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COMPARATIVE STUDY OF TE^{x}_{mn} VERSUS $TE_{mn} - TM_{mn}$ MODE ANALYSIS AND ITS APPLICATION TO WAVEGUIDE DISCONTINUITY MODELING

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ABSTRACT

This paper presents a comparative study of alternative techniques to be used in the mode matching method as applied to electromagnetic field modeling at waveguide discontinuities. It is found that a modified TE_{mn}-to-x-mode (TE^{x}_{mn}-or LSH_x-mode) approach becomes necessary for waveguide discontinuities in which resonant effects occur. A comparison between the conventional TE^{x}_{mn} mode matching technique, commonly known from literature, and the generalized analysis based on a linear superposition of TE^{z}_{mn} and TM^{z}_{mn} modes, shows conflicting results. The latter one is found to be in excellent agreement with the modified TE^{x}_{mn} mode analysis and with measurements on waveguide iris filters.

INTRODUCTION

Single or double plane waveguide discontinuities are integral design elements for a wide range of waveguide components. Accurate characterization of those discontinuity elements is required to design components such as waveguide transformers [1, 3, 4], iris and corrugated waveguide filters [1-3, 5, 6], evanescent-mode bandpass filters [7-9], horn antennas [10, 11] and polarizers [10-12].

The numerical characterization of waveguide junction discontinuities as shown in Fig. 1 is usually carried out by a $TE^{(z)}_{mn}$ - $TM^{(z)}_{mn}$ mode analysis. Based on the assumption that an incident TE_{10} wave would excite TE_{mn} and TM_{mn} modes, two vector potential functions are necessary to describe the four field components (E_x, E_y, H_x, H_y) to be matched at $z = 0$ [3-5]. From a physical standpoint, however, there is no reason why an incident TE_{10} wave should excite an electric field component in x-direction. Hence, the discontinuity can be calculated using a set of TE_{mn}-to-x modes commonly written as TE^{x}_{mn}. Since the condition $E_x = 0$ is implicitly incorporated, the number of modes is reduced to N_{TE} compared to $N_{TE} + N_{TM}$ for the TE-TM mode case [13]. This significantly speeds up the calculations and, at the same time, reduces the computer memory requirements. This procedure has been successfully applied in some special cases [8, 13, 14]. In cases, however, where resonant effects occur in the discontinuity plane, one major problem associated with this method becomes apparent: The resulting equation system contains two unknowns but requires three field components (E_y, H_x, H_y) to be matched.

In many applications, neglecting one of the two H-field (H_y) components does not lead to wrong results. In other cases, however, it does. Therefore, this paper focusses on a comparative study of the TE^{x}_{mn} versus the TE_{mn}-TM_{mn} mode analysis. Three different analysis methods are investigated:

- Method 1: The conventional TE^{x}_{mn} mode method where only E_y and H_x are matched.
- Method 2: The modified TE^{x}_{mn} mode procedure matching E_y and H_x or H_y alternatively.
- Method 3: The generalized TE-TM analysis matching the field components E_x, E_y, H_x, H_y.

The excellent agreement between the modified TE^{x}_{mn} routine (method 2), the TE-TM mode analysis (method 3) and measurements is demonstrated at the examples of waveguide iris filters.

THEORY

In this section, only the basic steps of the modified TE^{x}_{mn} mode formulation is presented. For details on the generalized TE_{mn} - TM_{mn} mode field description, which leads to the coupling matrices and the modal scattering matrix calculation, the reader is referred to [15].

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In the case of the TE^x_{mn} mode representation, the transverse field components in region $i = I, II$ (c.f. Fig. 1)

$$
\begin{aligned}
E^i_y &= \frac{\partial}{\partial z} A^i_{hx} \\
H^i_x &= \frac{j}{\omega\mu_o}[k_o^2 + \frac{\partial^2}{\partial x^2}] A^i_{hx} \\
H^i_y &= \frac{j}{\omega\mu_o}\frac{\partial^2}{\partial x \partial y} A^i_{hx}
\end{aligned}
\qquad (1)
$$

are derived from the x-component of a vector potential

$$
\begin{aligned}
A^i_{hx} &= 2\sum_{q=1}^{\infty}\sqrt{\frac{\omega\mu_o/k^i_{zq}}{F^i_\square[k_o^2 - (k^i_{xm})^2]}}\; T^i_{mn}(x,\, y) \\
&\cdot (V^i_q e^{-jk^i_{zq}z} - R^i_q e^{+jk^i_{zq}z})
\end{aligned}
\qquad (2)
$$

where V^i_q and R^i_q are the wave amplitudes traveling in the positive and negative z-direction, respectively, and mode indices m, n are related to q with respect to increasing cutoff frequencies.

Matching the field components of (1) at the common interface of regions I, II and truncating the infinite sums in (2) yields three matrix equations

$$
\begin{aligned}
E_y: &\quad \underline{V}^I + \underline{R}^I = \underline{L}_E(\underline{V}^{II} + \underline{R}^{II}) &(3)\\
H_x: &\quad \underline{L}_{Hx}(\underline{V}^I - \underline{R}^I) = \underline{V}^{II} - \underline{R}^{II} &(4)\\
H_y: &\quad \underline{L}_{Hy}(\underline{V}^I - \underline{R}^I) = \underline{V}^{II} - \underline{R}^{II} &(5)
\end{aligned}
$$

To calculate the modal scattering matrix of the double-step discontinuity

$$
\begin{bmatrix} \underline{R}^I \\ \underline{V}^{II} \end{bmatrix} = (\underline{S}) \begin{bmatrix} \underline{V}^I \\ \underline{R}^{II} \end{bmatrix} \qquad (6)
$$

only two equations of the set (3-5) are needed to separate the two unknowns $\underline{R}^I$ and $\underline{V}^{II}$. Matching E_y, H_x and

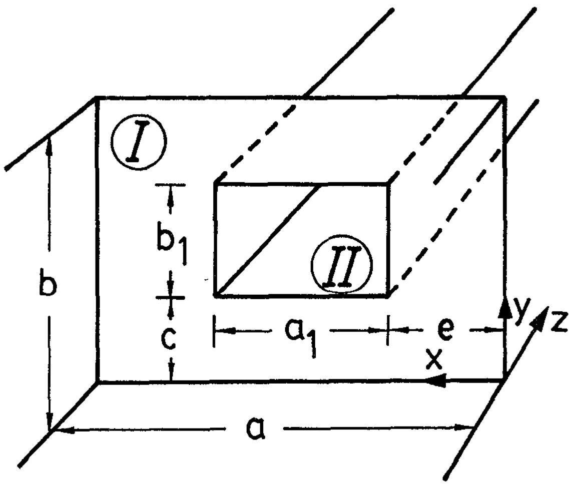

Fig. 1 Double-step discontinuity in rectangular waveguide.

ignoring H_y leads to excellent results in some special cases, e.g. [14], but fails for general double-step discontinuities. On the other hand, merely considering E_y and H_y for the matching conditions or, alternatively, H_x and H_y lacks information in the presence of TE^x_{m0} modes.

Therefore, the following procedure is proposed here: Besides matching E_y using (3), H_x and H_y are matched alternatively by creating a new matrix $(\underline{L}_H)$ where

$$
\begin{aligned}
(\underline{L}_H)qp &= (\underline{L}_{Hx})qp, && \text{if mode } q \text{ or mode } p \text{ is a } TE^x_{mo} \text{ type} \\
(\underline{L}_H)qp &= (\underline{L}_{Hy})qp, && \text{if neither mode } q \text{ nor mode } p \text{ is a } TE^x_{mo} \text{ type.}
\end{aligned}
\qquad (7)
$$

The modal scattering matrix of the double-step discontinuity calculated under the condition (7) shows excellent agreement with measured data. Compared with the TE-TM analysis, this new procedure considerably reduces the matrix sizes to be processed by the computer. Moreover, the fact that a smaller number of modes interacts between discontinuities than in the discontinuity plane itself can be easily implemented in this appraoch. By these measures, the processing time of the algorithm can be made up to five times faster.

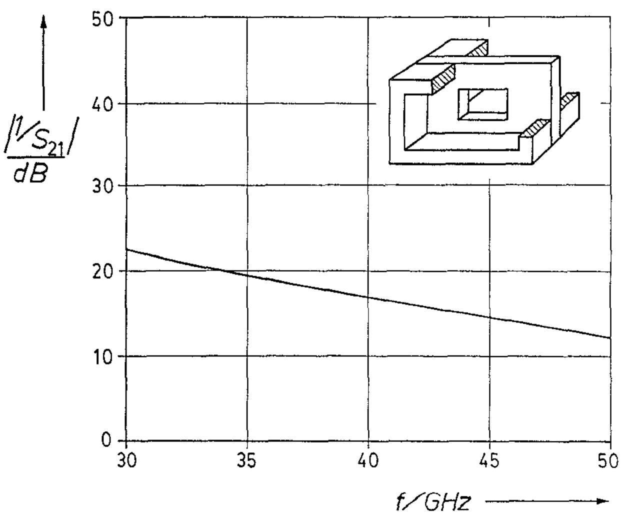

Fig. 2 Insertion loss of a non-resonant iris in rectangular waveguide ($a = 2\ b = 7.112$mm, $a_1 = b_1 = 2$mm, iris thickness = 0.5mm). Results of all methods investigated within plotting accuracy.

RESULTS

Waveguide iris filters can operate in two different ways. Firstly, half-wave resonators are connected by irises which act as coupling elements only. The transmission behavior of a single iris is shown in Fig. 2. The three methods investigated yield identical results within the plotting accuracy. Therefore, excellent agreement is usually obtained by all three methods when compared with measured data of an iris-coupled half-wave resonator filter. This is demonstrated in Fig. 3 using measurements carried out in [5]. However, the TE^{x}_{mn} procedures (methods 1, 2) require only 30 percent of the CPU time compared with the generalized TE-TM analysis (method 3).

Secondly, iris filters may be based on the resonances of the irises itself rather than half-wave waveguide cavities. In this mode of operation, the irises are required to function above cutoff, which occurs at 37.5 GHz in the example of Fig. 4. Exactly at this frequency, the first method reveals some instabilities due to the lack of H_y field matching, whereas the other two methods are in good agreement in modeling the resonance effect of the iris. Fig. 5 shows a comparison with measured data on this type of iris filter presented by Chen [2]. As expected from the results of Fig. 4, the TE^{x}_{mn} analysis matching E_y, H_x (method 1) is not suitable to accurately calculate the discontinuities involved. Methods 2 and 3 yield almost identical results which closely agree with measurements. However, the CPU time ratio of 1:5 in favour of the new TE^{x}_{mn} analysis (method 2) clearly demonstrates its advantage over the generalized TE-TM procedure.

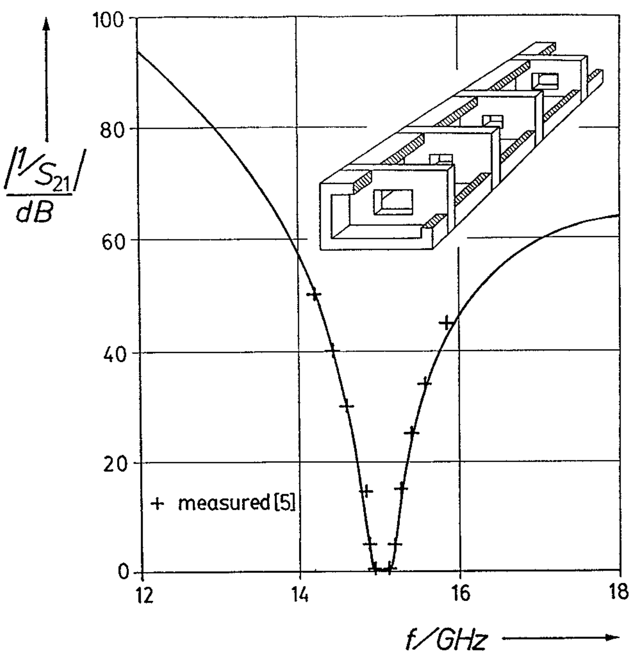

Fig. 3 Insertion loss of three-resonator iris-coupled Ku-band waveguide filter according to Tucholke [5] (— theoretical results obtained by all three methods; + + measured [5]).

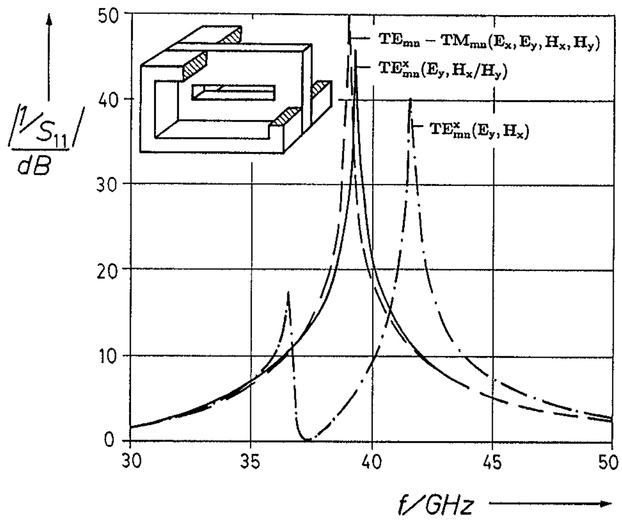

Fig. 4 Return loss of resonant iris in rectangular waveguide ($a = 2\ b = 7.112$mm, $a_1 = 4$mm, $b_1 = 1$mm, iris thickness = 0.5mm). Sets of modes used for analysis: - - - - TE-TM modes matching E_x, E_y, H_x, H_y (method 3); - · - · - TE^{x}_{mn} modes matching E_y, H_x (method 1); — TE^{x}_{mn} modes matching E_y, H_x/H_y (method 2).

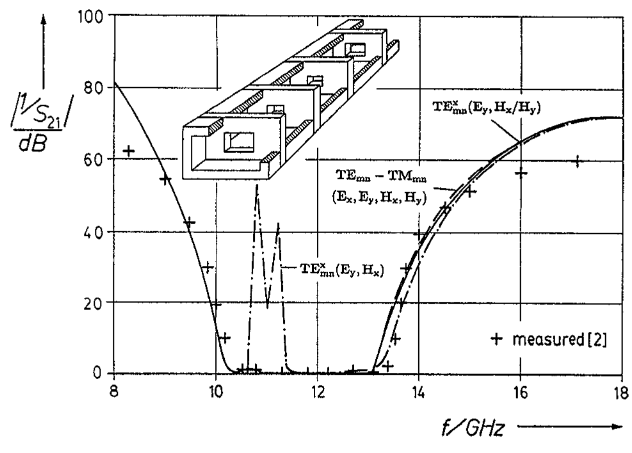

Fig. 5 Comparison between theories and measurements at the example of an X-band seven resonant-iris waveguide filter according to Chen [2] (+ + measured [2]; sets of modes as in Fig. 4).

CONCLUSIONS

A comparative study on three different methods for waveguide double-step discontinuity modeling is presented. The applicability of a new TE^{x}_{mn} mode approach matching E_y and H_z or H_y field components alternatively is demonstrated, whereas a method matching only E_y and H_z turned out to be limited to only a small variety of applications. With memory space savings of more than 50 percent, CPU time reductions down to 20 percent but with maintained accuracy compared with the generalized TE-TM procedure, the new TE^{x}_{mn} mode method offers an attractive solution for the computer-aided design of waveguide components involving double-step discontinuities.

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