EDGE-CONDITIONED VECTOR BASIS FUNCTIONS FOR THE ANALYSIS OF CORRUGATED ANTENNA FEEDS

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I. INTRODUCTION

Corrugated circular waveguides are essential components in modern antenna feeds where a high degree of symmetry in the radiation pattern or a very low crosspolarisation is required [1].

The presence of the periodic corrugations in the circular waveguide is reflected in the field distributions of the modes of the structure through the Floquet condition [1]. A common method of analysis of corrugated circular waveguides starts from an expansion of the electromagnetic fields in series of space harmonics [1]. The propagation constants of the Floquet modes are then determined from the boundary conditions at the interface between the corrugations and the uniform part of the waveguide. The concept of surface impedance is often used to reduce the complexity of the problem.

A disadvantage of this method is the fact that the propagation constants of the Floquet modes are determined from a determinantal non-linear equation. It is, unfortunately, true that such processes can be time consuming and even fail to locate closely packed roots.

In this paper the boundary-value problem, including the Floquet condition is formulated *exactly*. The propagation constants are determined from the *classical* eigenvalues of a matrix instead of a determinant. A large number of modes can be rapidly and accurately investigated. Furthermore, a priori knowledge on the solution, such as the edge condition at the metallic corrugations or the presence of other symmetries, can be straightforwardly included in the formulation. A new set of edge-conditioned vector basis functions is presented and used to accurately determine the entire dispersion diagram of modes with angular dependence of the form $\cos(\phi)$ or $\sin(\phi)$.

II. THEORY

The structure under consideration is depicted in Figure 1. It consists of a lossless corrugated circular waveguide of radius a. The inner radius of the corrugations of thickness t is b. The periodicity of the corrugations is p.

We focus attention on modes with angular dependence $\cos(\phi)$, other modes can be analyzed similarly. The modes of regions I and II are derived from electric and magnetic potentials which we denote by $\Phi_n^{Ite,tm}$ and $\Phi_n^{IIte,tm}$.

Let us assume that the exact distribution of the electric field at $z = 0$, $z = t$ and $z = p$ be denoted by three unknown vector functions $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$, respectively. The Floquet condition is satisfied through the choice

$$\mathbf{X}_3 = e^{-\theta p}\mathbf{X}_1 \qquad (1)$$

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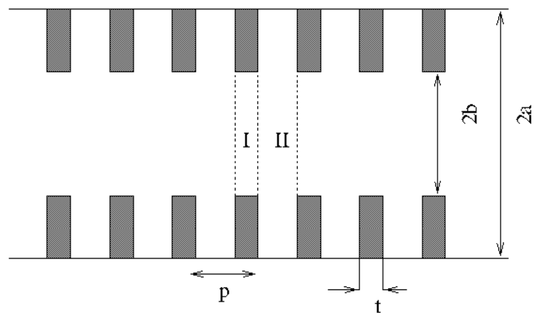

Figure 1: Cross section of a corrugate circular waveguide

where θ is the unknown propagation constant. To include the edge condition at the metallic wedges of the corrugations, we introduce the following set of basis functions

$$\mathbf{B}_i^{te} = \frac{\hat{\mathbf{e}}_\mathbf{z} \times \nabla_\mathbf{t} \Phi_i^{Ite}(\rho, \phi)}{[1 - (\frac{\rho}{b})^2]^{1/3}} \tag{2}$$

and

$$\mathbf{B}_i^{tm} = \frac{\nabla_\mathbf{t} \Phi_i^{Itm}(\rho, \phi)}{[1 - (\frac{\rho}{b})^2]^{1/3}}. \tag{3}$$

The unknown functions $\mathbf{X}_1$ and $\mathbf{X}_2$ are expanded in series of the form

$$\mathbf{X}_1 = \sum_{i=1}^{M} c_i^{te} \mathbf{B}_i^{te}(\rho, \phi) + \sum_{i=1}^{M} c_i^{tm} \mathbf{B}_i^{tm}(\rho, \phi) \tag{4}$$

and

$$\mathbf{X}_2 = \sum_{i=1}^{M} d_i^{te} \mathbf{B}_i^{te}(\rho, \phi) + \sum_{i=1}^{M} d_i^{tm} \mathbf{B}_i^{tm}(\rho, \phi). \tag{5}$$

To determine the expansion coefficients, we derive two coupled vector integral equations for the functions $\mathbf{X}_1$ and $\mathbf{X}_2$, which are then solved by the moment method [2]. It can be shown that the expansion coefficients satisfy the following generalized eigenvalue equation

$$\begin{bmatrix} Q_1 & Q_2 \\ Q_2 & Q_1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} Q_3 e^{-\theta p} & 0 \\ 0 & Q_3 e^{\theta p} \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = 0. \tag{6}$$

The entries of the submatrices $Q_i(2M \times 2M)$ in this equation involve sums over the modes of the two regions.
The generalized eigenvalue equation, equation (6), is not in a convenient form because of the appearance of two different functions of θ ($e^{\theta p}$ and $e^{-\theta p}$). It can be, however, easily transformed into the more convenient form,

$$[K]\,[v] + e^{-\theta p}\,[L]\,[v] = 0 \tag{7}$$

which can be straightforwardly solved using standard software packages.

III. RESULTS

To establish the validity of the approach, we determine the dispersion characteristics of a corrugated waveguide analyzed by Clarricoats and Olver [1].

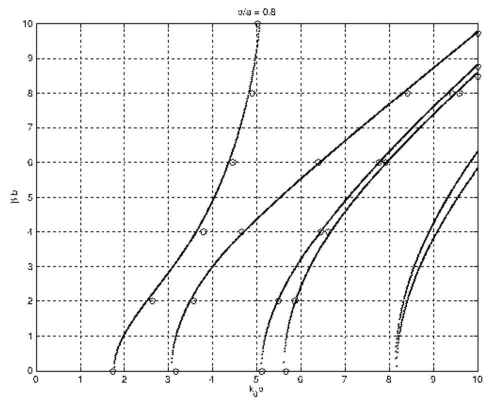

Figure 2: Dispersion characteristics of a corrugated waveguide. b/a=0.8, t=0, p=a/5.

Figure 2 shows the dispersion characteristics of the first 6 modes with the angular dependence $\cos(\phi)$ or $\sin(\phi)$. The circles are from reference [1] and the dots are from the present work using four basis functions.

The speed of the present method allows the calculation of the entire dispersion diagram ($k_0 - \beta$ diagram). Figure 3 shows a typical complete $k_0 - \beta$ diagram of the first mode for different values of the ratio $\frac{b}{a}$. It is worth noting the fact that the group velocity is *negative* for the first case when $b = a = 0.3a$ while it is positive for the remaining cases. This change in the sign of the group velocity is also reported in [1].

The convergence of the numerical solution versus the number of basis functions was also investigated. Figure 4 shows the dispersion diagram obtained from M = 1, 2 and 3 basis functions with the edge conditions. It can be clearly seen that only minor differences are observed between the three different curves. The presence of the bandgaps in the $k_0 - \beta$ diagram is a salient property of periodic structures [3]. This is clearly displayed in Figure 4.

IV. CONCLUSIONS

The propagation constants of Floquet modes are determined from the classical eigenvalues of a generalized matrix eigenvalue problem instead of a determinantal equation. A new set of edge-conditioned vector basis functions was presented and used to accelerate convergence of the numerical solution. Results obtained form the present work were compared with available data and excellent agreement was documented.

REFERENCES

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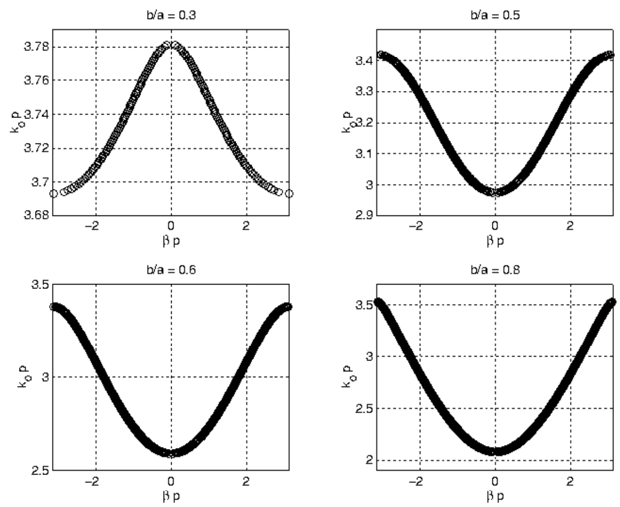

Figure 3: $k_0 - \beta$ diagram of the first mode of a corrugated circular waveguide. $t = 0.2a$ and $p = a$. Note the change in slope (group velocity) at $\beta = 0$.

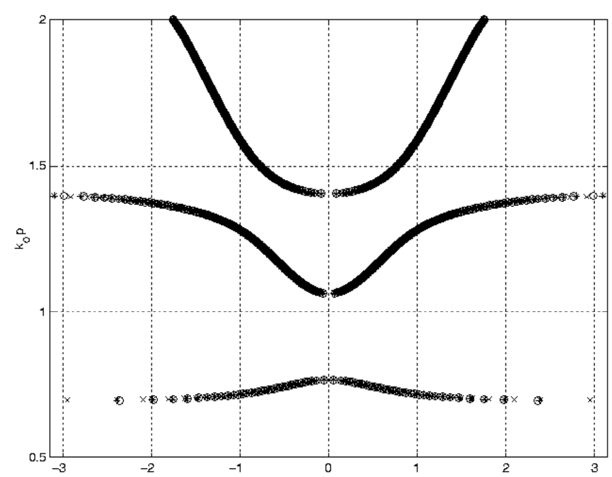

Figure 4: Convergence of the $k_0 - \beta$ diagram when $t = 0$, $p = a$ and $b = 0.3a$. M = 1 (x), M = 2 (*) and M = 3 (o) basis functions.