

**A SCATTERING-TYPE TRANSVERSE RESONANCE FORMULATION  
AND ITS APPLICATION TO OPEN, CONDUCTOR-BACKED AND  
SHIELDED SLOTLINE (M)MIC STRUCTURES**

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**ABSTRACT**

A new formulation of the transverse resonance technique is introduced and applied to the propagation characteristics calculation of MIC and MMIC slotline configurations. By utilizing a scattering-type representation of the transverse discontinuities involved, the influences of different boundary conditions as required for conductor-backed, shielded or even open structures can be easily incorporated. The computed results obtained with this method are found to be in excellent agreement with measurements as well as with previously published theoretical data on fundamental and higher-order mode characteristics. The software is operational on 386-compatible work stations.

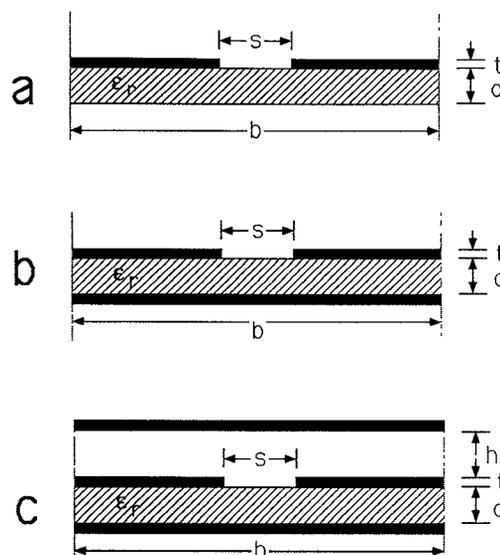
**I. INTRODUCTION**

The conventional transverse resonance method for transversely discontinuous waveguides [1] such as finlines or shielded microstrip is a well-known technique to calculate the propagation characteristics of quasi-planar transmission line structures. Although the restrictions to moderate and larger slotwidths [2, 3] can be reduced by using an improved transmission-matrix formulation for cascaded discontinuities [4], precisely defined boundary conditions are still required to formulate the resonance condition of a structure. Whereas many shielded configurations with electric-wall boundaries have been analyzed for their propagation constants and characteristic impedances, open structures have not yet been solved with this model, except for the groove guide in [5] where a fundamental-mode transverse resonance equivalent circuit approach is applied.

Modern MIC and MMIC components are realized in open, conductor-backed and shielded transmission line technology. The effects of shielding, conductor-backing and finite-extent ground planes have been addressed and analyzed in the literature [6-9]. Most of the investiga-

tion, however, are focussed on the low-dispersive coplanar waveguide rather than on slotline structures where the frequency dependence of the line characteristics is more pronounced. So far, only Heinrich [9, 10] has analyzed the slotline as applied to MMIC circuitry. The modal analysis used, however, involves upper and lower shieldings which have to be significantly spaced in order to be able to approximate open structures. This usually leads to numerical complications caused by large arguments of sinh, cosh or exp functions.

Therefore, this paper introduces a scattering-type formulation of the transverse resonance technique as applied to (M)MIC slotline structures (Fig. 1). Since the boundary values in the relevant directions are introduced in terms of scattering parameters rather than fixed conditions, open, conductor-backed and shielded structures



**Fig. 1** Slotline structures for (M)MIC applications; a) open, b) conductor-backed, c) shielded.

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only differ in their reflection coefficients at the boundaries. The basic resonance condition remains unchanged. Moreover, this method preserves the possibility of individually selecting the number of expansion terms in the different subregions as opposed to the transmission-like transverse resonance formulation in [2, 3].

## II. THEORY

According to the scattering-type formulation of the transverse resonance conditions, the electric and magnetic vector potentials are characterized by incident and reflected waves in  $x$ -direction: For region  $i = 0, I, II, IV$  (c.f. Fig. 2a)

$$A_{h_z}^i = \sum_{n=1}^N (A_n^i e^{-jk_{zn}^i x} + B_n^i e^{+jk_{zn}^i x}) \cdot \cos\left\{(2n-1)\frac{\pi}{b}y\right\} e^{-jk_x z} \quad (1)$$

$$A_{e_z}^i = \sum_{n=1}^N \frac{1}{jk_{zn}^i} (-C_n^i e^{-jk_{zn}^i x} + D_n^i e^{+jk_{zn}^i x}) \cdot \sin\left\{(2n-1)\frac{\pi}{b}y\right\} e^{-jk_x z} \quad ; \quad (2)$$

in region III, the sums read  $n = 0$  to  $M-1$ , the arguments of the sin and cos functions are replaced by  $2n\pi y/s$ , and the cos function is divided by  $\sqrt{1 + \delta_{on}}$  where  $\delta_{on}$  is the kronecker delta. In Fig. 2a, the amplitude coefficients  $A_n^i, C_n^i$  and  $B_n^i, D_n^i$  are combined to vectors  $\underline{F}^i$  and  $\underline{R}^i$ , respectively ( $i = 0$  to IV).

By matching the tangential field components at interfaces  $x = -d, 0$  and  $x = 0, t$ , the modal scattering-type representation of the dielectric substrate ( $\underline{S}_d$ ) and the metallization ( $\underline{S}_s$ ) are obtained (c.f. Fig. 2b). Note that since the amplitude coefficients are not power-normalized, the magnitudes of the elements of ( $\underline{S}_d$ ) and ( $\underline{S}_s$ ) may exceed the unity value. As  $\Delta x$  approaches zero (Fig. 2a), vectors  $\underline{F}^0$  and  $\underline{R}^0$  are related by

$$\underline{R}^0 = \underline{\Gamma}_{in} \underline{F}^0 \quad \text{and} \quad \underline{F}^0 = \underline{\Gamma}_{out} \underline{R}^0 \quad (3)$$

(c.f. Fig. 2b). Hence the resonance condition can be formulated as

$$[\underline{I} - \underline{\Gamma}_{in} \underline{\Gamma}_{out}] \underline{R}^0 = 0 \quad (4)$$

where  $\underline{I}$  represents the identity matrix. The propagation characteristics are determined by the zeros of the determinant in (4). Solving for  $\underline{R}^0$  and successively applying the scattering relations at the different interfaces finally allows the characteristic impedance to be calculated by the power-voltage definition, e.g. [10].

In (4), the reflection coefficient matrices  $\underline{\Gamma}_{in}, \underline{\Gamma}_{out}$  are calculated according to the boundary conditions in  $x$ -

direction. In case of an open structure,  $\underline{\Gamma}_L = \underline{\Gamma}_r = 0$  (Fig. 2b) and hence

$$\underline{\Gamma}_{in} = \underline{S}_{S11}, \quad \underline{\Gamma}_{out} = \underline{S}_{d22}. \quad (5)$$

For a conductor-backed slotline ( $\underline{\Gamma}_L = -1$ ),  $\underline{\Gamma}_{out}$  is replaced by

$$\underline{\Gamma}_{out} = \underline{S}_{d22} - \underline{S}_{d21} [\underline{I} + \underline{S}_{d11}]^{-1} \underline{S}_{d12}. \quad (6)$$

Together with (6), the conditions for the shielded structure are

$$(\underline{\Gamma}_r) = \begin{pmatrix} \underline{D} & \underline{0} \\ \underline{0} & \underline{D} \end{pmatrix} \quad (7)$$

$$\text{where } \underline{D} = \text{Diag} \{-e^{-j2k_{zn}^{IV} h}\} \quad (8)$$

$$\text{and } \underline{\Gamma}_{in} = \underline{S}_{S11} + \underline{S}_{S12} \underline{\Gamma}_r [\underline{I} - \underline{S}_{S22} \underline{\Gamma}_r]^{-1} \underline{S}_{S21}. \quad (9)$$

It should be noted that  $\underline{\Gamma}_{out}$  as well as the submatrices of ( $\underline{S}_d$ ) are purely diagonal. Therefore, the open and conductor-backed structures are less CPU-time intensive than the shielded case. As far as the computer algorithm is concerned, the open slotline represents the simplest configuration and does not experience any numerical limitations.

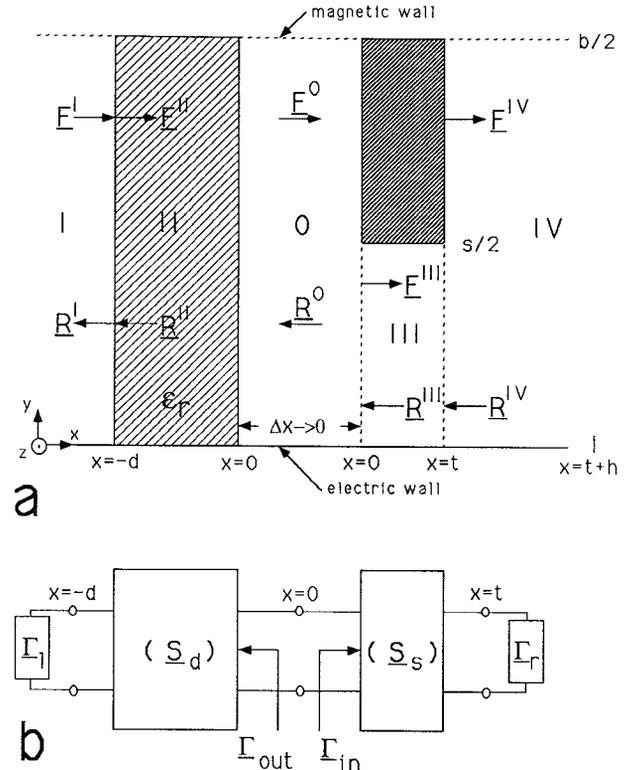


Fig. 2 Slotline geometries for the field theory treatment; a) subregions and wave amplitude vectors, b) scattering-type representation of structure.

### III. RESULTS

Fig. 3 compares the effective permittivity calculated with this method with measurements carried out in [11]. Although the finite strip width of the open slotline in [11] has been replaced by the magnetic wall model in this theory, good agreement is obtained. The two slotline models (finite strip and magnetic wall) have also been analyzed in [10] where the finite strip model values are found to be slightly higher than those resulting from the magnetic wall assumption. This tendency is confirmed in Fig. 4 when comparing the measured values (+) with the solid lines of this theory. The dashed line represents the results of Cohn's theory [12] for the open slotline. Since a ridge waveguide model is used in [12], Cohn's method fails at lower frequencies due to the fundamental mode cutoff effect.

A comparison between open, conductor-backed and shielded slotline propagation characteristics is shown in Fig. 4. Since the upper shielding in this case is four times the substrate thickness away from the slotline metallization, the differences between the shielded and conductor-backed structures can be neglected beyond 5 GHz. The open slotline, however, shows far lower propagation constants which is due to the fact that the electric field extends into the air region below the substrate. It should be noted that the values obtained by this method for the shielded structure are in perfect agreement with [13].

Fig. 5 shows the characteristics of a slotline configuration for MMIC applications. Since the distance  $h$  of the upper shielding is not specified for the reference values in [10], this structure has been calculated assuming the conductor-backed case. Excellent agreement is achieved for the propagation characteristics of the fundamental as well as the first higher-order mode when comparing the results of this method with the modal analysis values of the shielded structure in [10]. Reasonable agreement is also obtained for the characteristic impedance. The slight deviations are assumed to be due to loss considerations and the upper shielding in [10]. As has been shown for finline circuits in [3, 4] the characteristic impedance defined by voltage and power approaches zero as soon as higher-order modes start to propagate.

### IV. CONCLUSIONS

A versatile and powerful method for the calculation of (M)MIC transmission-line characteristics is introduced and demonstrated at the examples of slotline structures. The main new feature of this method is a scattering-type formulation of the transverse resonance technique which permits a simple procedure to include different bound-

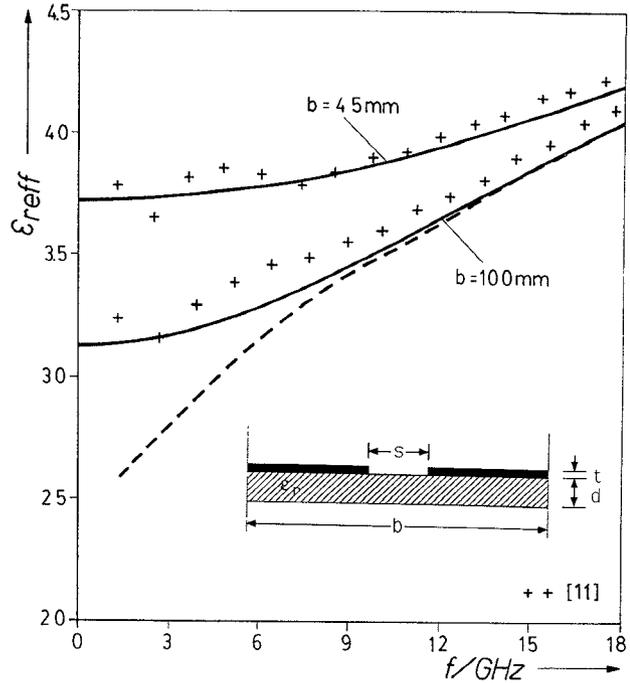


Fig. 3 Comparison of propagation characteristics obtained with this method (solid lines) with measurements in [11] (++) and Cohn's [12] results (dashed line);  $d = 0.635$  mm,  $t = 7.03$   $\mu$ m,  $s = 0.5$  mm,  $\epsilon_r = 9.7$ .

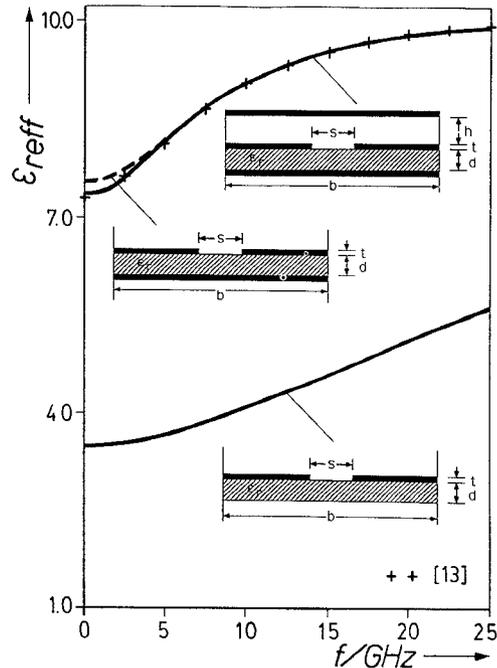


Fig. 4 Propagation characteristics of open, conductor-backed and shielded slotline MIC structure (++) [13]; —, - - - this theory);  $b = 10$  mm,  $d = s = 1$  mm,  $t = 3$   $\mu$ m,  $h = 4$  mm (shielded structure only),  $\epsilon_r = 10.2$ .

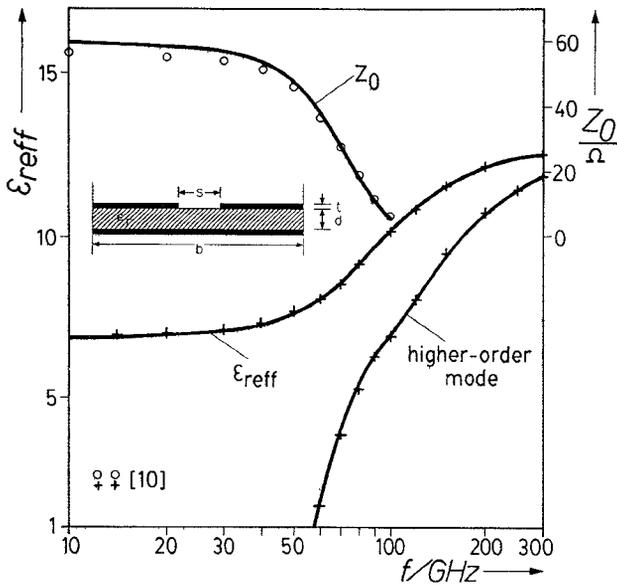


Fig. 5 Comparison of fundamental and higher-order mode characteristics with results presented in [10] for a slotline MMIC configuration (+ o [10], — this theory);  $b = 0.84$  mm,  $d = 0.6$  mm,  $s = 0.04$  mm,  $t = 3\mu\text{m}$ ,  $\epsilon_r = 12.9$ .

any conditions for open, conductor-backed and shielded configurations as required in modern (M)MIC circuit design. The results produced by this new method are verified by comparisons with measurements and theoretical data available in the literature. The related software is operational on modern 386-type work stations and does not require mainframe support.

## REFERENCES

- [1] Sorrentino, R., "Transverse Resonance Technique", in *Numerical Techniques For Microwave And Millimeter-Wave Passive Structures* (T. Itoh, ed), John Wiley & Sons, 1989, Ch. 11.
- [2] Vahldieck, R. and J. Bornemann, "A modified mode-matching technique and its application to a class of quasi-planar transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 916-926, Oct. 1985.
- [3] Bornemann, J. and F. Arndt, "Calculating the characteristic impedance of finlines by transverse resonance method", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 85-92, Jan. 1986.
- [4] Mansour, R.R. and R.H. MacPhie, "A unified hybrid-mode analysis of planar transmission lines with multilayer isotropic/anisotropic substrates", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1382-1391, Dec. 1987.
- [5] Oliner, A.A. and R. Lampariello, "The dominant mode properties of open groove guide: An improved solution", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 755-763, Sep. 1985.
- [6] Shigesawa, H., M. Tsuji and A.A. Oliner, "Conductor-backed slot line and coplanar waveguide: Dangers and full-wave analyses", in 1988 IEEE MTT-S Int. Microwave Symp. Dig., pp. 199-202, May 1988.
- [7] Jackson, R.W., "Considerations in the use of coplanar waveguide for millimeter-wave integrated circuits", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1450-1456, Dec. 1986.
- [8] Ghione, G. and C.U. Naldi, "Coplanar waveguides for MMIC applications: Effect of upper shielding, conductor backing, finite-extent ground planes, and line-to-line coupling", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 260-267, Mar. 1987.
- [9] Heinrich, W., "Full-wave analysis of conductor losses on MMIC transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1468-1472, Oct. 1990.
- [10] Heinrich, W., "The slot line in uniplanar MMIC's: Propagation characteristics and loss analysis", in 1990 IEEE MTT-S Int. Microwave Symp. Dig., pp. 167-170, May 1990.
- [11] Hoffmann, R.K., J.-P. Kurzweg and J.-P. Mutzig, "Measurement of the effective permittivity and of the resonator-Q of slot lines on ceramic substrate in the frequency range between 1 GHz and 18 GHz" (in German), *Frequenz*, vol. 27, pp. 32-40, Feb. 1973.
- [12] Cohn, S.B., "Slotline on a dielectric substrate", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768-778, Oct. 1989.
- [13] Nakatani, A. and N.G. Alexopoulos, "Toward a generalized algorithm for the modeling of the dispersive properties of integrated circuit structures on anisotropic substrates", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1436 - 1441, Dec. 1985.