NON-PERTURBATIVE FULLWAVE ANALYSIS OF LOSSY PLANAR CIRCUITS

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ABSTRACT: A non-perturbative analysis, including both metallic and dielectric losses, of planar circuits is presented. The Green's impedance dyadics are modified to account for metallic losses in the ground plane and the conducting surfaces. Dielectric constants are allowed complex values to describe their lossy properties. The complex resistive boundary condition is modified to take into account the fact that thin conductors distinguish between LSE and LSM modes. The theory describes lossy ground planes exactly and is exact for conductors of finite width in the limit of small thickness. Expression for the conductance, the resistance, the inductance and the capacitance matrices are presented as well as corresponding numerical results. Modal attenuation constants and dispersion curves are discussed.

1. INTRODUCTION

Computer simulation of lossy components and circuits used in microwave applications has not yet reached maturity in methodology or speed of execution. Metallic losses in planar circuits are commonly accounted for using perturbation theory where it is assumed that the field distribution of the lossy structure is not markedly different form the lossless approximation. Such an assumption holds at surfaces with small curvatures or large thickness, in which case the losses can be calculated through the concept of surface impedance. Conductors which are not thick enough to block incident electromagnetic fields, or circuits containing large curvatures, such as a the edge of a thin patch or microstrip, can not be adequately analyzed using the standard perturbation theory. This has been pointed out by Pregla for the case of lossy microstrip lines [1]. Dielectric losses are, however, straightforwardly, at least in principle, described by allowing the dielectric constants to assume imaginary parts.

Pond and coworkers combined the Spectral Domain Approach and the concept of complex resistive boundary condition to calculate metallic losses in a superconducting microstripline [2]. Basis functions which include the singularity at the edge were used in the Galerkin's solution. The presence of the edge condition in the formulation leads, however, to diverging matrix elements and infinite damping if the integrals are accurately evaluated. Kuo and Itoh used the same technique to solve a similar problem using subsectional basis functions which do not exhibit the edge condition [3]. The technique was also applied to patches by Cai and Bornemann [4]. In this presentation, we show how to extend the method to analyze thin planar circuits. The losses in the ground plane are exactly described by our formulation. The Greens' impedance dyadics are modified to include the metallic losses through new expressions for the surface impedance which we also present. We show that the surface impedance, being a response function, depends on the field distribution. In particular, a thin good conductor responds differently, i.e., has different surface impedances, to the LSE and LSM modes which are used in the Spectral Domain Imittance Approach. The capacitance and inductance matrices of coupled lines have been investigated by Tripathi [5] using the partial power definition of the characteristic impedances and by Amari using modal powers [6]. In this presentation we also present numerical results for the conductance and the resistance matrices which describe lossy systems. These matrices provide a convenient vehicle for analyzing the time response of systems of coupled lines.

2. THEORY

The effects of metallic losses can be accounted for by a an extension of the analysis presented by Pond et.al [2]. The surface impedance of the thin conductor depends on the field distribution in the structure. The LSE and LSM surface impedances are derived from the solution of Maxwell's equations for a conducting slab of thickness t and conductivity σ. The algebra is lengthy but otherwise straightforward. The modified Green's impedance dyadics, which relates the current density on the strips to the tangential electric field **outside** of the strips, take the following form:

$$\begin{pmatrix} E_x^{out} \\ E_z^{out} \end{pmatrix} = \begin{pmatrix} G_{xx} & G_{xz} \\ G_{zx} & G_{zz} \end{pmatrix} \begin{pmatrix} J_x \\ J_z \end{pmatrix} \qquad 1$$

where the elements G_{ij} are given by

$$G_{xx} = \frac{1}{\alpha^2+\beta^2}[\frac{\alpha^2}{Y_1^{LSM}+Y_2^{LSM}} + \frac{\beta^2}{Y_1^{LSE}+Y_2^{LSE}}] - \frac{G_s^{LSE}\beta^2+\alpha^2 G_s^{LSM}}{\alpha^2+\beta^2} \qquad 2.a$$

$$G_{xz} = G_{zx} = \frac{\alpha\beta}{\alpha^2+\beta^2} \times [\frac{1}{Y_1^{LSM}+Y_2^{LSM}} - \frac{1}{Y_1^{LSE}+Y_2^{LSE}} + G_s^{LSE} - G_s^{LSM}] \qquad 2.b$$

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$$G_{zz} = \frac{1}{\alpha^2+\beta^2}\left[\frac{\alpha^2}{Y_1^{LSE}+Y_2^{LSE}} + \frac{\beta^2}{Y_1^{LSM}+Y_2^{LSM}}\right] - \frac{G_s^{LSM}\beta^2+\alpha^2 G_s^{LSE}}{\alpha^2+\beta^2} \qquad 2.c$$

Where $Y_{1,2}^{LSE}$ and $Y_{1,2}^{LSM}$ are the LSE and LSM input admittances at the air-dielectric interface in the positive and negative x-directions. They include not only the dielectric losses but also the effect of the ground plane. The metallic terms G_s^{LSE} and G_s^{LSM} are given by

$$\frac{1}{G_s^{LSE}} = Y_s^{1,LSE} + Y_s^{2,LSE} \times \left[cosh(\gamma_c t) - \frac{Z_s^{1,LSE}}{Z_c} sinh(\gamma_c t)\right]. \qquad 3.a$$

and

$$\frac{1}{G_s^{LSM}} = Y_s^{1,LSM} + Y_s^{2,LSM} \times \left[cosh(\gamma_c t) - \frac{Z_s^{1,LSM}}{Z_c} sinh(\gamma_c t)\right] \qquad 3.b$$

$\gamma_c = (1+j)/\delta$ where δ is the skin depth and $Z_c = (1+j)/\sigma\delta$. $Y_s^{1,2,LSE}$ and $Y_s^{1,2,LSM}$ are the input admittances at the top of the conducting strip (1) and at its bottom (2). Details of the derivation will be discussed in the presentation, they are not presented here for lack of space. The terms G_s^{LSE} and G_s^{LSM} both approach the usual surface impedance $Z_s = (1+j)/\sigma\delta$ in the limit of thick conductor, i.e., $t/\delta \gg 1$. In the limit $t \to 0$, they approach the Green's impedance dyadics of a system with no metallic losses at the interface. When $t = 0$, the modified dyadics given by equations (2) all vanish as long as the conductivity of the conductors at the interface is finite. This reflects the fact that a true surface current can exist only in a perfect conductor [7]. To complete the solution, the current density is expanded over a set of basis functions which are nonzero only over the metallized surfaces. For lossy systems, the current density does not have the usual edge condition [3]. This does not, however, mean that it is not singular at the edges. The only requirement is that its singularity be not as strong as that of a lossless infinitely thin strip. The propagation constants and the current density are determined following the standard Galerkin's method. There is an important point which should be carefully handled when applying this approach to multiply connected conducting surfaces. In such a situation, the loss terms G_s should be interpreted as terms of appropriate submatrices since the current on a specific portion of the conducting surface contributes to metallic losses only on that portion.

The power transported by a mode can subsequently be computed from the integration of the complex Poynting vector over the cross section

$$P = \frac{1}{2}\int\int_S (\mathbf{E}\times\mathbf{H}^*).ds \qquad 4$$

To derive an equivalent circuit for the lossy structures, we introduce modal currents and voltages which are assumed to satisfy the transmission line equations

$$\frac{dV}{dz} = -[jL\omega + R]I \qquad 5$$

$$\frac{dI}{dz} = -[jC\omega + G]V \qquad 6$$

Furthermore, the modal voltages and currents are also required to conserve the average power transported by each mode, namely

$$[M_I]^T[M_V] = [P] \qquad 7$$

where [P] is the power diagonal matrix. Combining (5) to (7) and assuming propagating solutions with propagation constants $[\beta]$, the matrices $[L]$, $[C]$, $[R]$ and $[G]$ are seen to be given by

$$[C] = \frac{1}{c}[M_I]\left[\frac{Re(\sqrt{\epsilon_{eff}})}{2P}\right][M_I]^T \qquad 8$$

$$[L] = \frac{1}{c}[M_I]^{-1T}[2PRe(\sqrt{\epsilon_{eff}})][M_I]^{-1} \qquad 9$$

$$[R] = \frac{\omega}{c}[M_I]^{-1T}[-Im(\sqrt{\epsilon_{eff}})2P][M_I]^{-1} \qquad 10$$

$$[G] = \frac{\omega}{c}[M_I]\left[\frac{-Im(\sqrt{\epsilon_{eff}})}{2P}\right][M_I]^T \qquad 11$$

c is the speed of light in free space. Note that these last four expressions reproduce the results of the lossless case if the matrix $[\epsilon_{eff}]$ is assumed real.

3. NUMERICAL RESULTS

The method presented here is applied to the case of two coupled microstrip lines. Figure 1a shows the real part of the effective dielectric constant $\epsilon_{eff} = (\beta/k_0)^2$ versus the separation distance between the lines. The ground plane has conductivity $\sigma = 410^7 S/m$ and is 1 mm thick. The strips, which are 10 μm thick, have the same conductivity as the ground plane. All calculations are carried out at 1 GHz and $\epsilon_r = 10 - j10^{-4}$. Figure 1b is a plot of the attenuation constant versus the separation distance of the two lines. Note that the two curves approach each other in in the limit of large values of s as the lines become decoupled and the two modes degenerate into one. In addition, the attenuation of the odd mode is consistently smaller than that of the even mode. The main difference is due to the ground plane which affects more the even mode, the odd mode has its currents going out of one line into another with the ground plane playing a minor role. Needless to say that the total attenuation is interplay of the three contributions, the dielectric losses, losses in the strips and the ground plane. The relative thicknesses of the strips and the ground plane determine which mode is attenuated more. Figure 2a shows the elements of the resistance matrix $[R]$. The off-diagonal elements decrease with s since the lines become decoupled in this limit. The diagonal elements are however increasing with s. This behaviour is attributed to the finite conductivity of the ground

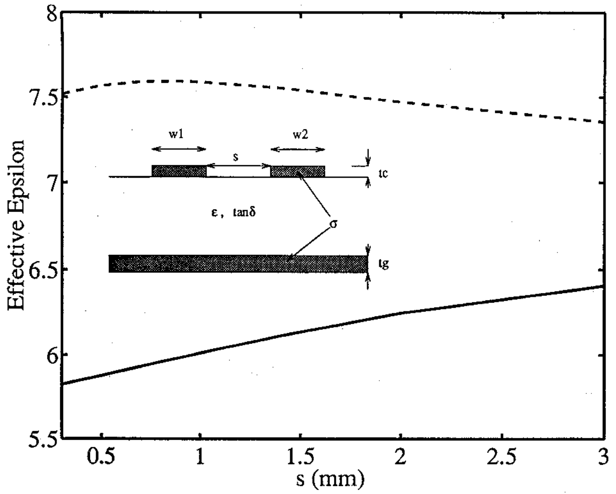

Figure 1a. Real part of effective dielectric constant of the even (dotted line) and odd (solid line) modes versus separation distance. $w_1 = w_2 = h = 1\text{mm}$, $tc = 10\mu\text{m}$, $tg = 1\text{mm}$, $\epsilon = 10$, $\tan\delta = 10^{-5}$.

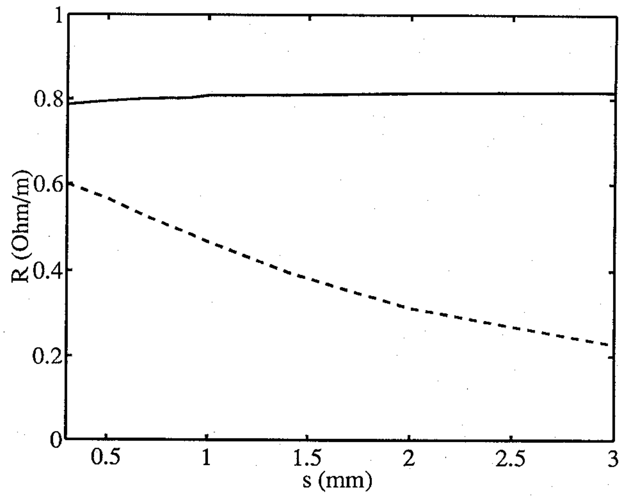

Figure 2a. Elements of resistance matrix versus separation distance. Solid line $R_{11} = R_{22}$, dotted line $R_{12} = R_{21}$. Dimensions and other parameters are same as in figure 1a.

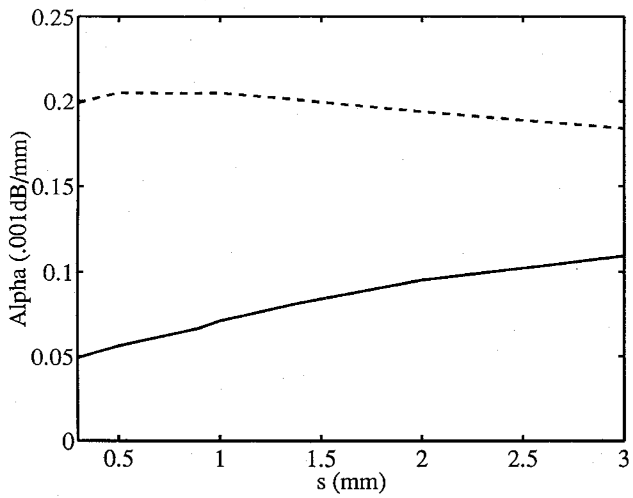

Figure 1b. Attenuation constant of the even (dotted line) and odd (solid line) modes versus separation distance. Dimensions and other parameters are same as in figure 1a.

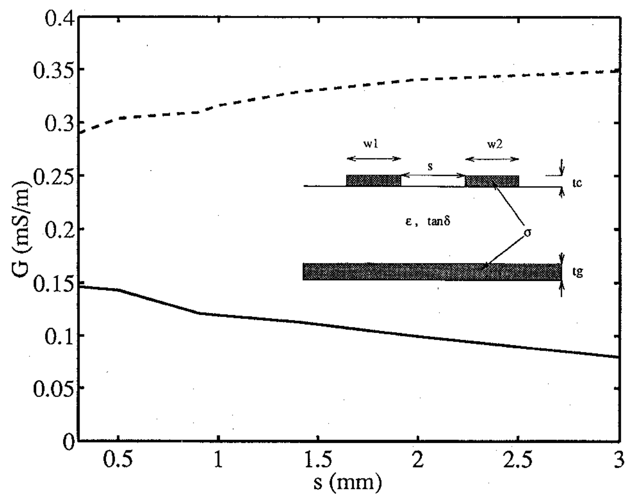

Figure 2b. Elements of conductance matrix versus separation distance.Solid line $G_{11} = G_{22}$, dotted line $G_{12} = G_{21}$. Dimensions and other parameters are same as in figure 1a.

plane. Similarly figure 2b shows the matrix $[G]$ versus the separation distance s. Its behaviour is similar to the $[R]$ matrix but the numerical values of G_{ij} are much lower because of the low losses in the dielectric ($Im(\epsilon) = -10^{-4}$). Numerical data for the inductance and capacitance matrices are also available and will be discussed during the presentation. They compare well with those in reference [6].

4. CONCLUSIONS

The method presented her can be used to analyze lossy planar circuits for both metallic and dielectric loss. Its power resides in the fact that the losses are included non-perturbatively. Problems associated with edges and extremely thin conductors are rigorously described. The conductance and resistance matrices of two symmetric coupled lines were presented.

REFERENCES

1. R. Pregla, "Determination of conductor losses in planar waveguide structures," IEEE Trans. Microwave Theory Tech., vol. 28, no. 4, pp. 433-434, Ap r. 1980.

2. I. M. Pond, C. M. Krowne, and W. C. Carter, "On the application of complex resistive boundary conditions to model transmission lines consisting of very thin superconductors," IEEE Trans. Microwave Theory Tech., vol. 37, no. 1, pp. 181-190, Jan. 1989.

3. C. W. Kuo and T. Itoh, "A flexible approach combining the spectral domain method and impedance boundary condition for the analysis of microstrip line," IEEE Microwave and Guided Wave Lett., vol. 1, pp. 172-174, Jul. 1991.

4. Z. Cai and J. Bornemann, "Generalized spectral-domain analysis for multilayered complex media and high-T_c superconductor applications," IEEE Trans. Microwave Theory. Tech., vol. 40, pp. 2251-2257, Dec. 1992.

5. V. K. Tripathi and H. Lee, "Spectral-domain computation of characteristic impedances and multiport parameters of multiple coupled microstrip lines," IEEE Transactions Microwave Theory Tech., vol. 37, pp. 215-222, Jan. 1989.

6. S. Amari, "Capacitance and inductance matrices of coupled lines from modal powers,", IEEE Transactions Microwave Theory Tech., vol. 41, pp. 146-150, Jan. 1993.

7. J. A. Stratton, Electromagnetic Theory, McGrawHill, New York 1941. p. 484.