

# ANALYSIS OF LOSSY ANISOTROPIC COPLANAR WAVEGUIDE WITH APPLICATION TO HIGH-T<sub>c</sub> SUPERCONDUCTORS

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## Abstract

A spectral domain immittance approach (SDIA) for the analysis of slow-wave and its characteristics of coplanar waveguide based on gyromagnetic substrates is presented with applications to high-T<sub>c</sub> superconductors. The loss analysis considers conductor, dielectric and gyromagnetic losses as well as the finite thickness of the metallizations. The method is verified by comparison with results available in the open literature.

## Introduction

Superconducting structures in coplanar technology involve both low loss and low dispersion and, therefore, offer attractive solutions in microwave circuits [1]. Of particular importance is the slow-wave effect which can considerably reduce the dimensions of active and passive components [2]. However, previous studies have not addressed the application of superconductors for slow-wave configurations. Also, the effects of finite conductor thickness have not been taken into account in the analysis of this kind of structure. Therefore, this paper focuses on the analysis of high-T<sub>c</sub> superconductor coplanar waveguide structures (c.f. Fig. 1). SDIA, which is extended to include anisotropic substrates, is combined with the concept of complex resistive boundary conditions [2] for the loss analysis.

## Theory

SDIA results in a simple solution for multilayered structures by decoupling the TE-wave and TM-wave components [3]. In the case of an anisotropic substrate, however, the key problem is to find the wave immittances of both TE and TM

waves for the commonly used magnetization in x or z direction (c.f. Fig. 1). If the substrate is magnetized in z direction, the permeability tensor can be expressed as:

$$\langle \vec{\mu} \rangle = \mu_0 \begin{pmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \quad (1)$$

where  $\mu$ ,  $\kappa$  and  $\mu_z$  are complex quantities to account for the losses in the magnetic material. The wave immittances of TE and TM waves can then be obtained from Maxwell's equations:

$$Y_{TE} = \frac{(\beta^2 \mu_\Delta + \alpha^2 \mu_z) \gamma_y + \alpha \mu_z \kappa (\alpha^2 + \beta^2) / \mu}{j \omega \mu_0 \mu_z \mu_\Delta (\alpha^2 + \beta^2)} \quad (2)$$

$$Y_{TM} = j \omega \epsilon_0 \epsilon_r / \gamma_y \quad (3)$$

with

$$\gamma_y^2 = \alpha^2 + \beta^2 (1 + \mu_z / \mu) / 2 - k_0^2 \epsilon_r (\mu_z + \mu_\Delta) / 2 \pm q / 2 \quad (4)$$

$$q^2 = \kappa_0^2 \epsilon_r (\mu_\Delta - \mu_z) - \beta^2 (1 - \mu_z / \mu) + 2 \kappa_0^2 \kappa^2 \beta^2 \epsilon_r \mu_z / \mu^2 \quad (5)$$

and

$$\mu_\Delta = (\mu^2 - \kappa^2) / \mu, \quad \kappa_0^2 = \omega^2 \mu_0 \epsilon_0 \quad (6)$$

In (2) - (6),  $\alpha$ ,  $\beta$ ,  $\gamma_y$  are the propagation constants in x, z and y directions, respectively. After obtaining the wave immittances of the gyromagnetic substrate, the multilayered dyadic

Green function is modified by considering a complex boundary condition [2] both for the normal conductors and Tc superconductors. For superconductors, a complex conductivity  $\sigma$  is considered [4]. Once the modified dyadic Green function has been obtained, the dispersion relation for the transmission line is calculated by applying Galerkin's procedure. The tangential current density or tangential electric field in the spectral domain is expanded in a set of Bessel functions which take the edge singularities into consideration [3]. The complex propagation constants are given by the roots of the characteristic equation.

## Results

Fig. 2 shows the propagation and loss characteristics of a single layer, conventional coplanar waveguide on ferrite substrate. As long as losses are neglected and the ferrite is biased in x direction, close agreement is obtained compared with the propagation constant values for infinitely thin conductor thickness in [5] (upper two dashed lines). In order to realistically approximate this idealized case, an extremely thin superconductor YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> (YBCO) with typical values  $\lambda_{\text{eff}} = 1500 \text{ \AA}$  at  $T/T_c = 0.5$  and  $\sigma_n = 210000 \text{ S/m}$  is introduced. As is demonstrated by the coarsely dotted line in Fig. 2, the propagation constant is reduced significantly compared to the case of ideal conductivity and zero conductor thickness assumed in [5]. For this structure the solid line shows the loss behavior which steadily increases with frequency. The dash-dotted line represents the normalized propagation constant of the conventional structure but with the magnetic bias in the z direction. Significant changes can be observed in comparison with the fine and medium dotted lines for x-direction magnetization.

The backward-wave characteristics of a slow-wave configurations for MMIC applicability is shown in Fig. 3. The slow-wave factor and the loss behavior of the coplanar waveguide on a lossy ferrite-dielectric layer are compared with results of Mesa [4] for the case of infinitely thin conductors. Excellent agreement is obtained. If a thin high-Tc superconductor is used, however, it is found that both slow-wave factor and loss are reduced compared to the case of  $t=0$ , with the influence on the loss characteristics slightly more pronounced.

Fig. 4 shows the slow-wave and loss characteristics of a reciprocal ferrite-dielectric layered coplanar waveguide structure with back metallization. The influence of demagnetized, partially magnetized and saturated magnetization levels in z-direction are investigated. A lossy copper ground plane with a conductivity of  $\sigma = 40000 \text{ S/mm}$  is considered. The highest values for the slow-wave factor as well as for the losses are obtained in the case of demagnetized ferrite. With increasing magnetic bias, both parameters decrease with this tendency slightly reduced towards higher frequencies.

## Conclusions

The spectral domain immittance approach (SDIA) is used for the analysis of multilayered coplanar waveguide slow-wave structures with high-Tc superconductor. The slow-wave effect shows a relatively low dependence on the material of the conductor, its thickness and the direction or the magnitude of the magnetic bias. However, the loss behavior can be greatly affected.

## References

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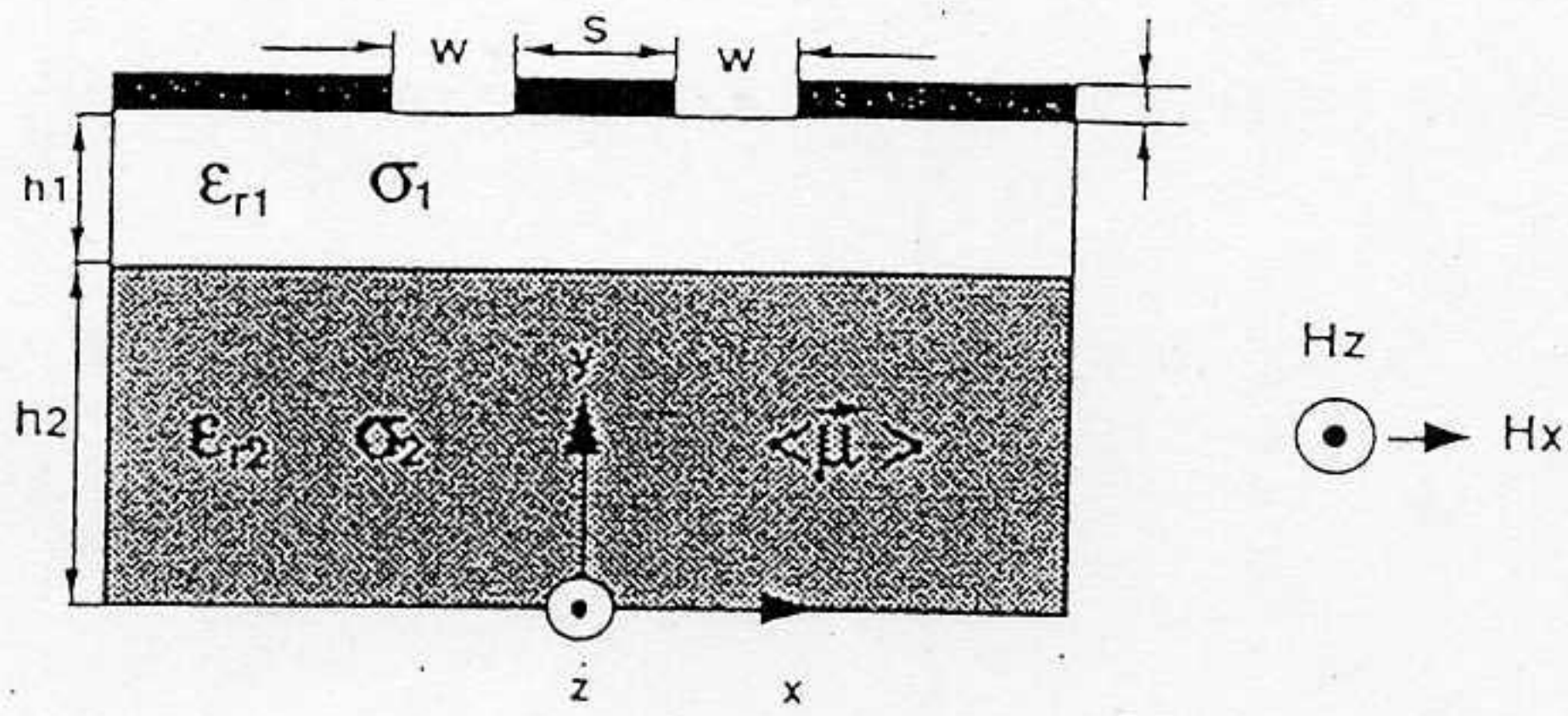


Fig. 1 Cross-section of a multilayered coplanar waveguide slow-wave transmission line

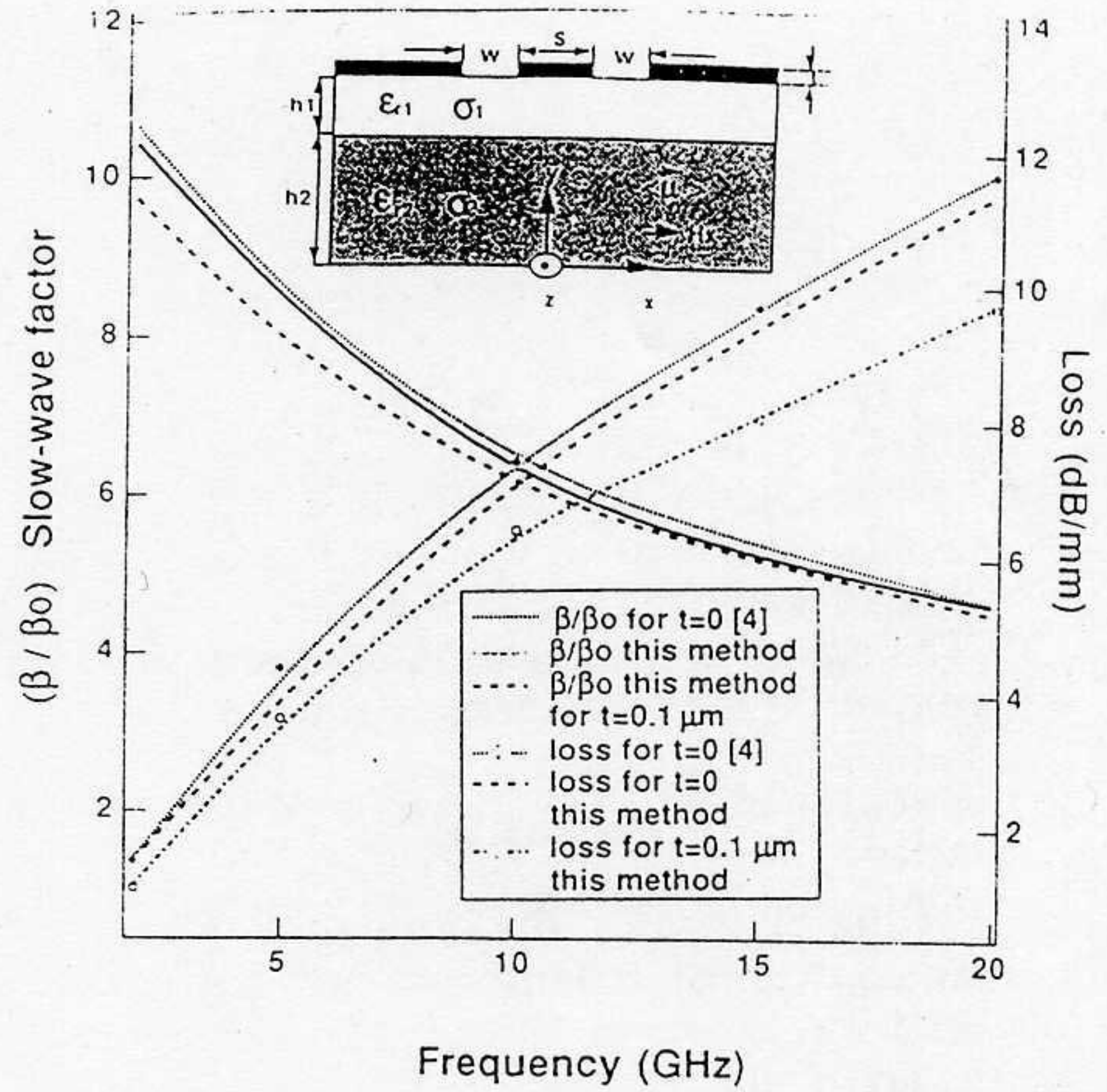


Fig. 3 Slow-wave factor and attenuation constant for multilayered CPW with ferrite magnetized in x direction. Dimensions:  $s=0.12\text{mm}$ ,  $w=0.1\text{mm}$ ,  $h_1=1\mu\text{m}$ ,  $h_2=0.1\text{mm}$ ,  $\epsilon_{r1}=14.9$ ,  $\epsilon_{r2}=4.3$ ,  $\sigma_1=0.4\text{S/m}$ ,  $\sigma_2=0.1\text{S/mm}$ ,  $M_s=725\text{G}$ ,  $H_x=500\text{Oers}$ ,  $\Delta H=37\text{Oers}$ . Superconductor:  $T/T_c=0.5$ ,  $\lambda_{\text{eff}}=1500\text{\AA}$ ,  $\sigma_n=210\text{S/mm}$

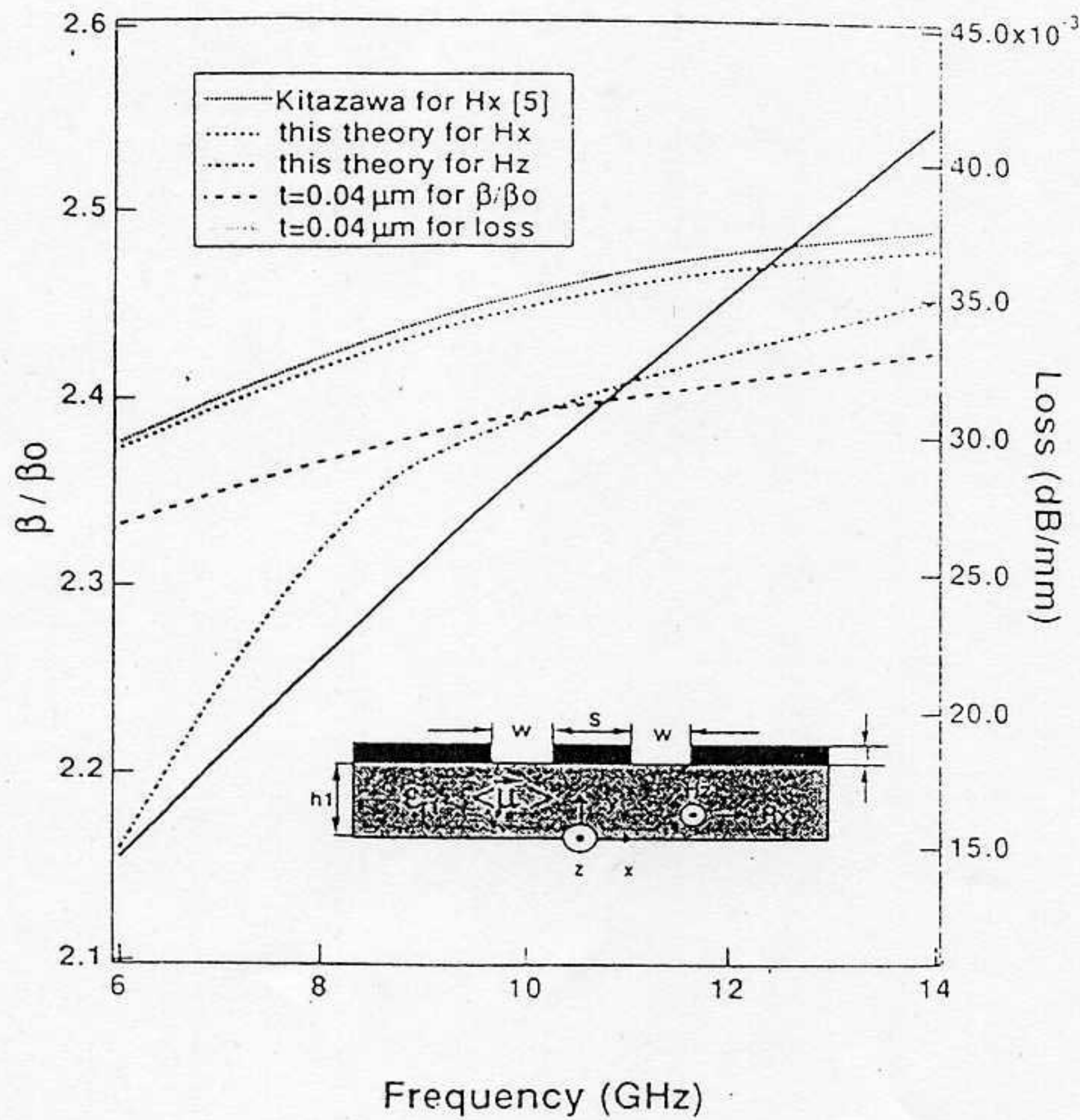


Fig. 2 Propagation characteristics of CPW with magnetized ferrite  $h=1\text{mm}$ ,  $w=1\text{mm}$ ,  $s=0.5\text{mm}$ ,  $\epsilon_r=11.6$ ,  $M_s=1800\text{A/cm}$ ,  $H_x=H_z=300\text{A/cm}$ . Superconductor:  $T/T_c=0.5$ ,  $\lambda_{\text{eff}}=1500\text{\AA}$ ,  $\sigma_n=210\text{S/mm}$ ,  $t=0.04\mu\text{m}$

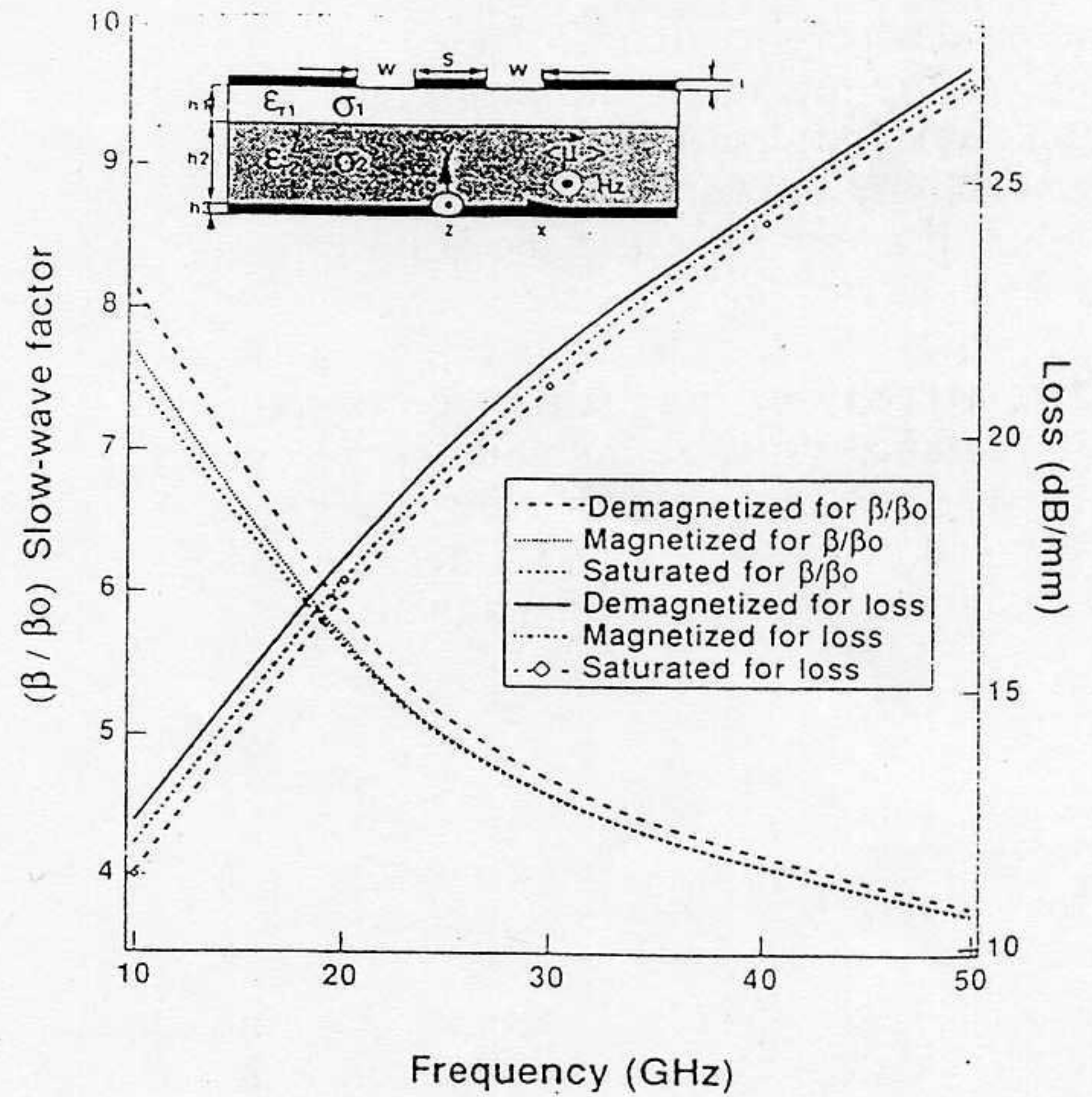


Fig. 4 Slow-wave factor and attenuation constant for multilayered CPW with magnetized ferrite in z direction. Dimensions:  $s=0.12\text{mm}$ ,  $w=0.1\text{mm}$ ,  $h_2=0.5\text{mm}$ ,  $h_3=0.1\text{mm}$ ,  $h_1=1\mu\text{m}$ ,  $\epsilon_{r1}=12$ ,  $\epsilon_{r2}=23$ ,  $\sigma_2=40000\text{S/mm}$ ,  $\sigma_3=0.1\text{S/mm}$ ,  $\sigma_1=0.4\text{S/m}$ ,  $M_s=870\text{G}$ ,  $M_z=550\text{G}$ ,  $H_z=2200\text{Oers}$ ,  $\Delta H=50\text{Oers}$ . Superconductor:  $T/T_c=0.5$ ,  $\lambda_{\text{eff}}=1500\text{\AA}$ ,  $\sigma_n=210\text{S/mm}$ ,  $t=0.04\mu\text{m}$