

# CROSS-POLARIZATION ANALYSIS OF LOSSY MICROSTRIP RESONATORS

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## I. INTRODUCTION

The cross-polarization characteristics of circular and square microstrip resonators has been investigated, e.g., in [1] and [2]. Both papers assume, first, an ideal (lossless) structure and, second, a cross-polarization pattern according to Ludwig's first definition [3]. This definition of cross polarization was chosen to compare the performance of the microstrip resonator to that of slots and dipoles [1]. However, with the increasing applications of microstrip patches in medium- and large-size aperture antenna arrangements, the use of Ludwig's first definition becomes arguable.

Therefore, this paper focuses on a comparison between the first and third definition of cross polarization as applied to linearly polarized rectangular microstrip resonators. It will be shown that, for the 45-degree skew angle, the third definition leads to higher cross-polarization values than the first one. The spectral-domain immittance approach is used to fully predict and simulate the behaviour of the patch. In particular, with suitable modifications, the method is able to include the losses in the analysis and, therefore, provides accurate and reliable results for the complete set of radiation characteristics.

## II. THEORY

The spectral-domain technique is well known for its efficient analysis of ideal microstrip resonators on lossless substrates [4]. Material parameters such as dielectric and metallic losses are incorporated in [5], and their effect on the principal-plane radiation patterns of patch radiators are demonstrated in [6]. However, only a single basis function is used in [4-6]. Using one basis function for the current distribution leads to a problem of minimizing a two-dimensional improper integral. With  $n$  basis functions, the dimension of the problem increases considerably, and  $3^n$  improper integrals have to be calculated. It is, therefore, understandable that the minimum number of basis functions was chosen in [4-6].

With respect to the cross-polarization analysis, however, it was felt necessary to investigate the dependence of pattern characteristics on the number of basis functions. The basis functions are selected according to [4]. The geometry of the microstrip resonator and that for the pattern calculations are depicted in Figs. 1 and 2, respectively. After solving for the complex resonant frequency, the Fourier transforms of the electric fields can be determined by

$$\tilde{E}_x(\alpha, \beta) = (\tilde{Z}_{11}(\alpha, \beta) - Z_s) \tilde{J}_x(\alpha, \beta) + \tilde{Z}_{12}(\alpha, \beta) \tilde{J}_z(\alpha, \beta) \quad (1)$$

$$\tilde{E}_z(\alpha, \beta) = \tilde{Z}_{21}(\alpha, \beta) \tilde{J}_x(\alpha, \beta) + (\tilde{Z}_{22}(\alpha, \beta) - Z_s) \tilde{J}_z(\alpha, \beta) \quad (2)$$

with

$$\begin{aligned} \alpha &= k_0 \cdot \sin \Phi \sin \theta \\ \beta &= k_0 \cdot \cos \Phi \sin \theta \end{aligned} \quad (3)$$

In (1) and (2),  $\tilde{J}_x, \tilde{J}_z$  are the Fourier transforms of the current distributions on the patch,  $\tilde{Z}_{mn}$  are the elements of the impedance Green's function, and  $Z_s$  is the surface impedance of the conducting patch [5]. When calculating the field in the E- or H-plane ( $\Phi=0$  or  $90^\circ$ , respectively; cf. Fig. 2), either  $\alpha$  or  $\beta$  vanishes. Thus  $\tilde{Z}_{12}, \tilde{Z}_{21}$  and  $J_x$ , which is an odd function, are zero and, therefore, there exist no cross-polar field in the principal planes.

Since a general expression for the co- and cross-polar field for arbitrary skew angles is difficult to formulate, the particular plane of  $\Phi=45^\circ$  is selected as an example. In this plane, the co- and cross-polar field components can be calculated according to Ludwig's first definition

$$E_{cp}^I = \left( \frac{1}{4} - \frac{\cos \theta}{2} + \frac{\cos 2\theta}{4} \right) \tilde{E}_x(\alpha, \beta) + \left( \frac{1}{4} + \frac{\cos \theta}{2} + \frac{\cos 2\theta}{4} \right) \tilde{E}_z(\alpha, \beta) \quad (4)$$

$$E_{xp}^I = \left( \frac{1}{4} + \frac{\cos \theta}{2} + \frac{\cos 2\theta}{4} \right) \tilde{E}_x(\alpha, \beta) + \left( \frac{1}{4} - \frac{\cos \theta}{2} + \frac{\cos 2\theta}{4} \right) \tilde{E}_z(\alpha, \beta) \quad (5)$$

or the third definition

$$E_{cp}^{III} = \frac{1 - \cos \theta}{2} \tilde{E}_x(\alpha, \beta) + \frac{1 + \cos \theta}{2} \tilde{E}_z(\alpha, \beta) \quad (6)$$

$$E_{xp}^{III} = \frac{1 + \cos \theta}{2} \tilde{E}_x(\alpha, \beta) + \frac{1 - \cos \theta}{2} \tilde{E}_z(\alpha, \beta) \quad (7)$$

Again, it should be noted that these expressions are valid only for the particular case of  $\Phi=45^\circ$ .

### III. RESULTS

Resonant frequencies for several lossless patch geometries have been calculated, and close agreement with [7] has been observed (not shown here). For the investigation of resonant frequencies and pattern characteristics, a patch with  $W=10$  mm,  $L=12$  mm,  $h=1.27$  mm,  $\epsilon_r=2.65$ ,  $\tan \delta=0.001$ ,  $t=5 \mu\text{m}$  and  $\sigma=58.13$  S/ $\mu\text{m}$  is chosen. With only a single basis function in z-direction, a resonance frequency  $f=7.284$  GHz is obtained. When adding a basis function in x-direction, this frequency moves to 7.163 GHz. A second basis function in x-direction leads to a resonance at 7.153 GHz. Looking at the radiation pattern for the different cases (Figs. 3 to 5), however, it can be seen that the influence on the pattern is rather minor. Due to the above mentioned properties of the current and impedance Green's functions, the pattern with respect to the E- and H-plane does not change at all (Fig. 3). Similar behaviour is observed in the copolar 45 degree plane (Fig. 4). This indicates that radiation pattern can be calculated to a satisfactory accuracy with only a single basis function. The differences in the resonant frequency

will be more important in practical applications with near-square patch geometries.

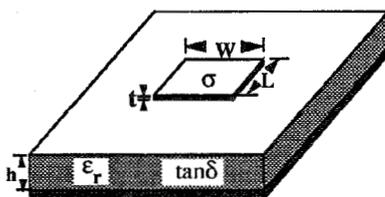
Figs. 4 and 5 also compare the patterns with respect to the different definitions for co- and cross-polar fields. From (4) - (7) it follows that, at least for the special case of  $\Phi=45^\circ$  considered here, Ludwig's third definition results in broader copolar patterns and higher cross-polar levels. This is a consequence of the definition-depending path, in which a theoretical and ideal linearly polarized receiving antenna would revolve around the microstrip resonator. While the maximum cross-polar level according to Ludwig's first definition is approximately -25dB at  $\theta=62^\circ$ , which is - considering the limitations in [2] - in reasonable agreement with tabulated data published in [2], the respective values for the third definition are -15dB and  $\theta=78^\circ$ .

### III. CONCLUSIONS

A modified spectral-domain approach is used to analyse lossy microstrip resonators with respect to resonant frequency and co- and cross-polar patterns. It is found that the number of basis functions considered has a small effect on the resonant frequency and only marginal effects on co- and cross-polar radiation patterns. Ludwig's first and third definition are compared in a 45-degree skew plane, and differences of 10dB in cross-polar levels are obtained.

### REFERENCES

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**Fig.1** Lossy microstrip patch resonator.

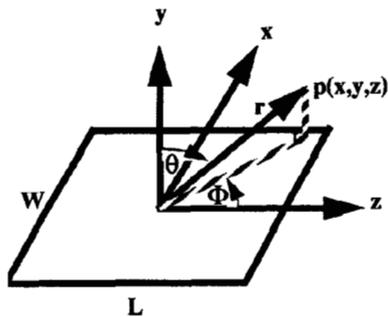


Fig. 2 Geometry for pattern calculation.

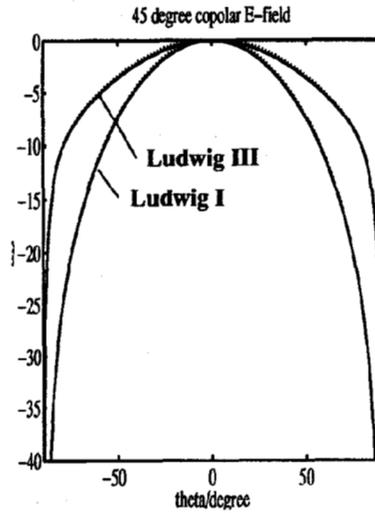


Fig. 4 45-degree co-polar patterns computed using one, two and three basis functions (results within plotting accuracy). Dimensions as in Fig. 3.

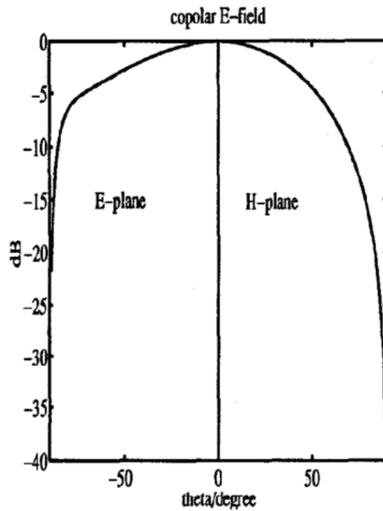


Fig. 3 Principal-plane co-polar radiation patterns of microstrip resonator. Identical results for one, two and three basis functions. Dimensions:  $W=10$  mm,  $L=12$  mm,  $h=1.27$  mm,  $\epsilon_r=2.65$ ,  $\tan\delta=0.001$ ,  $t=5\mu\text{m}$ ,  $\sigma=58.13$  S/ $\mu\text{m}$ .

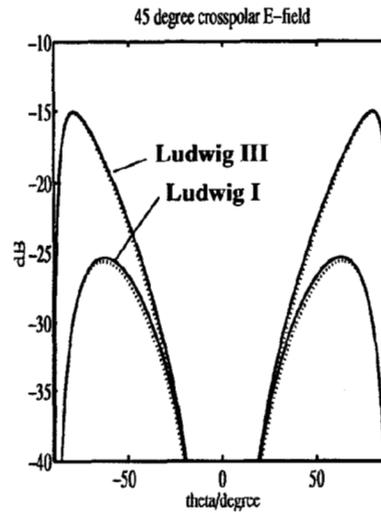


Fig. 5 45-degree cross-polar patterns computed using one (dotted line), two and three (solid line) basis functions. Dimensions as in Fig. 3.