TUNING-FREE ANALYSIS OF PLANAR RADIATORS BASED ON AN EXACT NUMERICAL EVALUATION OF THE TWO-DIMENSIONAL GENERALIZED EXPONENTIAL INTEGRAL

Martin Gimersky*, Smain Amari and **Jens Bornemann**

Laboratory for Lightwave Electronics, Microwaves and Communications (LLiMiC)
University of Victoria, Victoria, B.C., Canada V8W 3P6

I. INTRODUCTION

The two-dimensional generalized exponential integral has been widely used in many applications that involve planar transmission lines and radiating structures. When evaluating mutual and self-impedances of individual elements of a planar structure, calculation of the integral

$$I_1 = \int_{x_1<0}^{x_2>0} \int_{y_1<0}^{y_2>0} \frac{e^{-jk\sqrt{x'^2+y'^2}}}{\sqrt{x'^2+y'^2}} dy'dx' \qquad (1)$$

in one form or another, has to be considered: if the distance between the two elements with respect to the wavelength is large, the point-source approximation can be used; if the target element cannot be classified as positioned in the far zone of the source element, the integral has to be actually calculated. The integral I_1 corresponds to the usage of pulses in the role of subsectional expansion functions in the method-of-moments formulation; if different expansion functions are desired, the integrand of I_1 needs to be properly modulated and the limits of integration may change, where the origin - point (x,y) =(0,0) - may be located inside, or on the boundary, of the integration region.

The integrand of I_1 contains a singularity at the origin of the coordinate system. When evaluating I_1 through the forthcoming slice-and-dice scheme, the singularity cannot be rigorously integrated over. The approach that has become standard in antenna engineering is to evaluate integral

$$I_2 = \int_{x_1<0}^{x_2>0} \int_{y_1<0}^{y_2>0} \frac{e^{-jk\sqrt{x'^2+y'^2+a^2}}}{\sqrt{x'^2+y'^2+a^2}} dy'dx' \qquad (2)$$

instead, which in effect introduces an offset in the third coordinate of the Cartesian system. However, there is no physical justification for the introduction of any offset, as the integration is supposed to stand for sampling the electric field intensity in the infinitesimally small elevation above the target-element surface. Introducing the offset a, however small, does not correspond to physical reality; furthermore, it - artificially and unnecessarily - adds to the system an unknown variable, which somehow has to be determined.

Proper determination of a, assuming one still can use the word 'proper' in

0-7803-2719-5/95/$4.00 © 1995 IEEE

this context, has almost become a discipline in itself, thus rendering a software package 'more' or 'less' efficient. Without contributing to the discussion, one can safely conclude that the determination of the offset a, customarily called tuning [1], requires skills and extra computation, which both are time consuming. One way of determining a is to observe the effects of varying a on the radiation pattern and, through stability analysis and experience, find the usually small range of a where the radiating structure behaves as expected. This approach, however, is not reliable, for the calculation of a radiation pattern involves a double integration, which smoothens the pattern, even if the utilized current distribution is not very accurate. A better technique to determine the offset is to follow how the changing a affects the input impedance of the structure, namely its real part. As a varies, typically there is a range where the real part of the input impedance is positive and stable; the most rational choice to make is to select the value of a that corresponds to the center of that range.

In the following section, we will show that there is absolutely no need for introducing and tuning an offset, since the integral I_1 can be calculated exactly. The validity of the technique will be demonstrated by a comparison with measurements.

II. INTEGRATION TECHNIQUE AND NUMERICAL RESULTS

Rather than evaluating I_1 directly, we start with the calculation of I_2 and then find the solution for the case of $a=0$. This approach is perfectly valid, as

$$I_1 = \lim_{a \to 0} I_2 \tag{3}$$

The general configuration for integrals of the type of I_1 and I_2 is depicted in Fig. 1. Instead of performing integration in the Cartesian coordinate system, we evaluate I_2 in polar coordinates, resulting in

$$I_2 = \int_0^{2\pi} \int_0^{\rho_c(\theta)} \frac{e^{-jk\sqrt{\rho^2 + a^2}}}{\sqrt{\rho^2 + a^2}} \rho d\rho d\theta \tag{4}$$

where $\rho = \sqrt{x'^2 + y'^2}$ and $\rho_c(\theta)$ is the radial distance from the origin of the coordinate system to the contour of the integration region. It is obvious that this technique is directly applicable also to cases where the origin of the coordinate system is located outside of the integration region - in these cases the lower integration limit over ρ would be some positive value $\rho_{cl}(\theta)$ instead of zero. After the change of variables

$$u = \sqrt{\rho^2 + a^2} \tag{5}$$

we get:

$$I_2 = \frac{j}{k} \int_0^{2\pi} \left(e^{-jk\sqrt{\rho_c(\theta)^2 + a^2}} - e^{-jk|a|} \right) d\theta \tag{6}$$

Finally,

$$I_1 = \lim_{a \to 0} I_2 = \frac{j}{k} \int_0^{2\pi} (e^{-jk\rho_c(\theta)} - 1)\, d\theta \tag{7}$$

Upon inspection, the integral poses no numerical difficulties. The above equation is independent of the shape of the contour describing the boundary of the integration region. For the rectangular contour of Fig. 1, $\rho_c(\theta) = x_2/(\cos\theta)$ for the right vertical line, $\rho_c(\theta) = y_2/(\sin\theta)$ for the upper horizontal line, $\rho_c(\theta) = x_1/(\cos\theta)$ for the left vertical line, and $\rho_c(\theta) = y_1/(\sin\theta)$ for the lower horizontal line.

Using this technique, we calculated the frequency dependance of the input reflection coefficient of a rectangular patch antenna ([2], length 3.85cm, width 3.18cm, substrate thickness 1.568mm, substrate relative permittivity 2.34) fed at the center of the shorter edge of the patch. The method of moments [3], with non-overlapping pulses as subsectional expansion functions is used. The model is a simple wire-grid one, with two sets of wires: one set parallel with the longer edge of the patch; the other perpendicular to the first one, parallel with the patch's shorter edge. Magnitude and phase of the calculated input reflection coefficient are presented in Figs. 2a and 2b, respectively; they are in a significantly better agreement with measurements than the data of [4], which were obtained by the standard technique, utilizing tuning. The integral of (7) was calculated by the Gaussian integration, in 12 points; further increase of the number of integration points resulted only in minute changes that were within the plotting accuracy.

III. CONCLUSIONS

It is demonstrated that it is possible to accurately calculate the two-dimensional generalized exponential integral in the method-of-moments formulation. By performing the integration in polar coordinates over the radial variable ρ, the singularity existing in Cartesian coordinates is removed from the integrand. This in effect means that the commonly applied introduction of an offset into the integrand and subsequent tuning of that offset can be eliminated entirely. As a result, the calculated current distribution is unambiguous and exact. A comparison between calculated and measured input impedance data of a patch antenna shows good agreement and, hence, verifies the integral evaluation, even if a simplified model of non-overlapping subsectional expansion functions is assumed. The proposed integral calculation requires minimal computational effort; only 12 points in the Gaussian quadrature are needed to achieve results accurate to better than 0.1 percent.

REFERENCES

[1] B.J. Rubin and S. Daijavad, "Radiation and Scattering from Structures Involving Finite-Size Dielectric Regions," *IEEE Trans. Antennas Propagation,* vol. 38, pp. 1863-1873, Nov. 1990.

[2] R.W. Dearnley and A.R.F. Barel, "A Broad-Band Transmission Line Model for a Rectangular Microstrip Antenna," *IEEE Trans. Antennas Propagation,* vol. 37, pp. 6-15, Jan. 1989.

[3] R.F. Harrington, *Field Computation by Moment Method.* Malabar, FL: Krieger Publishing Co., Chapter 4, pp. 62-81, 1987.

[4] M. Gimersky and J. Bornemann, "Utilization of Symmetry Properties for the Pattern Analysis of Mutually Coupled Patch Radiators," *Radioengineering*, vol. 1, pp. 11-16, May 1992.

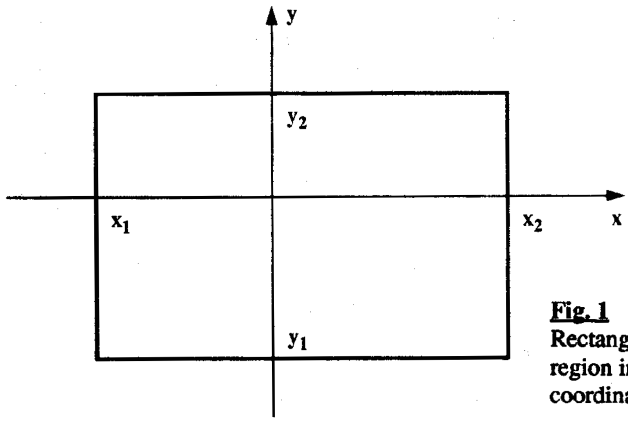

Fig. 1
Rectangular integration region in the Cartesian coordinate system.

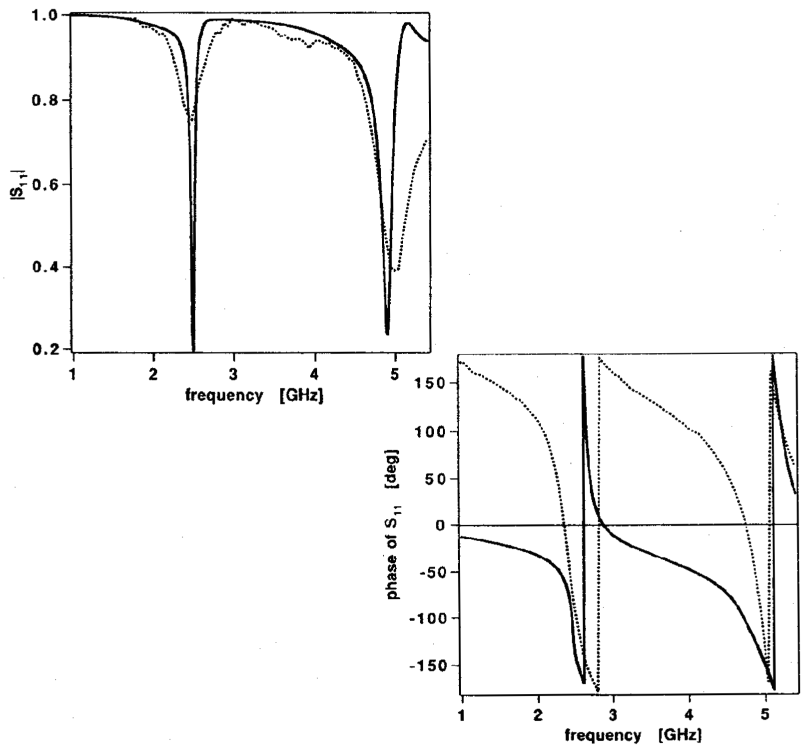

Fig.2 Input reflection coefficient of the patch of [2]: length 3.85cm, width 3.18cm, substrate thickness 1.568mm, substrate relative dielectric constant 2.34 -- a) amplitude, b) phase; solid line: calculated, dashed line: measured.