Optimal Investment for Retail Company in Electricity Market

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***Abstract*—Considering an optimal investment problem for a retailer in electricity market, the objective is to seek the optimal investment decision that maximizes the weighted sum of the expected return and the variance of wealth. Unlike existing works, the price fluctuation of both the wholesale and retail side of electricity market is considered, and the retailer can invest its wealth in electricity market and traditional financial market simultaneously. Hence, there is a complicated wealth dynamic, which is the main challenge in our work. In this paper, by utilizing the method of Lagrange multiplier and the classical Tchebycheff inequality, we first show that the investment problem is a quadratic programming problem in terms of the decision variable, and thus has a unique optimal solution. Then, a closed-form optimal solution is derived by solving the stationary equation and comparing the feasible solution interval. Based on the optimal solution, we find the key price, which will affect the investment is the wholesale price rather than the retail price. Moreover, with a similar analysis approach, we also provide the optimal solution considering a more general model, which allows the retailer to purchase the electricity temporarily to avoid the supply shortage. Extensive simulations demonstrate the better performance of the proposed solution over the Kelly strategy widely used in the financial market.**

***Index Terms*—Electricity market, optimal investment, risk management, smart grid.**

I. Introduction

SMART GRID, as the next-generation power grid, enables environmentally friendly electricity generation and smart electricity trading mechanisms, and endows both suppliers and consumers with full visibility and pervasive control over their assets and services [1]–[5]. Hence, the emerging electricity market enables numerous revolutionary features, such as various electricity suppliers (e.g., solar or wind generator) and real-time electricity price, such that the market becomes more competitive and diverse than the traditional electricity market [6]–[9]. Consequently, it presents more opportunities for retailers, e.g., electricity retail companies, to make profit by flexible electricity trading with the grid.

In the emerging electricity market, retail companies purchase electricity from power companies in the wholesale market and sell it to end-use customers in the retail market [10]–[12]. Hence, retail companies need to manage two sets of contracts–one with power companies (i.e., the supply side) and the other with end-use customers (i.e., the demand side). At the supply side, the price for wholesale electricity, say wholesale price, can be predetermined by a retailer and a power company through a bilateral contract, e.g., a contract in which a mutual agreement has been made between the parties. This wholesale price depends on the amount of electricity and the time a retailer intends to purchase. Since the competition in the wholesale market is inevitable and electricity demands change with time, the wholesale price is uncertain and fluctuates over time [2]. Thus, in the wholesale market, the retailer is exposed to the risk caused by the fluctuation of the wholesale price in the wholesale market. At the demand side, the price for retail electricity, say retail price, also fluctuates with time as the real-time price is used in the retail market [10], [13], [14]. Meanwhile, the retailer is obliged to serve the varying demand of its customers, i.e., the electricity demands of customers should be met by the retailer who is selected as the supplier for the customers. Hence, in the retail market, the retailer is exposed to the uncertainties of both retail price and customer demands, and thus needs to perform risk assessment in electricity trading [15]. Therefore, in the emerging electricity market, a retail company needs to predict the electricity prices in both the wholesale and retail markets (both supply side and demand side) so as to develop an optimal purchasing strategy.

However, the existing literatures mainly focus on one of the electricity markets, only the wholesale or retail one. In this paper, to find optimal investment decisions for a retailer, we formulate the investment problem using a classical Markowitz framework in finance [16], i.e., using mean and variance to model investment return and risk, respectively. The effectiveness of this framework on investment problems is reconfirmed recently by Markowitz [17], which motivates us to use this framework to model the investment problem in electricity market. The framework has been applied to model and solve the

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electricity procurement problem in electricity market [18], [19]. Then, the uncertainties in both wholesale and retail sides of the emerging electricity markets are considered in our model. Meanwhile, note that in reality, an electricity retail company can also invest its wealth in the financial market, and thus our model allows the retailer to invest in both the financial market (e.g., stock market) and the electricity market [e.g., electric vehicle (EV) charging market]. By utilizing the method of Lagrange multiplier and the classical Tchebycheff inequality, we first prove that the formulated investment problem is a convex quadratic programming problem, and then we obtain a closed-form optimal solution by solving the problem with the method of convex optimization. The major contributions of this work are summarized as follows.

1) To the best of our knowledge, this is the first work that investigates the optimal investment decision for a retail company considering the fluctuations of both wholesale price and retail price. Our model allows the retailer to invest in the financial market and the electricity market simultaneously, the results reveal how the estimated return in financial market affects the decision in the electricity market.
2) We obtain the optimal solution to the proposed problem with quadratic programming, which can guarantee that the probability of supply deficiency is lower than a given small bound. Then, considering a more general model allowing temporary purchase to avoid supply deficiency, we also give the relevant optimal investment decision to the retail company.
3) Compared with the traditional investment problem, we reveal a surprising property that a high risk may have a low return for proposed problem. We also observe that the optimal investment decision is not affected by the retail price in the electricity market, which is totally different from that in traditional financial markets.

This paper is organized as follows. Section II discusses the related work. In Section III, we introduce the system model and problem formulation. Section IV presents the optimal solution of the proposed problem. Simulation results are presented in Section V, followed by the concluding remarks and further discussions in Section VI.

II. Related Work

Many efforts have been devoted to studying the emerging electricity market, e.g., electricity pricing and scheduling [6]–[8], [20]–[23], risk management [15], [24], [25], and investment decision [26], [27]. For example, the works in [20] and [21] were concerned with setting the real-time retail price to maximize the aggregate surplus of users and retailers subjecting to the supply–demand balancing. Denton *et al.* [25] studied how to measure and manage the market risks in operations by using option models and stochastic optimization techniques.

On the other hand, the optimal investment problem becomes a hot research topic recently, which is also the main concern in this paper. The objective is to maximize the profit while minimize the risk of the investment in electricity market. The famous mean–variance framework (Markowitz methodology) in finance has been widely used to model and solve the investment problem (e.g., electricity procurement) in electricity market [18], [19], [28]. For example, the authors in [28] used a mean-variance framework to address the best decision of peak/off peak forward contracts. In addition, the value-at-risk (VaR) and conditional VaR (CVaR) methodologies used in finance can also be used to model the investment risk in electricity market [23]. Recently, taking new power generation into consideration, the authors in [26] presented a novel model for optimization of investments in the electricity market, where the model can calculate optimal investment strategies under both centralized social welfare and decentralized profit objectives. A stochastic programming method is presented to study how to determine the optimal price based on fixed pricing and the amounts of purchased electricity from the market and forward contracts [29].

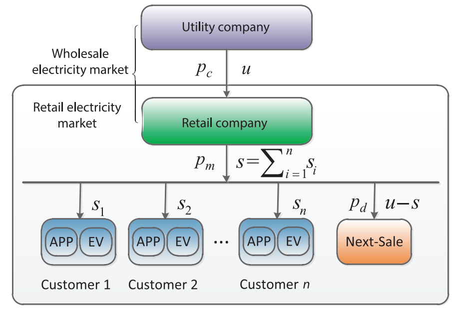

Fig. 1. Simplified illustration of electricity market.

However, most of these works focused on one of the electricity markets, i.e., only wholesale or retail one. Meanwhile, the existing works on investment problems in the electricity market ignored the impact from the financial market. To address these problems, this paper formulates a novel investment problem by utilizing a typical mean-variance portfolio model. Compared with the existing works, the main novelties of this paper include that enabling the retail company to invest in both the financial and electricity markets, revealing new properties of the related investment problem, and providing closed-form optimal solutions.

III. Market Model and Problem Formulation

A. Electricity Market Model

An electricity market generally consists of a wholesale electricity market and a retail electricity market. In the wholesale electricity market, there exist competing generators who offer their electricity output to retailers, while in the retail electricity market, there exist competing electricity retailers who can be chosen by the end-users for their suppliers. As shown in Fig. 1, suppose that there are a utility company (e.g., generator), a retail company (supplier), and customers (end-users) in the electricity market. The retail company purchases electricity from utility company in the wholesale market, and then sells it to customers in the retail market.

Let p_c be the unit price for the retail company purchasing electricity from the utility company in the wholesale electricity market, named *wholesale price*. This price can be

TABLE I
STATISTICS OF RANDOM VARIABLES

Variables	p_c	p_m	p_d	s	r_0
Mean	p_c	e_m	e_d	e_s	e_r
Variance	0	σ_m^2	σ_d^2	σ_s^2	σ_r^2

predetermined by a retail company and a utility company through a bilateral contract, and thus we assume that the *wholesale price* p_c is determined and known to the retail company when it purchases electricity from the utility company. Let p_m be the unit price for the retail company selling electricity to customers in the retail electricity market, named *retail price*. Referring to [30], the retail real-time price program has been applied in some places and it would be a trend in the future. We thus assume that real-time pricing is used in the retail market and p_m is a random variable. Let $s = \sum_{i=1}^{n} s_i$ be a random variable denoting the total amount of demand electricity for all customers during a fixed time period (e.g., 1 h or more), where each s_i is the demand of customer i for $i = 1, 2, \ldots, n$. Suppose that a retail company purchases a certain amount of electricity, denoted by u, from the utility company at each trading, and then we name u as the investment decision variable for the retail company (here u is not the total generated electricity and thus could be less than s). Assume that there is a fixed trading period between retail company and utility company. After a fixed time period of sale, the retailer still has $u - s$ electricity not been sold. The unit price of the remaining electricity, denoted by p_d, depends on the *wholesale price* or the price it can be sold out at that time when the retail company needs to make the next round investment decision. We model p_d as a random variable since the future *wholesale price* is not known to the retail company and may depend on the electricity generation and requirement at that time. In this paper, we assume that it is not necessary for a retail company to store the electricity and the electricity is generated and stored in the power generator. Each buying process of a retail company just guarantees that the retail company can get a certain amount of electricity from the generation with a wholesale price.

Note that the relevant statistics of the above random variables can be regressed from their historical data in the market. Thus, we assume that the mean and variance of all random variables considered in this paper are provided. We summarize all of them in Table I. Since p_c is assumed to be a constant, its mean and variance are p_c and 0, respectively. The mean and variance of the random variables are positive numbers denoted by e and σ^2, respectively, and it is assumed that these random variables are mutually independent.[1]

B. Problem Formulation

Assume that a manager of the retail company (the investor) joins the electricity market with an initial wealth w_0. The manager allocates part of his wealth to purchase electricity from the wholesale electricity market for reselling it in the retail market, and allocates the remainder wealth to one asset in the financial market (e.g., put it in a bank or buy a kind of stock). Assume that the return of the asset in the financial market is r_o which is also a random variable. After one trading period, the total wealth hold by the manager is denoted by w_1. Then, the wealth dynamic is given by

$$\begin{aligned} w_1 &= (w_0 - p_c u) r_0 + p_m s + p_d(u - s) \\ &= w_0 r_0 + u(p_d - p_c r_o) + s(p_m - p_d) \\ &= w_0 r_0 + u r_1 + s r_2 \end{aligned} \quad (1)$$

where $r_1 = p_d - p_c r_0$ and $r_2 = p_m - p_d$.

Let $\boldsymbol{r} = [r_0, r_1, r_2]^T$. Since the mean and variances of random variables r_0, p_m, and p_d are given, it is easy to obtain the mean and covariance of random vector $\boldsymbol{r}$, i.e.,

$$\mathbf{E}[\boldsymbol{r}] = [e_r, e_d - p_c e_r, e_s(e_m - e_d)]^T \quad (2)$$

and

$$\mathbf{Cov}[\boldsymbol{r}] = \begin{bmatrix} \sigma_r^2 & -p_c\sigma_r^2 & 0 \\ * & \sigma_d^2 + p_c^2\sigma_r^2 & -\sigma_d^2 \\ * & * & \sigma_m^2 + \sigma_d^2 \end{bmatrix}. \quad (3)$$

A retail company needs to provide a sufficient electricity to customers, such that the demands of customers are satisfied. Hence, the retail company usually should purchase enough electricity from the utility company at each trading period to meet the requirement of customers, i.e., $u \geq s$. Otherwise, the retail company may need to purchase the electricity with a much higher price (e.g., *retail price*) from other retail companies or utility companies to meet the demands of customers. In case that the customers' electricity demands cannot be satisfied by a retail company, they may switch to other retail companies and thus the retail company will lose its customers. Considering this issue, it is better for a retail company to purchase a larger amount of electricity than demands at each trading period. However, note that the total demands s is a random variable, $u \geq s$ cannot be guaranteed completely. Thus, we give a constraint to guarantee a reliable service to the end-users as follows:

$$\Pr[u - s \leq 0] \leq \varepsilon \quad (4)$$

i.e., the decision u should ensure that the probability of $u < s$ is less than a given small value ε, where $\Pr[u - s \leq 0]$ is the probability of short supply. Meanwhile, the decision u generally should satisfy

$$0 < u \leq \frac{w_0}{p_c}. \quad (5)$$

Although the remanning electric quantity $u - s$ can be used for next-round selling, it does not mean that purchasing electricity as much as possible is the best decision as the unit price p_d is a random variable which could be lower than p_c. This is undesirable to the retail company, and thus the overinvestment in electricity may bring some risk to the company. Moreover, by purchasing a less amount of electricity, the

[1]Since most of the demand depends on the customers' preferences especially when the real-time price is not very high, the price has little affection on the demand. Thus, we assume that the demand s is independent of the price p_m. When the affections of the price on the demand cannot be ignored, e.g., when there are some price incentive policy, the investment problem would be more challenging and will be left as the future work.

manager can have more money to invest in financial market. Therefore, it is interesting to seek an investment decision for a retail company at each trading period to optimize given certain objectives. Mathematically, we formulate a mean–variance investment problem which can be posed in one of the following two forms:

$$\begin{aligned} &\max_u \mathbf{E}[w_1] \\ &\text{s.t.} \quad \mathbf{Var}[w_1] \le \varrho, \\ &\quad (1), (4), \text{and } (5) \end{aligned} \tag{6}$$

or

$$\begin{aligned} &\min_u \mathbf{Var}[w_1] \\ &\text{s.t.} \quad \mathbf{E}[w_1] \ge \epsilon, \\ &\quad (1), (4), \text{and } (5). \end{aligned} \tag{7}$$

By varying the value of ϱ in (6) or ϵ in (7), the set of efficient solutions of them can be generated. By utilizing the method of Lagrange multiplier, an equivalent formulation to either (6) or (7) is given as

$$\begin{aligned} &\max_u J(u) = \alpha\mathbf{E}[w_1] - \mathbf{Var}[w_1] \\ &\text{s.t. } (1), (4), \text{and } (5) \end{aligned} \tag{8}$$

where $\alpha > 0$ is a weight for the expected return of wealth. A larger α will increase the importance of the expected return while decreasing the importance of the risk for the investment. The objective function in (8) is a tradeoff between the expected return and the associated risk. Clearly, if u^* is the optimal solution of (8), then it is the optimal solution of (6) with $\varrho = \mathbf{Var}[w_1]|_{u^*}$ or of (7) with $\epsilon = \mathbf{E}[w_1]|_{u^*}$. Since we can vary the values of α to meet different value of ϱ or ϵ, we only consider (8) in this paper.

The main differences between the above problems and traditional financial investment problem, e.g., [31] and [32], are given as follows.

1) We have a more complicated wealth dynamic function, as more random variables, the prices, p_m and p_d, and the demand s will affect the return of the investment in an electricity market, while there is only one random variable r_0 will affect the return of each investment in a financial market.
2) We have more complicated investment risk, which depends not only on the fluctuation of the prices of investment assets (e.g., electricity and stock) but also on the fluctuation of the electricity demands. As a result, the common sense in the financial market, a higher investment risk is associated with a higher return, may no longer hold.
3) Considering the special requirement in power market, the customers demands usually should be met; there is an additional constraint [the constraint (4) in (8)] to the investment in the problem. Under this additional constraint, it will be observed that the optimal decision does not depend on the retail price p_m, which is totally different from that in the financial market.

Problem (8) is a mixed investment problem as it includes two different markets: 1) the financial market; and 2) the electricity market. Therefore, it is interesting while challenging to investigate (8) to help a retail company to make an optimal decision, and analyze how and which the key factors will affect the investment in electricity market.

IV. Closed-Form Optimal Solution

In this section, we first analyze the relationship between return and risk of the investment. Then, we provide a closed-form optimal solution for (8). Based on the obtained optimal solution, we reveal some important properties.

A. Relationship Between Return and Risk

Taking the expectation on both sides of (1), we obtain the expected wealth return as

$$\mathbf{E}[w_1] = w_0 e_r + u(e_d - p_c e_r) + e_s(e_m - e_d) \tag{9}$$

which is a linear function of decision variable u. Clearly, $\mathbf{E}[w_1]$ is an increasing function of u when $e_d - p_c e_r > 0$, but is a decreasing function of u when $e_d - p_c e_r \le 0$. $\mathbf{E}[w_1]$ is an increasing function of e_s when $e_m - e_d > 0$, and a decreasing function of e_s when $e_m - e_d \le 0$. Hence, a higher investment or a higher requirement in electricity market cannot guarantee a higher return, it depends on the sign of $e_d - p_c e_r$ and $e_m - e_d$.

Given $\mathbf{Var}[w_1] = \mathbf{E}[w_1^2] - \mathbf{E}^2[w_1]$, the variance satisfies

$$\begin{aligned} \mathbf{Var}[w_1] &= \mathbf{E}[(w_0 r_0 + u r_1 + s r_2)^2] - \mathbf{E}^2[w_0 r_0 + u r_1 + s r_2] \\ &= w_0^2 \mathbf{Var}[r_0] + u^2 \mathbf{Var}[r_1] + \mathbf{Var}[s r_2] \\ &\quad + 2 w_0 u \mathbf{Cov}[r_0 r_1] + 2 w_0 \mathbf{E}[s] \mathbf{Cov}[r_0 r_2] \\ &\quad + 2 u \mathbf{E}[s] \mathbf{Cov}[r_1 r_2]. \end{aligned} \tag{10}$$

Then, by using (3), we have

$$\begin{aligned} \mathbf{Var}[w_1] &= w_0^2 \sigma_r^2 + u^2 \left(\sigma_d^2 + p_c^2 \sigma_r^2\right) + \mathbf{Var}[s r_2] \\ &\quad - 2\left[w_0 u p_c \sigma_r^2 + u e_s \sigma_d^2\right]. \end{aligned} \tag{11}$$

Note that

$$\begin{aligned} \mathbf{Var}[s r_2] &= \mathbf{E}[s^2]\mathbf{E}\left[r_2^2\right] - \mathbf{E}^2[s]\mathbf{E}^2[r_2] \\ &= (\mathbf{Var}[s] + \mathbf{E}^2[s])(\mathbf{Var}[r_2] + \mathbf{E}^2[r_2]) \\ &\quad - \mathbf{E}^2[s]\mathbf{E}^2[r_2] \\ &= \sigma_s^2\left(\sigma_m^2 + \sigma_d^2\right) + \sigma_s^2 (e_m - e_d)^2 \\ &\quad + e_s^2\left(\sigma_m^2 + \sigma_d^2\right) = B \end{aligned} \tag{12}$$

which is a computable constant. Hence, (11) is simplified as

$$\begin{aligned} \mathbf{Var}[w_1] &= \left(\sigma_d^2 + p_c^2 \sigma_r^2\right) u^2 - 2\left(e_s \sigma_d^2 + w_0 p_c \sigma_r^2\right) u \\ &\quad + B + w_0^2 \sigma_r^2. \end{aligned} \tag{13}$$

Clearly, $\mathbf{Var}[w_1]$ is a quadratic function of u. Taking the derivative of $\mathbf{Var}[w_1]$ over u, we have

$$\frac{\mathrm{d}\mathbf{Var}[w_1]}{\mathrm{d}u} = 2u(\sigma_d^2 + p_c^2 \sigma_r^2) - 2(e_s \sigma_d^2 + w_0 p_c \sigma_r^2). \tag{14}$$

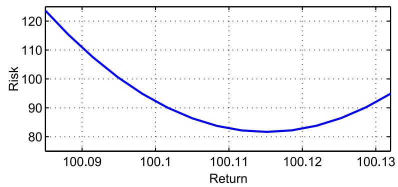

Fig. 2. Relationship between return and risk.

Solving $\frac{\mathrm{d}\mathbf{Var}[w_1]}{\mathrm{d}u} = 0$, one obtains a stationary point $\tilde{u}$ of $\mathbf{Var}[w_1]$ satisfying

$$\tilde{u} = \frac{w_0 p_c \sigma_r^2 + e_s \sigma_d^2}{\sigma_d^2 + p_c^2 \sigma_r^2}. \tag{15}$$

Since the coefficient of u^2 satisfies $(\sigma_d^2 + p_c^2\sigma_r^2) \geq 0$, $\mathbf{Var}[w_1]$ is a decreasing function of u when $u < \tilde{u}$, while it is an increasing function of u when $u > \tilde{u}$. Hence, at the stationary point, $\mathbf{Var}[w_1]$ reaches the smallest value, i.e., the risk of the investment is minimized when $u = \tilde{u}$.

Based on the above discussions, we have the following lemma, which provides the properties of the investment.

Lemma 1: Considering (20), we have the following points.

1) If $e_d - p_c e_r > 0$, a larger u leads to a higher $\mathbf{E}[w_1]$. When $u \leq \tilde{u}$, it has a lower $\mathbf{Var}[w_1]$; otherwise, it has a higher $\mathbf{Var}[w_1]$.
2) If $e_d - p_c e_r \leq 0$, a larger u leads to a lower $\mathbf{E}[w_1]$. When $u \leq \tilde{u}$, it has a lower $\mathbf{Var}[w_1]$; otherwise, it has a higher $\mathbf{Var}[w_1]$.

From Lemma 1, one sees that a higher return may have a lower risk, which contradicts to the fact in traditional financial investment that high return means high risk.

Example 1: Given the statistics of the random variables in Table I, where the detail values will be provided in the simulation section, we can utilize (9) and (13) to calculate the expected return and variance (risk) under different decision u. Then, we obtain the relationship between return and risk, which is shown in Fig. 2. It is observed that there is a stationary point with the lowest risk, which is not corresponding to the lowest return, and the same risk may be corresponding to a low return and a high return. Hence, it is important to seek an optimal decision for (8), which will be discussed in the following section.

B. Optimal Solution

From the above section, the expected wealth return is a linear function of u and the variance of wealth is a quadratic function of u. Thus, the objective function is also a quadratic function of u, which means that (8) is a quadratic programming problem.

However, we cannot solve (8) directly using quadratic programming as (4) is not a convex constraint. Fortunately, by Tchebycheff inequality, we have

$$\Pr[u - s \leq 0] \leq \frac{\mathbf{Var}[u-s]}{(\mathbf{E}[u-s] - 0)^2}.$$

Based on the above inequality, one can set a small ε as a upper bound for $\frac{\mathbf{Var}[u-s]}{\mathbf{E}^2[u-s]}$, such that the constraint (4) holds, i.e., $\Pr[u - s \leq 0] \leq \frac{\mathbf{Var}[u-s]}{\mathbf{E}^2[u-s]} \leq \varepsilon$. Note that

$$\begin{aligned} \frac{\mathbf{Var}[u-s]}{\mathbf{E}^2[u-s]} \leq \varepsilon &\Leftrightarrow \mathbf{Var}[u-s] \leq \varepsilon \mathbf{E}^2[u-s] \\ &\Leftrightarrow \mathbf{E}[(u-s)^2] \leq (1+\varepsilon)\mathbf{E}^2[u-s] \\ &\Leftrightarrow \sigma_s^2 - \varepsilon(u - e_s)^2 \leq 0. \end{aligned} \tag{16}$$

Solving inequality $\sigma_s^2 - \varepsilon(u-e_s)^2 \leq 0$ gives the results that $u \geq e_s + \sqrt{\frac{\sigma_s^2}{\varepsilon}}$ or $u \leq e_s - \sqrt{\frac{\sigma_s^2}{\varepsilon}}$. Since $\Pr[u - s \leq 0]$ is a decreasing function of u, we obtain $\Pr[u - s \leq 0] \leq \varepsilon$ when $u \geq e_s + \sqrt{\frac{\sigma_s^2}{\varepsilon}}$, i.e., (4) can be guaranteed by setting $u \geq e_s + \sqrt{\frac{\sigma_s^2}{\varepsilon}}$. From (5), it follows that the probability $\Pr[u - s \leq 0]$ has a smallest upper bound ε, which satisfies

$$\varepsilon = \frac{\sigma_s^2}{\left(\frac{w_0}{p_c} - e_s\right)^2}$$

i.e., if $\varepsilon < \frac{\sigma_s^2}{\left(\frac{w_0}{p_c} - e_s\right)^2}$, then it needs $u > \frac{w_0}{p_c}$ to guarantee (4), which contradicts to (5). Thus, it is reasonable to assume that the ε in (4) satisfy $\varepsilon > \frac{\sigma_s^2}{\left(\frac{w_0}{p_c} - e_s\right)^2}$. Then, we have

$$e_s + \sqrt{\frac{\sigma_s^2}{\varepsilon}} < \frac{w_0}{p_c}. \tag{17}$$

Let

$$A_1 = e_s + \sqrt{\frac{\sigma_s^2}{\varepsilon}} \tag{18}$$

be the lowest requirement of the electricity investment. Then

$$A_1 \leq u \leq \frac{w_0}{p_c} \tag{19}$$

can guarantee the constraints of (4) and (5). Therefore, (8) can be simplified as

$$\begin{aligned} \max_{u} \quad & J(u) = \alpha \mathbf{E}[w_1] - \mathbf{Var}[w_1] \\ \text{s.t.} \quad & (1) \text{ and } (19). \end{aligned} \tag{20}$$

For (20), note that the objective function is a quadratic function of u, and both (1) and (19) are linear constraints. By referring [33], we state the following theorem, which shows that (20) is a convex optimization problem.

Theorem 1: Quadratic programming: Problem (20) is a convex quadratic programming problem and it has a unique global optimal solution.

Based on the above theorem, (20) can be solved as follows. Substituting (1) into $J(u)$ in (20), we obtain the detail expression of $J(u)$, and then we obtain the stationary point of the objective function by solving $\frac{\partial J(u)}{\partial u} = 0$. Comparing the value of the stationary point to the constraint (19), we can obtain the optimal solution u^*. Define

$$A_0 = \frac{\frac{\alpha}{2}(e_d - p_c e_r) + \left(e_s \sigma_d^2 + w_0 p_c \sigma_r^2\right)}{\sigma_d^2 + p_c^2 \sigma_r^2} \tag{21}$$

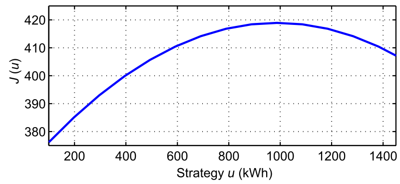

Fig. 3. Relationship between u and $J(u)$.

which is the stationary point of $J(u)$ and we have the following theorem, which provides the closed-form optimal solution for (20).

Theorem 2: Optimal solution: Considering (20), then u^* satisfies as follows.

1) If $A_0 < A_1$, $u^* = A_1$.
2) If $A_0 > \frac{w_0}{p_c}$, $u^* = \frac{w_0}{p_c}$.
3) Otherwise, $u^* = A_0$.

Proof: Submitting (1) into $J(u)$, we have

$$\begin{aligned} J(u) &= \alpha[w_0 e_r + u(e_d - p_c e_r) + e_s(e_m - e_d)] \\ &\quad - \left(\sigma_d^2 + p_c^2\sigma_r^2\right) u^2 + 2\left(e_s\sigma_d^2 + w_0 p_c \sigma_r^2\right) u \\ &\quad - B - w_0^2\sigma_r^2 \\ &= -\left(\sigma_d^2 + p_c^2\sigma_r^2\right) u^2 \\ &\quad + \left[\alpha(e_d - p_c e_r) + 2\left(e_s\sigma_d^2 + w_0 p_c\sigma_r^2\right)\right] u \\ &\quad + \alpha\left[w_0 e_r + e_s(e_m - e_d)\right] - \left(B - w_0^2\sigma_r^2\right)\right]. \end{aligned} \tag{22}$$

Taking the derivative of $J(u)$ over u, we have

$$\begin{aligned} \frac{\mathrm{d}J(u)}{\mathrm{d}u} &= -2\left(\sigma_d^2 + p_c^2\sigma_r^2\right) u \\ &\quad + \alpha(e_d - p_c e_r) + 2\left(e_s\sigma_d^2 + w_0 p_c\sigma_r^2\right). \end{aligned} \tag{23}$$

Solving $\frac{\mathrm{d}J(u)}{\mathrm{d}u} = 0$, we obtained $u = A_0$, which means that the objective function $J(u)$ has the maximum value when $u = A_0$. From (23), one also infers that $\frac{\mathrm{d}J(u)}{\mathrm{d}u} < 0$ when $u < A_0$, and $\frac{\mathrm{d}J(u)}{\mathrm{d}u} > 0$ when $u > A_0$. Since u should satisfy $A_1 \le u \le \frac{w_0}{p_c}$ due to constraint (19), one can obtain the optimal solution for (20) as given in the theorem. ■

Since (20) is a simplified problem of (8), the optimal solution given in Theorem 2 is also the optimal solution for (8). Then, we study the relationship between decision variable u and the objective function $J(u)$, where the value of $J(u)$ is calculated from (22). The result is shown in Fig. 3. Under the same setting of parameters as in Example 1 (provided in Section V), it is observed from Fig. 3 that there exists an optimal decision $u = 978.46$ such that $J(u)$ reaches the maximum value. However, since $A_1 = 1022.8$ [obtained from (18) by setting $\varepsilon = 5\%$] in this case, it follows from Theorem 2 that the optimal decision for (8) is actually $u^* = 1022.8$.

C. Further Discussion

From the above analysis, we have three important insights into the optimal decision u^* of (8).

First, the optimal decision u^* does not depend on the *retail price* p_m. Note that the possible values of optimal solution u^* are A_0, A_1, and $\frac{w_0}{p_c}$. All of them are independent of price p_m, and thus the optimal decision does not depend on the retail price, which contradicts to the intuition in financial market that a higher retail price of an asset will motivate the investors to invest more of this asset. In theory, from (9) and (13), one infers that the contributions of retail price p_m to the objective function in (8) are $e_s e_m$ in expected return and B in variance. Hence, different u does not affect the contributions of retail price p_m on both return and risk, and thus the optimal u^* will not be affected by p_m. In fact, this is because that in our modeling, all customers' electricity requirement should be met, i.e., u must be larger than s, and the leftover electricity will affect the return/profit, and thus the return depends on p_d rather than p_m. If we remove the constraint that all customers' electricity requirement should be met, then p_m will play a role, which will be discussed in Remark 2.

Second, if $e_d - p_c e_r = 0$, the optimal decision u^* will not be affected by the weight α, which means that different balances between return and risk will have the same optimal solution for (8). We have the following corollary.

Corollary 1: If $e_d - p_c e_r = 0$, then the optimal solution u^* for (8) satisfies

$$u^* = \max\{\tilde{u}, A_1\}.$$

Proof: When $e_d - p_c e_r = 0$, we have

$$A_0 = \frac{\left(e_s\sigma_d^2 + w_0 p_c\sigma_r^2\right)}{\sigma_d^2 + p_c^2\sigma_r^2} = \tilde{u}.$$

Note that

$$\begin{aligned} \max\left\{\frac{e_s\sigma_d^2}{\sigma_d^2}, \frac{w_0 p_c\sigma_r^2}{p_c^2\sigma_r^2}\right\} &= \frac{\max\left\{\frac{e_s\sigma_d^2}{\sigma_d^2}, \frac{w_0 p_c\sigma_r^2}{p_c^2\sigma_r^2}\right\}\left(\sigma_d^2 + p_c^2\sigma_r^2\right)}{\left(\sigma_d^2 + p_c^2\sigma_r^2\right)} \\ &= \frac{\max\left\{e_s\sigma_d^2, \frac{w_0 p_c\sigma_r^2}{p_c^2\sigma_r^2}\sigma_d^2\right\} + \max\left\{\frac{e_s\sigma_d^2}{\sigma_d^2}p_c^2\sigma_r^2, w_0 p_c\sigma_r^2\right\}}{\left(\sigma_d^2 + p_c^2\sigma_r^2\right)} \\ &\ge \frac{e_s\sigma_d^2 + w_0 p_c\sigma_r^2}{\left(\sigma_d^2 + p_c^2\sigma_r^2\right)} = \tilde{u}. \end{aligned} \tag{24}$$

Then, we have

$$\tilde{u} \le \max\left\{\frac{e_s\sigma_d^2}{\sigma_d^2}, \frac{w_0 p_c\sigma_r^2}{p_c^2\sigma_r^2}\right\} \le \max\left\{e_s, \frac{w_0}{p_c}\right\}.$$

From (17), we infer that $e_s < \frac{w_0}{p_c}$. Hence, from Theorem 2, u^* should be the larger one among $\tilde{u}$ and A_1. ■

Note that both $\tilde{u}$ and A_1 are independent of α, and thus it follows from the above corollary that u^* is not affected by the weight. In theory, from (1), one sees that when $e_d - p_c e_r = 0$, the dynamic of wealth w_1 does not depend on u, and thus will not be affected by the return of the wealth. This is the main reason why different weights setting to return and risk may have the same optimal solution. In addition, note from (15) that $\tilde{u}$ is the stationary point of variance, where the variance reaches the smallest value. From (19), A_1 is the lowest value that u can be. This corresponds to the fact that the optimal decision should be with the lowest variance (risk), when the value of u will not affect the expected return. In this case, we only need to compare the risk of two markets and put all additional money (beyond satisfying the demand) to the place with a lower risk.

Third, if the price of p_d is deterministic, the retail company will invest all its wealth to purchase electricity from wholesale company when $e_d - p_c e_r \geq 0$, and the wealth for investing in the electricity market will be decreased with α when $e_d - p_c e_r < 0$. We have the following corollary.

Corollary 2: Suppose that the price p_d is deterministic, i.e., $\sigma_d^2 = 0$.

1) If $e_d - p_c e_r \geq 0$, then $u^* = \frac{w_0}{p_c}$.
2) If $e_d - p_c e_r < 0$, then

$$u^* = \max\left\{\frac{w_0}{p_c} + \frac{\alpha(e_d - p_c e_r)}{2p_c^2\sigma_r^2}, A_1\right\}.$$

Proof: Since p_d is deterministic and $\sigma_d^2 = 0$, we have

$$A_0 = \frac{\frac{\alpha}{2}(e_d - p_c e_r) + w_0 p_c \sigma_r^2}{p_c^2\sigma_r^2} = \frac{w_0}{p_c} + \frac{\alpha(e_d - p_c e_r)}{2p_c^2\sigma_r^2}.$$

Then, if $e_d - p_c e_r \geq 0$, we have $A_0 \geq \frac{w_0}{p_c}$, and then it follows from Theorem 2 that $u^* = \frac{w_0}{p_c}$. If $e_d - p_c e_r < 0$, we have $A_0 < \frac{w_0}{p_c}$, and then $u^* = \max\{A_0, A_1\}$. ■

Actually, $p_d = e_d$ when p_d is deterministic. From (9), it follows that when $e_d - p_c e_r \geq 0$, the expected return $\mathbf{E}[w_1]$ is an increasing function of u. From (13) and (15), one infers that $\tilde{u} = \frac{w_0}{p_c}$ when $\sigma_d^2 = 0$ and $\frac{d\mathbf{Var}[w_1]}{du} < 0$ when $u < \tilde{u}$, which means that the risk is a decreasing function of u when $u < \tilde{u}$. Thus, the optimal decision for a retail company is investing all its money in the electricity market, which corresponds to the first result in the above corollary. Intuitively, this is the case that p_d is so high that it beats the yield of the financial market. In this case, the retailer should put all his wealth in buying the electricity, at the condition that the value gained exceeds the storage cost.

From the above insights, one infers that the key prices affecting the decision are wholesale prices p_c and p_d rather than retail price p_m. The sign of $e_d - p_c e_r$ is also an important factor which will affect the decision. Therefore, the optimal investment decision not only depends on the wholesale price in the electricity market but also on the asset's return in the financial market.

D. A More General Model

Note that in (8), there is a constraint (4) to ensure that the probability of supply shortage is less than a given small bound. Hence, the optimal purchase decision should satisfy $u^* \geq A_1 \geq e_s$, and p_d is thus the price of excessive electricity which depends on the future wholesale price.

In this section, we consider a more general case that when $u < s$, the retail company can temporarily purchase electricity probably at a higher price than the current wholesale price or retail price, to meet the demands of customers. This is a more general model especially when the utility company can provide temporally purchasing service to the retail company. We thus consider a price model of p_d as follows:

$$p_d = \begin{cases} p_b, & u < s \\ p_w, & u \geq s \end{cases} \tag{25}$$

where p_b and p_w are two random variables which denote the contract price and the real-time wholesale price, respectively. Let e_b and e_w be the mean of them, and σ_b^2 and σ_w^2 be the variance of them, respectively. Under this general model, we can remove the shortage prevention constraint used in the previous sections.

Suppose that the retail company can purchase electricity temporarily when $u < s$, and then we remove the constraint of (4) in (8) as the customers' demands can be always satisfied under this assumption. Under (25), the wealth dynamic (1) does not change. Then, consider the following optimization problem:

$$\begin{aligned} &\max_u \ J(u) = \alpha\mathbf{E}[w_1] - \mathbf{Var}[w_1] \\ &\text{s.t. } (1),\ (5),\ \text{and } (25). \end{aligned} \tag{26}$$

Note that p_d is a piecewise function, so we first consider the optimal solutions for (29) under different constraints to the feasible interval of u. Thus, consider two subproblems of (29) as follows:

$$\begin{aligned} &\max_u \ J(u) = \alpha\mathbf{E}[w_1] - \mathbf{Var}[w_1] \\ &\text{s.t. } (1), 0 \leq u \leq e_s,\ \text{and } p_d = p_b \end{aligned} \tag{27}$$

and

$$\begin{aligned} &\max_u \ J(u) = \alpha\mathbf{E}[w_1] - \mathbf{Var}[w_1] \\ &\text{s.t. } (1),\ e_s < u \leq \frac{w_0}{p_c},\ \text{and } p_d = p_w. \end{aligned} \tag{28}$$

For these two optimization problem, since the wealth dynamic function does not change under the general model, except that the mean and variance of p_d will change when u is in different interval. Hence, we have the stationary points of the objective functions in these two problems, respectively, as

$$A_0^1 = \frac{\frac{\alpha}{2}(e_b - p_c e_r) + \left(e_s\sigma_b^2 + w_0 p_c \sigma_r^2\right)}{\sigma_b^2 + p_c^2\sigma_r^2}$$
$$A_0^2 = \frac{\frac{\alpha}{2}(e_w - p_c e_r) + \left(e_s\sigma_w^2 + w_0 p_c \sigma_r^2\right)}{\sigma_w^2 + p_c^2\sigma_r^2}$$

Let u_1^* and u_2^* be the optimal solutions for (27) and (28), respectively. With a similar proof of Theorem 2, one infers

$$u_1^* = \begin{cases} 0, & A_0^1 < 0, \\ e_s, & A_0^1 > e_s, \\ A_0^1, & \text{otherwise} \end{cases}$$
$$u_2^* = \begin{cases} e_s, & A_0^2 < e_s, \\ \frac{w_0}{p_c}, & A_0^2 > \frac{w_0}{p_c}, \\ A_0^2, & \text{otherwise.} \end{cases}$$

Then, let $p_d = p_b$ and $u = u_1^*$, substituting (1) into the objective function of (27), we obtain the maximum value of $J(u)$, denoted by $J(u_1^*)$. Similarly, let $p_d = p_w$ and $u = u_2^*$, substituting (1) into the objective function in (28), we obtain the maximum value of $J(u)$, denoted by $J(u_2^*)$. The following theorem provides the optimal solution for (29).

Theorem 3: Optimal solution: Considering (29), the u^* satisfies

$$u^* = \arg\max_{u=u_i^*, i=1,2.} J(u).$$

Note that A_0^1 and A_0^2 have the same form as A_0, which means that the stationary points of the objective functions in (27) and (28) have the same form as that in (8). We can use a similar method used for (8) to solve (29).

Remark 1: Considering the case that the short supply is allowable, i.e., the demands of the customers may not be satisfied. In this case, when $u < s$, we have

$$\begin{aligned} w_1 &= (w_0 - p_c u) r_0 + p_m u \\ &= (w_0 - p_c u) r_0 + p_m s + p_m (u - s) \end{aligned}$$

and when $u \geq s$, we have

$$w_1 = (w_0 - p_c u) r_0 + p_m s + p_w (u - s).$$

Hence, the wealth dynamic in this case is equivalent to that in the above model by setting $p_b = p_m$ in (25). Then, in this case, the optimal solution A_0^1 is a function of e_m and σ_m, i.e., p_m plays a role as we claimed in the above section.

Remark 2: When the retail company needs to store the electricity buying from the generator, the storage cost should be considered in the optimal investment problem. Specifically, taking the storage into consideration, a cost function will be added in the objective function to denote storage cost, and the bound of the storage results in a new constraint on u. Hence, we can formulate the following optimization problem:

$$\begin{aligned} &\max_u \; J(u) = \alpha \mathbf{E}[w_1] - \mathbf{Var}[w_1] - C(u) \\ &\text{s.t. } (1), (19), \text{and } u \leq B \end{aligned} \tag{29}$$

where $C(u)$ is the storage cost function and B is the upper bound of the storage. If $C(u)$ is a convex function, e.g., linear function, then the proposed method can still be adopted to solve (29) since it is a convex optimization problem and the method of Lagrangian multiplier is still effective. But if $C(u)$ is a nonconvex function or it still depends on the real-time storage of the electricity, (29) becomes much more challenging, which will be left as our future works.

V. Performance Evaluation

In this section, we evaluate the performance of the optimal solution for the proposed investment problems. All the results were obtained using Monte Carlo simulation with MATLAB 7.0.

We used 1 week (the last week in August 2014) daily hours' price data extracting from [34], and the standard linear regression was employed to obtain the relevant statistics of the random demands. Then, we obtain the mean and variance of electricity hourly price from 1:00 to 24:00, denoted by $m(t)$ and $v(t)$ for $t = 1, 2, \ldots, 24$, respectively. The mean and variance are shown in Fig. 4, where each circle on the line and each vertical line represent the mean and the variance of the price at a time, respectively. Based on $m(t)$ and $v(t)$, we set the relevant statistics of r_0, p_c, p_m, and p_d as follows. Set $e_r = 1 + 10^{-4}$, $p_c = m(t)$, $e_m = m(t) e_s + 10^{-4} \theta_m(t)$, and $e_d = m(t) e_s + 5\theta_d(t) 10^{-5}$ for $t = 1, 2, \ldots, 24$, where $\theta_m(t)$ and $\theta_d(t)$ are selected randomly from $[0, 1]$. We set the same value of variances to all random prices and r_0, i.e., $\sigma_m^2 = \sigma_d^2 =$

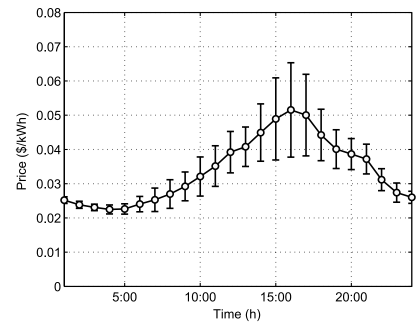

Fig. 4. Mean and variance of hours' prices.

$\sigma_r^2 = v(t)$ (different settings of σ_r^2 will not affect the following simulation results). The α and ε are 5 and 0.2, respectively. Assume that $w_0 = 100$ (one can see 100 as one unit investment) and $e_s = 1000 \left(1 - 10^{-5} \theta_s(t)\right)$ with $\sigma_s^2 = 10$, where e_s equals $\frac{w_0}{4p_c}$. These settings of parameters are also applied in Example 1 with $t = 1$. The following simulations were conducted based on the above statistics obtained from real-price data processing to be realistic.

Suppose that the investor makes a decision at the beginning of each hour based on the current wealth, w_{t-1}, i.e., let $w_0 = w_{t-1}$ in our model at each time t. We apply our solution given in Theorem 2 as the investment decision at each time t, marked with "proposed strategy." We compare our method with the method in [35], marked with "Kelly strategy," which is a typical model-free investment approach applied in financial markets. For Kelly strategy, the optimal strategy u^* satisfies $u^* = (1 + r)(\hat{\sigma})^{-1} \hat{r} - r$, where r is the return of a riskless bond, $\hat{r}$ and $\hat{\sigma}$ are the mean and variance of the excess returns [35], and we set $\hat{r} = r_0 - r = r_0 - 1$ (i.e., $r = 1$) in this paper, where r_0 was defined in Table I. Using proposed strategy and Kelly strategy, we can observe two wealth series $w(t)$ for $t = 1, 2, \ldots, 24$. With $w(t)$, investment performance measures such as return, risk, and Sharpe ratio (see its definition in [36]) are calculated.

We compare proposed strategy and Kelly strategy in terms of the optimal decisions, excess returns, risks, and Sharp ratios. The compare results are shown in Fig. 5.

1) From Fig. 5(a), we can see that under proposed strategy, the investor will purchase much more electricity at each time than that under Kelly strategy, as the value of each $u^*(t)$ is always larger in proposed strategy. It should be pointed out that under proposed strategy, we have $u^*(t) \geq A_1$ hold for every time t, which cannot be guaranteed using Kelly strategy. This is why each $u^*(t)$ is almost larger than 10^3 (the value of e_s) in proposed strategy, and why we need the constraint (4) in (8).
2) The comparison results on return and risk are shown in Fig. 5(b) and (c), respectively. It is observed that under proposed strategy, the excess return is always larger while the risk is always lower than those under Kelly strategy, which means that proposed strategy is much better than Kelly strategy. Meanwhile, the advantage on return of proposed strategy will increase with investment times. Especially, when $t = 24$, the excess return under

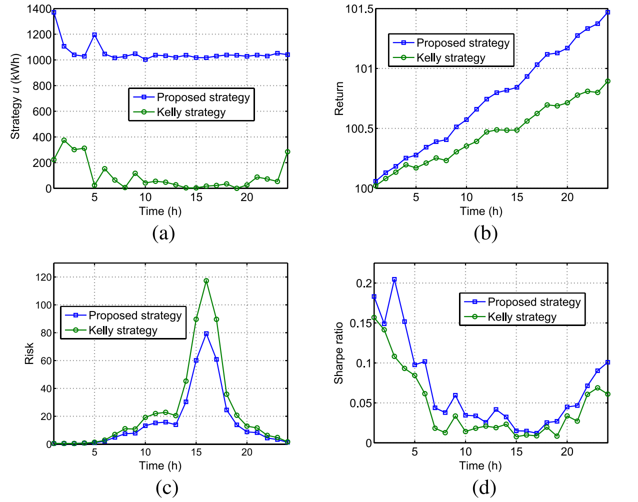

Fig. 5. Performance comparison. (a) Decision. (b) Return. (c) Risk. (d) Sharpe ratio.

proposed strategy is about 1.5, while it is about 0.7 for Kelly strategy, i.e., our method can achieve more than twice excess return than Kelly strategy. For the risk, for both kinds of strategies, they have the same tendency as the mean and variance of hourly price shown in Fig. 4, since a higher mean or variance of price renders a higher investment risk.

3) Proposed strategy also has a higher Sharp ration, which means that our method can obtain more return per unit of risk. Therefore, proposed strategy can completely outperform the widely used Kelly strategy.

VI. Conclusion

In this paper, we have investigated the optimal investment decision problem for a retail company in emerging electricity market.

1) We have formulated an investment problem using classical mean-variance model to help the retail company make investment decisions. Unlike the existing energy trading, it enables the investor investing in both the financial and electricity markets simultaneously.
2) Using quadratic programming, we have derived the closed-form optimal solution to the proposed problem no matter whether the temporal purchase is allowed or not.
3) We have revealed the new properties of proposed problem comparing to traditional investment problem, i.e., a high risk may have a low return, and when the retail price is independent of the demands, the optimal investment decision is not affected by the retail price in the electricity market. Simulations driven by real-time price data have been used to demonstrate the superior performance of the proposed solution.

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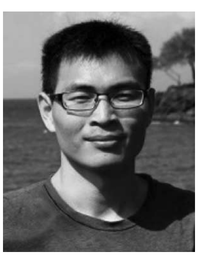

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