of window size of each flow should be enlarged by $Q_{\min}/N = 2$ packets, and the upper bound of the queue length should be enlarged by $Q_{\min} = 20$ packets. We compare the theoretical bounds with both the average window size among all flows and its time average of window size over a round. Both the numerical results with Matlab and simulation results with NS-2 show that although the window variation of AIMD$(1/5, 7/8)$ in steady state is smaller, it takes longer time for AIMD$(1/5, 7/8)$ flows to converge to the steady state. Simulation results also demonstrate the tightness of the upper bound of window size. Another interesting observation is that although the upper bound of queue length is not tight comparing to the time average of queue length, it is close to the maximum instantaneous queue length in steady state.

Considering that the future Internet might contain mixed traffic with different AIMD parameters, we further study the performance of the AIMD/RED system with heterogeneous flows. Fig. 3 shows the window trace and queue length when TCP flows and AIMD($1/5$, $7/8$) flows share the bottleneck. Parameters are firstly chosen as C=10000 packet/sec, K_p=0.005, and $R = 0.05$ sec for 5 TCP flows competing with 5 AIMD($1/5$, $7/8$) flows. For comparison, we also choose C=20000 packet/sec, K_p=0.005, and $R = 0.05$ sec for 10 TCP flows and 10 AIMD($1/5$, $7/8$) flows. As shown in Fig. 3, TCP window size oscillates stronger than that of AIMD($1/5$, $7/8$), but their average window sizes are close to each other.

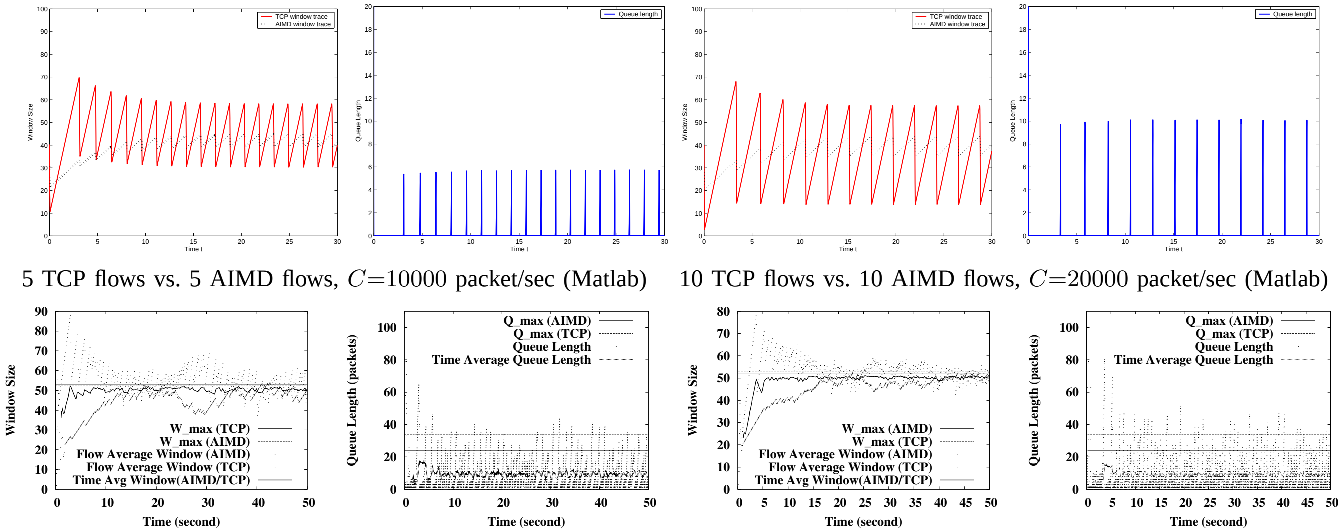
Notice that we use the bounds for TCP or AIMD($1/5$, $7/8$) flows only for comparison. The simulation results show that the time average window size of all flows are below the UB of TCP flows. This suggests that we can use the upper bounds of AIMD flows with the smallest value of β among the mixed traffic to determine the upper bounds of heterogeneous flows systems.

Theoretical bounds for heterogeneous-flow AIMD/RED system can be obtained by applying similar approach in the report. But it should be noticed that the upper bound of window size derived for the heterogeneous-flow system by this method is usually not as tight as that for the homogeneous-flow system, because the interactions among heterogeneous flows are implicitly expressed in the fluid-flow model and not easy to calculate.

B. *Impact of system parameters*

In the following, we study how the parameters N, R, C and K_p affect the bounds of window size and queue length. We choose $(\alpha,\ \beta)$ pair to be (1, $1/2$) and ($1/5$, $7/8$), and obtain the results with different network parameters as shown in Tables I and II.

1) Round-trip delay and link capacity: First, comparing rows 1 and 2 in both tables. By enlarging the delay from 0.02 sec to 0.05 sec (by 2.5 times), the upper bound of window sizes only increases by 1.54 times and 1.86 times for TCP and AIMD$(1/5, /, 7/8)$, respectively, which means a larger delay reduces the relative oscillation amplitude of window size. In addition, the upper bound of queue length is decreasing. Similar trend can be found if comparing rows 4 and 5 in both tables. This is a surprising result. From [3], a longer delay may drive the system from stable to unstable. We can explain it as follows. A larger delay means that the window size increasing speed (in terms of packet per second) during the additive increase period is smaller, and the AIMD flows will overshoot the network capacity in a slower pace; thus, the upper bound of window size is closer to the optimal operating point, and the maximum queue length is smaller. Similar results are found if we compare rows 4



5 TCP flows vs. 5 AIMD flows, C=10000 packet/sec (Matlab) 10 TCP flows vs. 10 AIMD flows, C=20000 packet/sec (Matlab)

5 TCP flows vs. 5 AIMD flows, C=10000 packet/sec (NS-2) 10 TCP flows vs. 10 AIMD flows, C=20000 packet/sec (NS-2)

Fig. 3. Heterogeneous flows, K_p=0.005, and $R = 0.05$ sec

TABLE I
AIMD/RED SYSTEM BOUNDS WITH (α, β)=(1, 1/2)

No.	N	R	C	K_p	$(W^*,\ q^*)$	UB		LB_2		UQ	
						Num	**Ana**	**Num**	**Ana**	**Num**	**Ana**
1	10	0.02	1000	0.01	(2, 37.5)	4.04	4.41	1.52	0.09	51.25	147.50
2	10	0.05	1000	0.01	(5, 6)	6.60	6.80	2.13	0.32	28.15	43.30
3	20	0.05	2000	0.005	(5, 12)	6.60	6.80	2.12	0.38	56.34	78.0
4	10	0.05	1000	0.005	(5, 12)	6.82	7.10	2.78	0.66	39.10	54.60
5	10	0.4	1000	0.005	(40, 3/16)	41.30	42.02	14.14	0.11	9.85	23.2
6	10	0.05	10,000	0.005	(50, 3/25)	51.03	51.15	16.98	0.18	8.35	14.05
7	20	0.05	20,000	0.005	(50, 3/25)	51.00	51.20	8.91	0.068	14.89	23.12
8	100	0.05	10,000	0.005	(5, 12)	6.28	6.41	0.72	0.04	153.16	241.6
9	1000	0.1	1,000,000	0.001	(100, 3/20)	101.00	101.02	0.026	0.0002	576.95	1024.15
10	10000	0.1	1,000,000	0.001	(10, 15)	11.04	11.05	0.02	$1.6*10^{-4}$	6731.3	10785.0
11	10000	0.1	1,000,000	0.005	(10, 3)	11.017	11.023	0.005	$6.9*10^{-6}$	5941.6.5	10349.4
12	10000	0.1	1,000,000	0.01	(10, 3/2)	11.011	11.016	0.002	$1.8*10^{-6}$	5713.5	10247.9

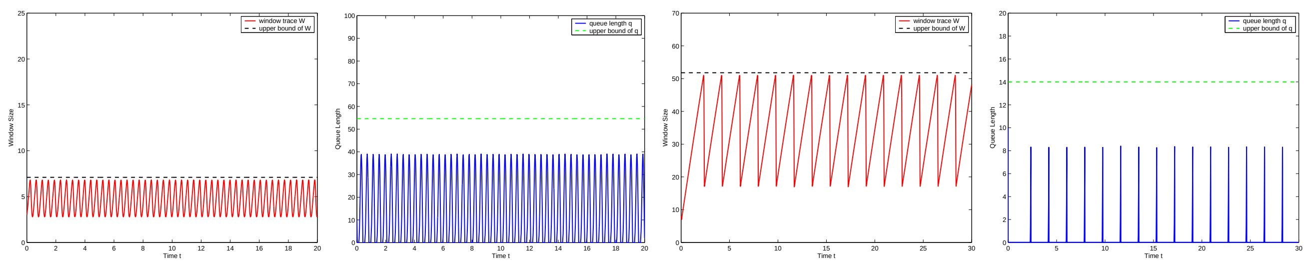
and 6 in both tables. By enlarging the link capacity by 10 times, the upper bound of window size is increased by 7.5 and 8.9 times, for TCP and AIMD $(1/5, /, 7/8)$, respectively. Although enlarging the link capacity may drive the system from stable to unstable [3], the oscillating amplitude of window size (relative to the equilibrium W^*) and queue length will actually decrease. The window and queue traces of 10 TCP flows in a link with 1000 packet/sec and $10,000$ packet/sec are depicted in Fig. 4. The conclusion is that larger values of delay and link capacity will actually reduce the oscillating amplitude of window size and queue length, and significantly reduce the maximum queueing delay.

2) Number of flows: Comparing rows 3 and 4, or rows 6 and 7 in Tables I and II, we conclude that if we increase the number of flows and the link capacity proportionally, the bounds of window size are almost un-affected. With twice the flows multiplexed in a twice capacity link, the upper bound of queue length increases less than twice. Therefore, the queuing delay bound is slightly reduced because of the multiplexing gain.

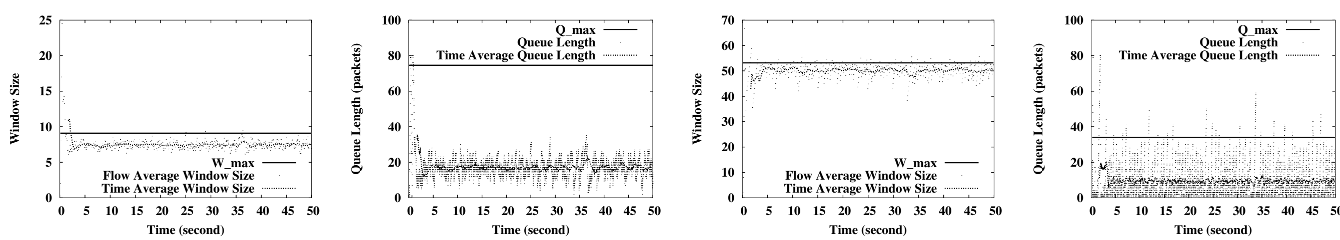
Comparing rows 6 and 8 in Tables I and II, if we increase the number of flows in the same link, the $N \cdot UB$ becomes larger. In other words, the oscillation of window size will increase significantly if the number of flows in a link increases, and

TABLE II
AIMD/RED SYSTEM BOUNDS WITH (α, β)=(1/5, 7/8)

No.	N	R	C	K_p	(W^*, q^*)	UB		LB_2		UQ	
						Num	**Ana**	**Num**	**Ana**	**Num**	**Ana**
1	10	0.02	1000	0.01	(2, 37.5)	2.81	3.03	1.76	0.59	55.39	135.50
2	10	0.05	1000	0.01	(5, 6)	5.50	5.63	4.19	1.77	17.64	31.20
3	20	0.05	2000	0.005	(5, 12)	5.51	5.65	4.19	1.65	35.3	65.2
4	10	0.05	1000	0.005	(5, 12)	5.62	5.80	4.27	2.10	29.13	48.70
5	10	0.4	1000	0.005	(40, 3/16)	40.25	40.29	36.79	5.38	3.10	5.21
6	10	0.05	10,000	0.005	(50, 3/25)	50.23	50.26	45.93	6.31	2.48	3.85
7	20	0.05	20,000	0.005	(50, 3/25)	50.23	50.26	43.99	3.24	4.28	7.10
8	100	0.05	10,000	0.005	(5, 12)	5.34	5.46	3.76	1.39	66.59	83.8
9	1000	0.1	1,000,000	0.001	(100, 3/20)	100.20	100.21	39.26	0.025	127.34	211.15
10	10000	0.1	1,000,000	0.001	(10, 15)	10.22	10.23	2.02	0.02	1666.5	2361.4
11	10000	0.1	1,000,000	0.005	(10, 3)	10.208	10.211	0.07	0.00097	1354.8	2158.8
12	10000	0.1	1,000,000	0.01	(10, 3/2)	10.205	10.207	0.015	0.00024	1265.7	2111.5



$C = 1000$ packet/sec (Matlab) $C = 10,000$ packet/sec (Matlab)



$C = 1000$ packet/sec (NS-2) $C = 10,000$ packet/sec(NS-2)

Fig. 4. Bounds of TCP window size and queue length with different C

the queueing delay will also increase significantly. This can be understood as N AIMD(α, β) flows will increase their windows by $N\alpha$ packets per RTT, and the larger the increasing rate during Additive Increase stage, the more significantly the flows will overshoot the link capacity. This suggests that we should limit the number of TCP/AIMD connections in a link or promote to use more conservative AIMD parameter pairs to ensure that the queueing delay (and also the loss rate) is less than certain threshold.

3) K_p: Comparing rows 2 and 4 in Tables I and II, for a smaller value of K_p, the RED parameter will result in a larger bounds of both window size and queue length.

The last four rows of Tables I and II are the upper bounds of the TCP/AIMD window size and queueing delay in a highly multiplexed, high bandwidth (tens of Gbps), and long delay (0.1 sec RTT) link. It can be seen for TCP flows, the queuing delay can be bounded to 10.785 ms if the K_p is chosen to be 0.001. The delay bound can be slightly reduced to 10.349 ms and 10.248 ms if K_p is increased to 0.005 and 0.01, respectively. The results show that although K_p can be adjusted to control the queueing delay in the system, the impact is limited for high bandwidth cases. Limiting the number of flows or using more conservative AIMD pairs are more effective in reducing queueing delay. For instance, if the number of flows is reduced to 100 or 1000, the queueing delay bound can be reduced to 0.241 ms or 1.079 ms, respectively. If using an AIMD parameter pair of ($1/5$, $7/8$), the queueing delay for 10000 flows with $K_p = 0.001$ can be bounded to 2.361 ms only.

V. Related Work

Internet stability analysis have received wide attention recently [1]–[4]. For delay-free marking scheme, the fluid-model of the AIMD/RED system has been proved to be asymptotically stable [15] by applying the method of Lyapunov function. It is well known [3] that the system may become asymptotically unstable in the presence of time delays. In [16], sufficient conditions for the asymptotic stability of AIMD/RED system with feedback delays are given in terms of linear matrix inequalities. However, simulation results show that even though the system is not asymptotically stable, it oscillates around the steady state periodically. Motivated by this phenomenon, we demonstrate in this report that the delayed AIMD/RED system is bounded from above and below.

The boundedness issue has been studied in [6]–[8] by applying Lyapunov-like method for some TCP-like congestion control algorithms. [9] justified the use of deterministic model for Internet congestion control and [5] gave the upper bound on the transmission rate for two kinds of TCP-like traffic. However, to the best of our knowledge, the theoretical upper and lower bounds of window size and queue length of AIMD/RED system considering feedback delays have not been reported in the literature. Since the bounds are closely related to system performance, which is critically important to obtain in-depth understanding of the whole system, we study the problem in this report.

VI. Conclusion

In this report, we have derived bounds of window size and queue length of the AIMD/RED system. Our main findings are 1) larger values of delay and link capacity will actually reduce the oscillating amplitude of window size and queue length from their equilibrium in steady state; 2) if we proportionally increase the link capacity and number of TCP/AIMD flows, the queueing delay will be slightly reduced, so the multiplexing gain slightly increases; and 3) although AIMD flows can adapt their sending rates according to available bandwidth, larger number of flows leads to longer queueing delay in the AIMD/RED system. Thus, we should limit the number of AIMD connections in a link or promote to use more conservative AIMD parameters to bound the queueing delay and loss. The theorems given in the report can also help to predict and control the system performance for future Internet with higher data rate links multiplexed with more flows with different parameters.

There are many interesting research issues worth further investigation: a) how to deploy effective admission control for TCP/AIMD flows to bound delay and loss; b) how to adapt AIMD parameter pair to ensure that the system can converge to the equilibrium quick enough and to control the queueing delay and loss in the network; and c) how to extend the work to heterogeneous flows over multiple bottleneck links cases.

Acknowledgement

This work has been supported in part by research grants from the Natural Science and Engineering Council of Canada.

References

[1] R. Johari and D. K. H. Tan, "End-to-End Congestion Control for the Internet: Delays and Stability," *IEEE/ACM Transactions on Networking*, vol. 9, no. 6, pp. 818-832, Dec. 2001.
[2] C. V. Hollot and Y. Chait, "Non-linear stability analysis of a class of TCP/AQM networks," *Proc. of IEEE Conference on Decision and Control*, Vol. 3, pp. 2309-2314, Orlando, Florida USA, Dec. 2001.
[3] S. Low, F. Paganini, J. Wang, S. Adlakha and J. C. Doyle, "Dynamics of TCP/RED and a scalable control," *IEEE Infocom'02*, vol. 1, pp. 239-248, Jun. 2002.
[4] C. V. Hollot, V. Misra, D. Towsley and W. B. Gong, "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows," *IEEE Trans. on Automatic Control*, Vol. 47, No. 6, pp. 945-959, June 2002.
[5] S. Shakkottai, R. Srikant and S. Meyn, "Bounds on the Throughput of Congestion Controllers in the Presence of Feedback Delay," *IEEE Transactions on Networking*, vol. 11, no. 6, pp. 972-981, Dec. 2003.
[6] Z. Wang and F. Paganini, "Boundedness and Global Stability of a Nonlinear Congestion Control with Delays," *IEEE Transactions on Automatic Control*, vol. 51, no. 9, pp. 1514-1519, Sept. 2006.
[7] L. Ying, G. E. Dullerud and R. Srikant,"Global Stability of Internet Congestion Controllers with Heterogeneous Delays," *IEEE Transactions on Networking*, vol. 14, no. 3, pp. 579-591, Jun. 2006.
[8] S. Deb and R. Srikant, "Global Stability of Congestion Controllers for the Internet," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 1055-1060, Jun. 2003.
[9] S. Shakkottai and R. Srikant, "How Good are Deterministic Fluid Model of Internet Congestion Control?" *IEEE Infocom'02*, vol. 2, pp. 497-505, Jul. 2002.
[10] V. Misra, W. B. Gong and D. Towsley, "Fluid-Based Analysis of a Network of AQM Routers Supporting TCP Flows," *Proc. of ACM/SIGCOMM*, pp. 151–160, 2000.
[11] S. Floyd and K. Fall, "Promoting the use of end-to-end congestion control in the Internet," *IEEE/ACM Transactions on Networking*, vol. 7, no. 4, pp. 458-472, Aug. 1999.
[12] S. Floyd, M. Handley and J. Padhye, "A Comparison of Equation-Based and AIMD Congestion Control", May 2000. Available http://www.aciri.org/tfrc/tcp-friendly.TR.ps.
[13] Y. R. Yang and S. S. Lam, General AIMD Congestion Control. Technical Report TR-2000-09, University of Texas, May 2000. A shorter version appeared in Proceedings of ICNP'00, Osaka, Japan, November 2000.
[14] L. Cai, X. Shen, J. Pan and J. W. Mark, "Performance Analysis of TCP-Friendly AIMD Algorithms for Multimedia Applications," *IEEE Trans. on Multimedia*, Vol. 7, No. 2, pp. 339–355, April 2005.
[15] L. Wang, L. Cai, X. Liu and X. Shen, "AIMD Congestion Control: Stability, TCP-friendliness, Delay Performance," *Tech. Rep.*, Mar. 2006. Available http://www.ece.uvic.ca/~cai/tech-2006.pdf
[16] L. Wang, L. Cai, X. Liu, and X. Shen, "Stability and TCP-friendliness of AIMD/RED Systems with Feedback Delays," accepted by *Computer Networks*. Available http://www.math.uwaterloo.ca/~l44wang/publications/Computer_Networks_2007.pdf
[17] F. P. Kelly, "Models for a self-managed Internet," *Philosoph. Trans. Royal Soc. London*, vol. 358, pp. 2335-2348, 2000.
[18] S. Liu, T. Basar and R. Srikant, "Pitfalls in the Fluid Modeling of RTT Variations in Window-Based Congestion Control," *Proc. of IEEE INFOCOM*, vol. 2, pp. 1002-1012, Miami, Florida USA, March 2005.
[19] D. G. Luenberger, *Introduction to Dynamic Systems: Theory, Models, and Application,* May 1979. ISBN: 0-471-02594-1
[20] H. K. Khalil, *Nonlinear Systems,* Upper Saddle River, N.J.: Prentice Hall, 2002.
[21] S. Floyd and S. McCanne, "Network Simulator," LBNL public domain software. Available via ftp from ftp.ee.lbl.gov. NS-2 is available in http://www.isi.edu/nsnam/ns/.