SPECTRAL-DOMAIN ANALYSIS OF PATCH RADIATORS ON LOSSY FERRITE SUBSTRATES

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Abstract A spectral-domain analysis for patch radiators on lossy ferrite substrates is presented. The theoretical model includes the losses of the substrate as well as different directions of applied d.c. magnetic bias. Moreover, the conductivity of the patch and its finite thickness is taken into account by incorporating a complex resistive boundary condition. The influence of direction and magnitude of the magnetic bias on resonance frequency and Q factor is investigated. It is demonstrated that comparable dependencies are obtained for square patches while, in the rectangular case, a bias in the narrower direction of the patch yields slightly less tunability. In applications on relatively thin ferrite substrates, the Q factor can be increased by one order of magnitude - without significantly affecting the resonance frequency - if the conventional conductor is replaced by superconducting material.

I. INTRODUCTION

Patch radiators are well known for their excellent performance in light-weight and low-cost array architectures. Many dielectric substrates commonly used to support patch radiators exhibit a significant amount of anisotropy that can either degrade the performance of printed circuits and antennas or, alternatively, have a beneficial effect once it is fully included in the design procedure [1]. Consequently, a substrate, whose material parameters can be influenced by outside parameters, offers - to a certain degree - the potential for tuning capabilities. Patch antennas on anisotropic media have been studied before, e.g. [2]-[3]. More recently, microstrip antennas on biased ferrite substrates with emphasis on linearly and circularly polarized radiation have been investigated [4], [5]. However, the field theoretical models presented neglect effects due to the finite thickness and the finite conductivity of the radiating patch and the losses of the ferrite substrate, which play an increasingly important part in the design of microstrip-integrated antenna systems.

Therefore, this paper focuses on a technique to analyze finite-thickness as well as finite-conductivity patch radiators on lossy ferrite substrate material. The theoretical model allows the d.c. magnetic bias to be applied in any direction of the cartesian coordinate system. The influence of magnitude and direction of the magnetic bias on resonance frequencies and Q values is investigated. A modified spectral-domain immittance approach is used to formulate the three-dimensional characteristic equation system. A decoupling procedure for the electric and magnetic field leads to closed form impedance dyadic Greens functions in the spectral domain. As required and as is necessary and essential for reliable modern integrated circuit design, the theoretical model includes the loss properties of the ferrite tensor elements, the finite metallization thickness of the patch as well as its finite conductivity. The flexibility of the approach allows the patch to be realized as conventional, e.g., copper conductors or as superconductors in order to improve the radiation efficiency [6].

II. THEORY

The geometry of a patch radiator with an imperfect conductor on lossy ferrite substrate is shown in Fig. 1. The general form of the ferrite tensor in the case of magnetization in y, z and x direction can be expressed as

$$\langle\vec{\mu}\rangle = \mu_o\begin{bmatrix}\mu & 0 & j\kappa\\ 0 & \mu_y & 0\\ -j\kappa & 0 & \mu\end{bmatrix} \qquad \langle\vec{\mu}\rangle = \mu_o\begin{bmatrix}\mu & j\kappa & 0\\ -j\kappa & \mu & 0\\ 0 & 0 & \mu_z\end{bmatrix} \qquad \langle\vec{\mu}\rangle = \mu_o\begin{bmatrix}\mu_x & 0 & 0\\ 0 & \mu & j\kappa\\ 0 & -j\kappa & \mu\end{bmatrix} \tag{1}$$

respectively, where μ, $\mu_{x,y,z}$, and κ may be complex quantities to account for the losses in the material [7], [8].

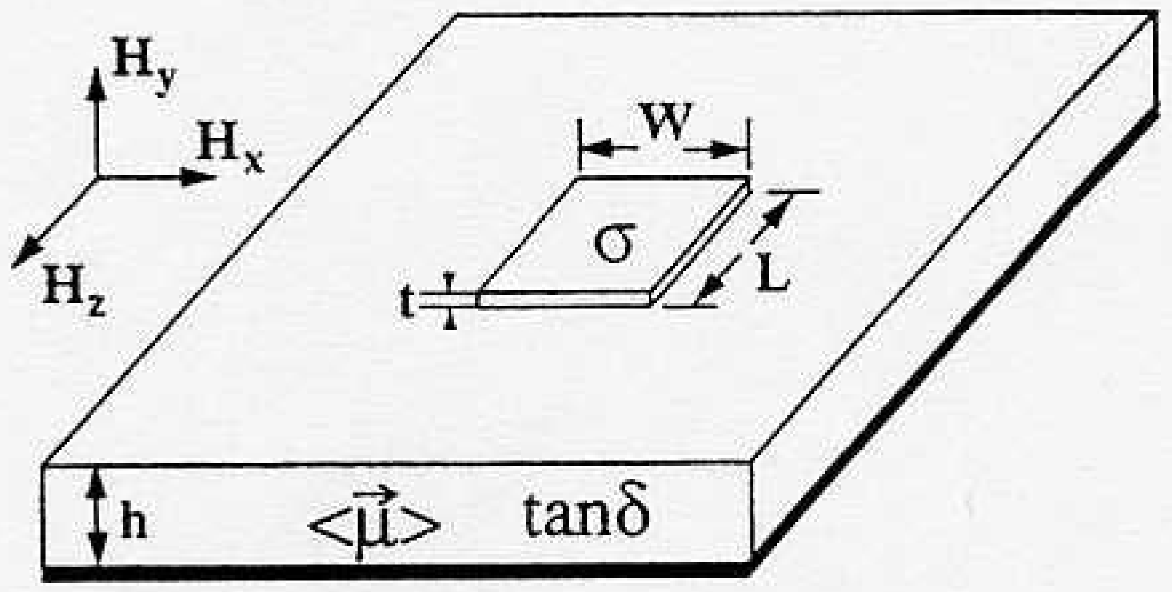


Fig. 1 Geometry of patch radiator.

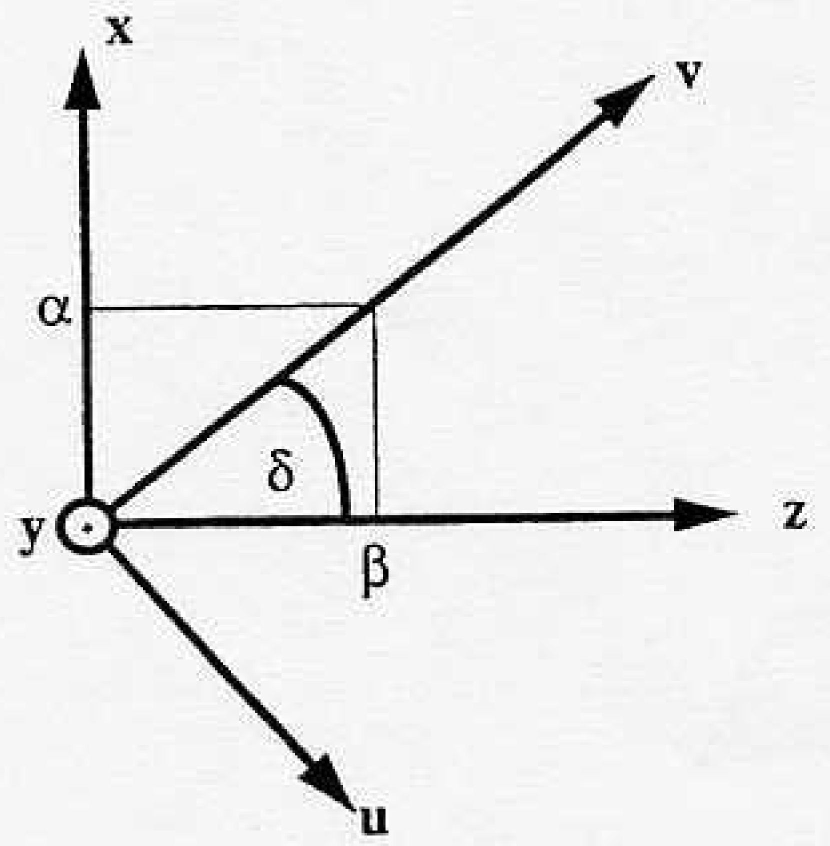


Fig. 2 Transform of coordinate systems.

By using the concept of the spectral-domain immittance approach [9], the six-component electromagnetic field considered in a coordinate system (x,y,z) can be decomposed into TM-to-y and TE-to-y waves in a (u,v,y)-system (Fig. 2).

$$u = z\sin\delta - x\cos\delta \tag{2}$$

$$v = z\cos\delta + x\sin\delta \tag{3}$$

$$\delta = \arccos\left(\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\right) \tag{4}$$

The complex elements of the permeability tensor are incorporated into Maxwell's equations which are readily applied in the transformed domain. This leads to decoupled TE and TM waves and directly yields the wave immittances of TE and TM waves with their associated propagation constants $\gamma_{m,e}$. For magnetic bias in y direction, we obtain

$$\gamma_m^2 = -k_o^2\varepsilon_r\mu_\perp + \alpha^2 + \beta^2 \tag{5}$$

$$\gamma_e^2 = \frac{\mu}{\mu_y}(-k_o^2\varepsilon_r\mu_\perp + \alpha^2 + \beta^2) \tag{6}$$

where k_o is the free-space wave number, α, β are the propagation constants in x, z directions, respectively, and

$$\mu_\perp = (\mu^2 - \kappa^2)/\mu \tag{7}$$

Consequently, the the related admittances are given by

$$Y_{TM} = \frac{j\omega\varepsilon}{\gamma_m} \qquad Y_{TE} = \frac{\gamma_e}{j\omega\mu_o\mu_\perp} \tag{8}$$

Following the same steps for a magnetic bias in z direction, we find

$$Y_{TE} = \frac{(\beta^2\mu_\perp + \alpha^2\mu_z)\gamma_e + \alpha\mu_z\kappa(\alpha^2+\beta^2)/\mu}{j\omega\mu_o\mu_z\mu_\perp(\beta^2+\alpha^2)} \tag{9}$$

$$Y_{TM} = j\omega\varepsilon_o\varepsilon_r/\gamma_m \tag{10}$$

$$\gamma_{m,e} = \left(-\frac{q_{m,e}}{2} + \sqrt[\frac{1}{3}]{\left(\frac{q_{m,e}}{2}\right)^2 + \left(\frac{p_{m,e}}{3}\right)^2}\right) + \left(-\frac{q_{m,e}}{2} - \sqrt[\frac{1}{3}]{\left(\frac{q_{m,e}}{2}\right)^2 + \left(\frac{p_{m,e}}{3}\right)^2}\right) \tag{11}$$

with

$$q_m = 2a^3/27 - ab/3 + c \qquad p_m = b - a^2/3 \tag{12}$$

$$q_e = -k_o^2\mu_z\varepsilon_r\kappa a/\mu \qquad p_e = (k_o^2\mu_z\varepsilon_r - \alpha^2 - \beta^2\mu_z/\mu) \tag{13}$$

$$a = -k_o^2\varepsilon_r\kappa/2 \qquad b = k_o^2\mu_z\varepsilon_r - \alpha^2 - \beta^2 \qquad c = k_o^2\kappa\varepsilon_r(\alpha^2 - \mu_z\varepsilon_r)/\alpha \tag{14}$$

For a biasing field in the x direction, the positions of α, β are exchanged in (9), (10), μ_z is replaced by μ_x, and

$$q_m = k_o^2\kappa\varepsilon_r\beta/2 \qquad p_m = k_o^2\mu_x\varepsilon_r - \alpha^2 - \beta^2 \tag{15}$$

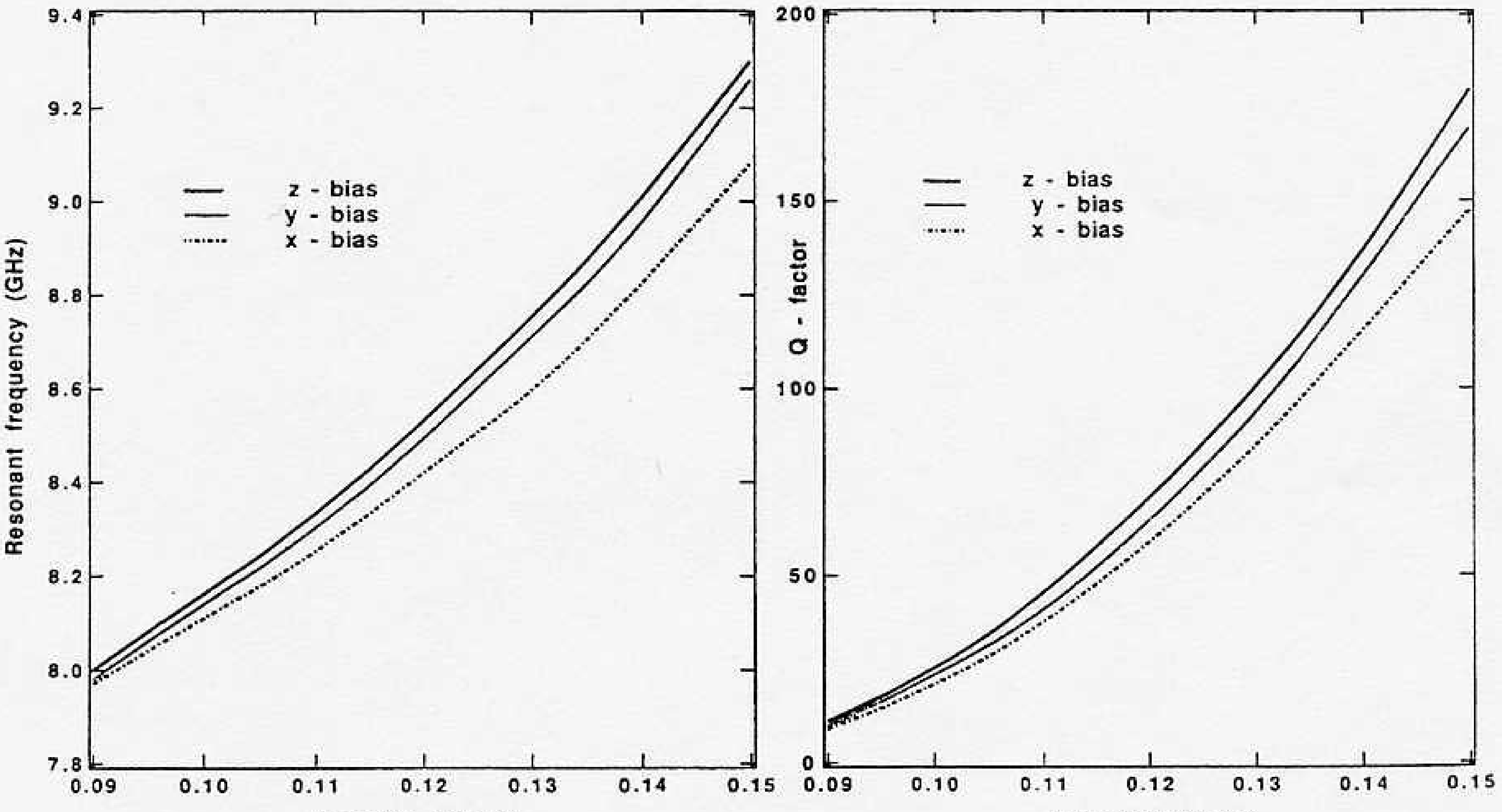
$$q_e = k_o^2\kappa\varepsilon_r\beta/\mu \qquad p_e = k_o^2\varepsilon_r - \alpha^2/\mu - \beta^2 \tag{16}$$

in (11).

The finite thickness and conductivity of the conducting patch is taken into account by utilizing the complex resistive boundary condition [10] which is capable of modelling conventional metal as well as superconductors. After applying Galerkins procedure and expanding the tangential current densities in sets of Bessel functions, the complex resonance frequencies $\omega_c=\omega_r+j\omega_i$ are given by the roots of the characteristic equation which is evaluated using numerical integration techniques [2]. Once ω_c is determined, the Q factor of the patch antenna is calculated as the ratio ω_r/ω_i.

III. RESULTS

The influence of a change in magnitude and direction of the d.c. magnetic bias on the resonance frequency and the Q factor of a rectangular patch radiator on ferrite substrate is shown in Figs. 3. As expected, the resonance frequency increases with the external field (Fig. 3a). The tuning range for z-direction bias is almost identical to that in y direction while the x-direction bias offers a lightly lower tuning range. A similar behavior is observed for the Q factor (Fig. 3b) which also increases with magnetic bias. This effect considerably changes the bandwidth of the patch while being tuned and needs to be considered by design engineers.



a **b**

Fig. 3 Influence of direction and amplitude of magnetic bias on (a) resonant frequency and (b) Q-factor of a rectangular patch. Parameters: $\varepsilon_r=16.6$, $\tan\delta=10^{-3}$, W=2.5mm, L=8.0mm, h=1.0mm, t=0.2mm, $\sigma=40S/\mu m$, $H_s=0.16T$.

For comparison with Figs. 3, Figs. 4 show the same parameters for a square patch. The resonance frequencies are slightly lower due to the enlarged patch area. The main difference, however, is that identical tuning ranges are obtained for the three investigated directions of magnetic bias (Fig. 4a). Consequently, the Q factor does not significantly depend on the direction of bias either (Fig. 4b).

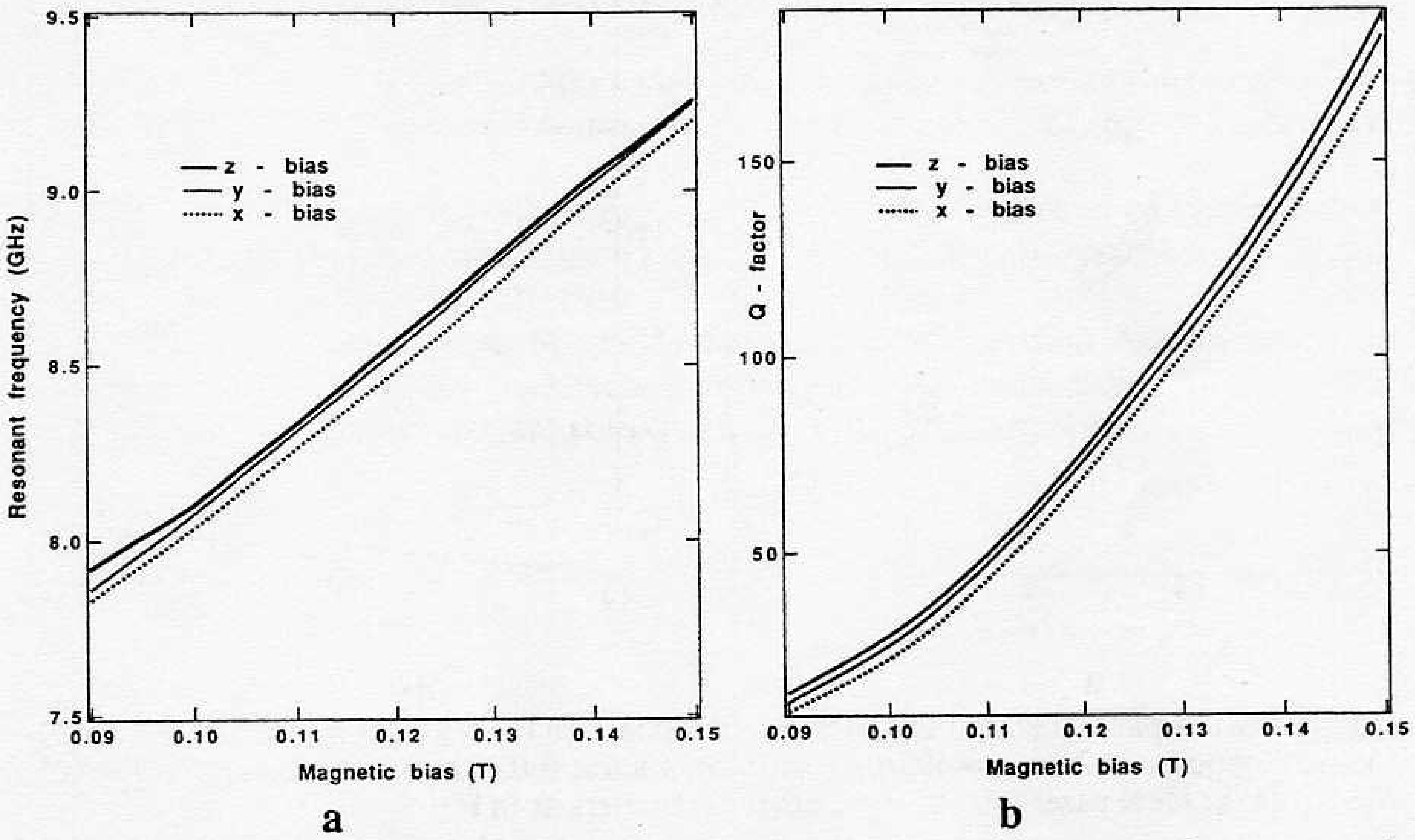
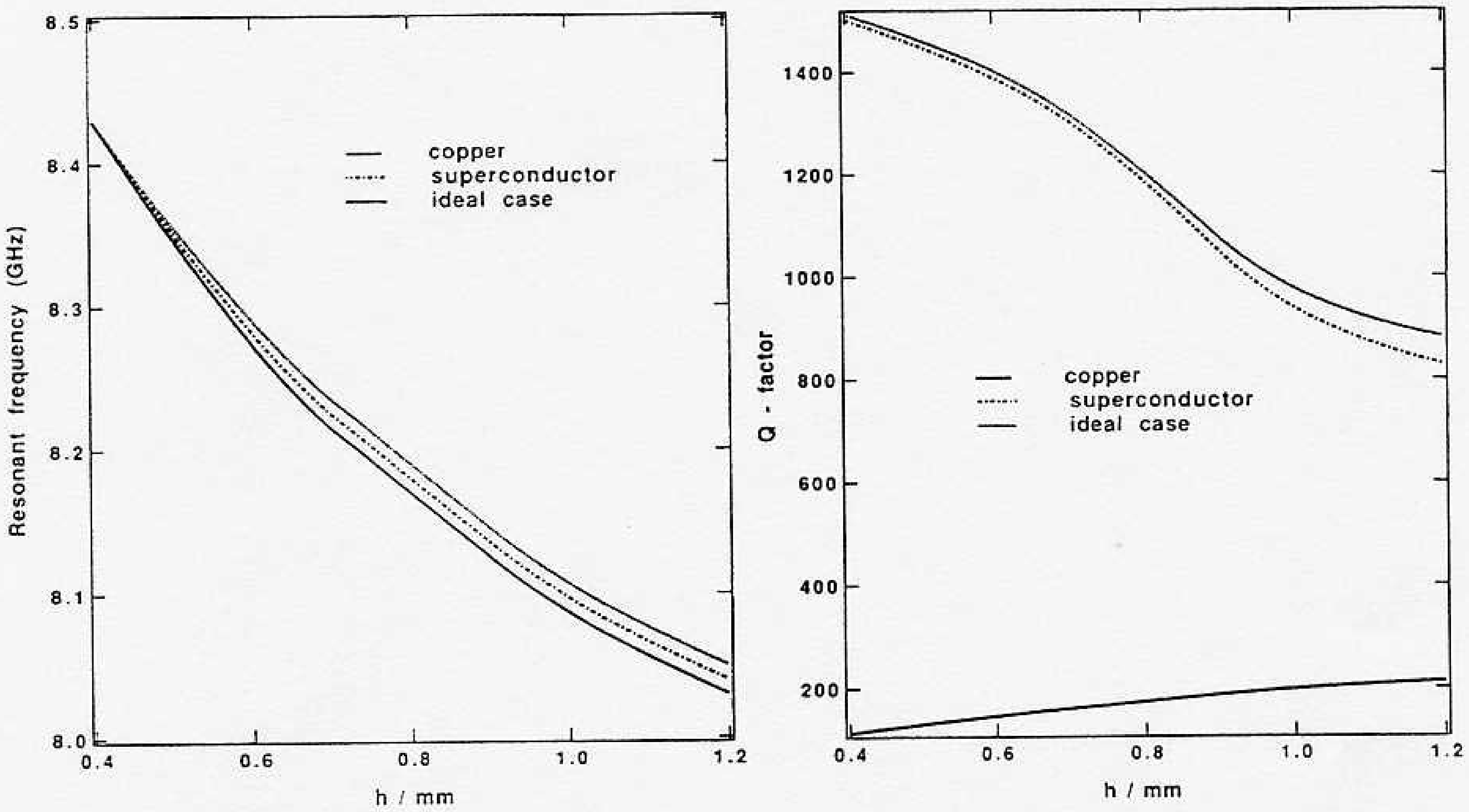


Fig. 4 Influence of direction and amplitude of magnetic bias on (a) resonant frequency and (b) Q-factor of a square patch (W=L=8.0mm, other parameters as in Fig. 3).

The effect of the substate thickness on the resonance frequency and Q factor are shown in Figs. 5 with different conductors as parameter. The resonance frequencies decrease with increasing substrate height, as expected, and are relatively insensitive to the different conductor materials considered (Fig. 5a). The differences with respect to the Q factor are obvious (Fig. 5b). The superconducting patch comes close to the ideal case, and the Q factor decreases as the amount of losses contributed by the ferrite increases. The Q factor behavior of the copper patch, however, seems to be reversed. This is due to the fact that the conductor thickness is of the same order of magnitude as the substrate. Consequently, the conductor losses contribute overproportionally to the low Q factor. With further increasing substrate height, the solid curve in Fig. 5b shows a behavior similar to those of the ideal and superconductor since the substrate material then carries the majority of losses. For the relatively small substrate thicknesses displayed, however, the difference between a copper and a superconducting patch is one order of magnitude in Q-factor.

IV. CONCLUSIONS

An analysis technique for patch radiators on lossy ferrite substrates is presented. The method is based on a modified spectral-domain approach which allows the finite patch thickness as well as its finite conductivity to be taken into account. The tuning



a **b**

Fig.5 Effect of patch material and substrate thickness on (a) resonant frequency and (b) Q-factor. Copper: t=0.2mm, σ=40S/μm; superconductor: t=0.4μm, σ_n=200S/mm, T/T_c= 0.5, λ_{eff}=1500Å; ideal case: t=0, $\sigma \rightarrow \infty$; other parameters as in Fig. 3.

ranges and Q factors of square patches are found to be comparable for different directions of applied d.c. magnetic bias. A comparison between conventional and superconducting patches shows that the Q factor in the latter case can be improved by one order of magnitude. The influence of the conducting material on the resonance frequency is found to be extremely low.

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