Digital Signal Processing

Signals

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Outline

Applications

Applications of Digital Signal Processing

1. Entertainment
2. Hospitals
3. Telecommunication
4. Smart vehicles

Definitions

What is a signal?

Definition

A physical quantity that carries information

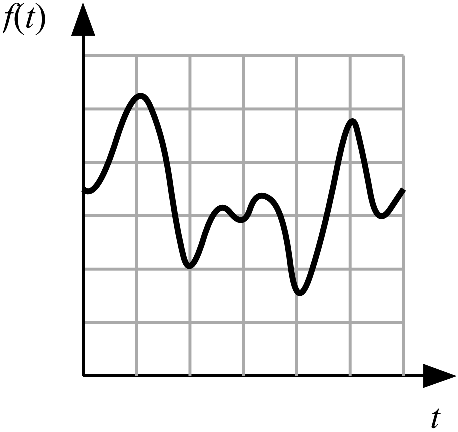
What is Signal Processing?

Definition

Performing operations on a signal using hardware or software

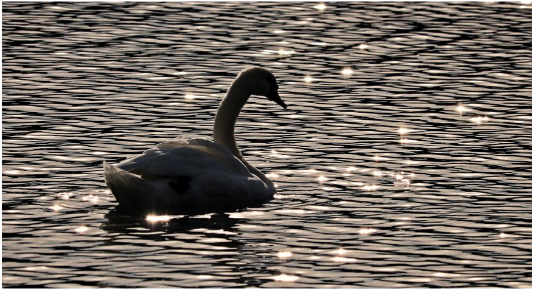
Signals

One-Dimensional (1-D) Signals



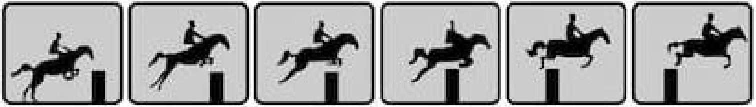
Need 1-D transducer (e.g. IR detector, accelerometer, gyro, etc.)

Two-Dimensional (2-D) Signals



Need 2-D transducer (e.g. digital camera)

Three-Dimensional (3-D) Signals

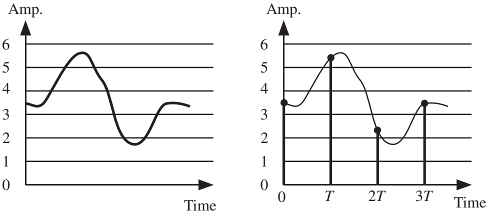


Need 3-D transducer (e.g. video recoder)

Signal Types

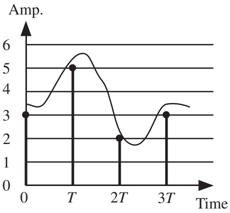
Discrete Versus Continuous Signals

Analog



(a) (b)

Digital



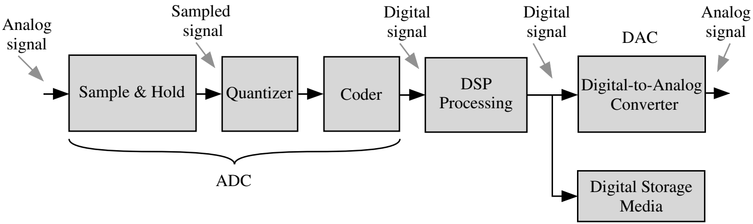
(c)

Types of Signals

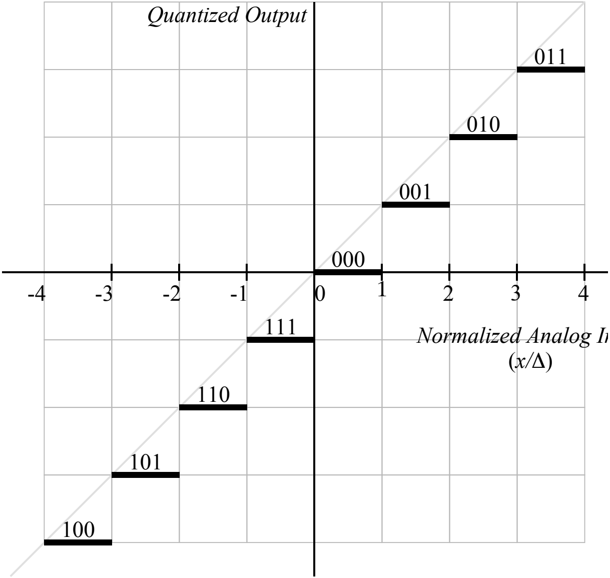
	Continuous-Time	**Discrete-Time**
Continuous-Value	Analog signal as obtained from some sensor or transducer	Signal after sampling but before quantization.
Discrete-Value (Quantized)	Not possible. Quantizer delay prevents this mode of operation.	Digital signal after sampling and quantization and coding

ADC/DAC

Analog-to-Digital and Digital-to-Analog Conversion



Quantization & Coding: Using Truncation



Error Analysis of Analog Signal Quantization: Truncation

1. Normalized signal is in the range

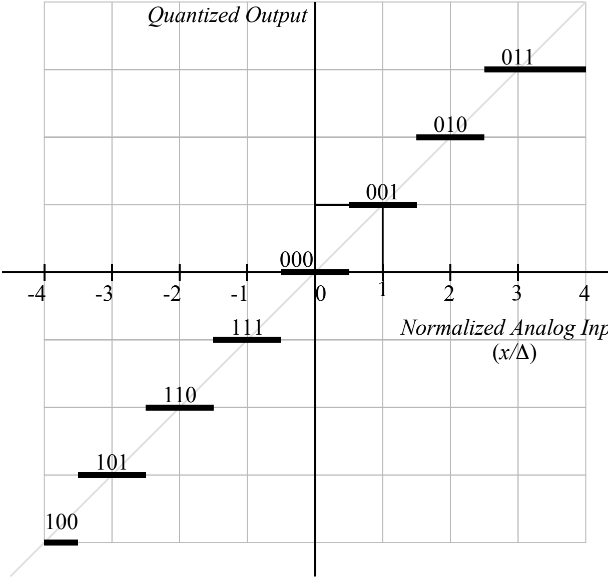
$$Q_i \leq x/\Delta < Q_{i+1}$$

2. Error range is $0 \leq \varepsilon < \Delta$

3. Uniform error distribution implies

$$\mu = 0.5\Delta \qquad \text{and} \qquad \sigma^2 = \Delta^2/12$$

Quantization & Coding: Using Rounding



Error Analysis of Analog Signal Quantization: Rounding

1. Error range is $-\Delta/2 \leq \varepsilon < \Delta/2$
2. Uniform error distribution implies

$$\mu = 0 \qquad \text{and} \qquad \sigma^2 = \Delta^2/12$$

Signal to Noise Ratio

and

Signal Energy

Signal-to-Noise Ratio (SNR)

$$SNR = 10 \log \frac{E(x^2)}{E(\epsilon^2)}$$

For uniformly distributed signal, expected value $E(\cdot)$ defined as:

$$E(x) = \frac{1}{X} \int_0^X x \, dx$$

General Expression for Signal Energy

We can prove the following

$$E(x^2) = \mu^2 + \sigma^2$$

Signal Energy: $E(x^2)$

Energy of a uniformly distributed signal x is

$$E(x^2) = \frac{1}{V_0} \int_0^{V_0} x^2 \, dx$$

and we get

$$E(x^2) = V_0^2/3$$

Can you find that using μ and σ^2?

Error Energy: $E(\epsilon^2)$

1. Assuming N quantization levels, $\Delta = V_0/N$
2. For truncation: $E(\epsilon^2) = \Delta^2/3 = V_0^2/(3N^2)$
3. For rounding $E(\epsilon^2) = \Delta^2/12 = V_0^2/(12N^2)$

ADC SNR

1. Truncation:

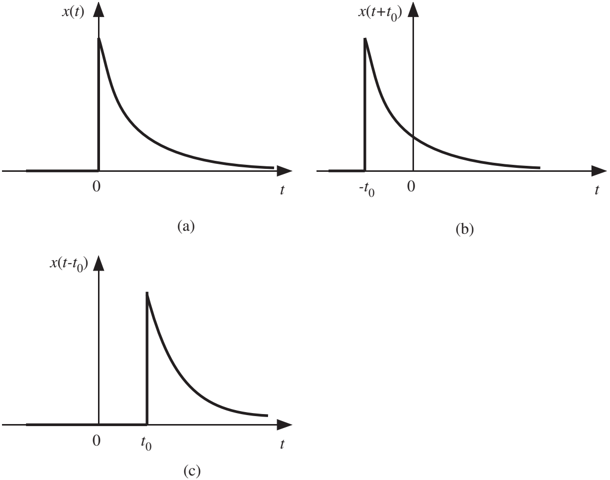
$$SNR = 20 \log N$$

2. Rounding (Best option):

$$SNR = 20 \log N + 10 \log 4$$

Time Shifting a Signal

Time Shifting a Signal



(a) $x(t)$: No shift (b) $x(t+t_0)$: Advance (c) $x(t-t_0)$: Delay

Special Functions

Sinusoidal Signal: Canonic or Standard Form

$$x(t) = A\cos(\omega t + \phi) \quad = \quad A\cos[\omega(t + t_0)]$$

A: Amplitude

ω: Radian frequency (rad/s)

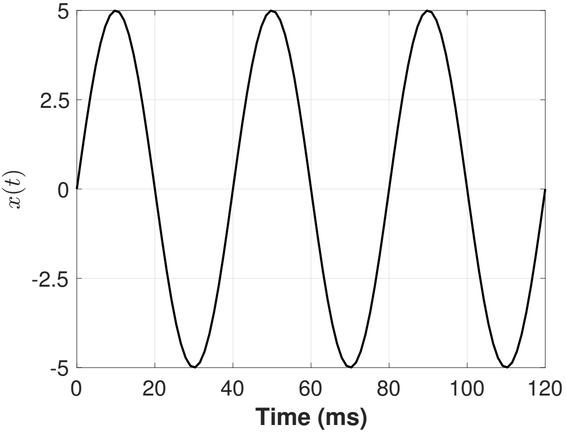
ϕ: Phase (rad)

t_0: time shift

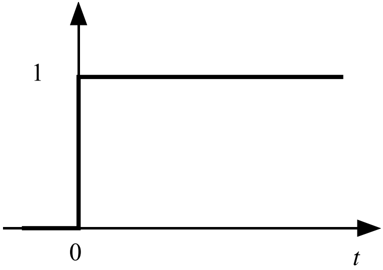
Sinusoid: $x(t) = A\cos[\omega(t+t_0)] = A\cos(\omega t+\phi)$

Example

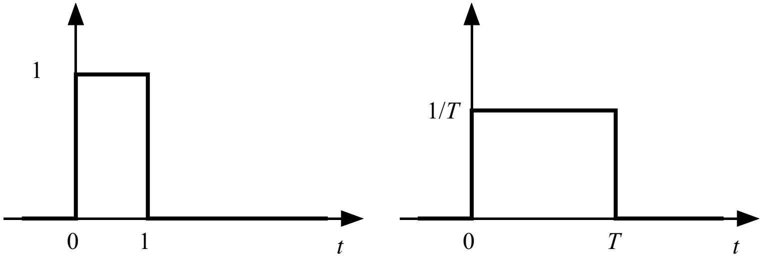
Find the parameters of this sinusoid.



Unit Step (Heaviside) Function $u(t)$



Unit Pulse Function



(a) (b)

Dirac Delta Function $\delta(t)$

$$\delta(t) = \lim_{T \to 0} \begin{cases} 0, & t < 0 \\ 1/T, & 0 \leq t < T \\ 0, & t \geq T \end{cases}$$