A SPACE-HARMONIC FREE FULL-WAVE ANALYSIS OF CORRUGATED ANTENNA FEEDS

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1 Introduction

Corrugated circular waveguides are essential components in modern antenna feeds where a high degree in the symmetry of radiation pattern or a very low crosspolarisation is required [1].

The periodicity of the structure allows us to limit the analysis to a single unit cell provided the Floquet condition

$$F(z+p) = e^{-\gamma p} F(z), \tag{1}$$

is satisfied. Here, p is the period of the structure, γ is the propagation constant and $F(z)$ is a generic component of the electromagnetic field. In addition to the usual boundary conditions, the main difficulty in the investigation of corrugated waveguides comes from the enforcement of the above condition. A popular approach to achieving this consists in rewriting the function $F(z)$ as $e^{-\gamma z}G(z)$ where $G(z)$ is a periodic function with the period of the structure p; it can be expanded in a standard Fourier series. The individual terms in the expansion are known as the space harmonics [1]. The propagation constants of the Floquet modes are then determined from the boundary conditions at the interface between the corrugations and the inner uniform part of the waveguide. The concept of surface impedance is often used to reduce the complexity of the problem. Details of the method can be found in [1].

An drawback of this method of analysis stems from the fact that the propagation constants of the Floquet modes are determined from a determinant non-linear equation. A root finding iterative algorithm is necessary. It is unfortunately true that such iterative processes can be time consuming and even fail to locate closely packed roots. Furthermore, the presence of complex modes in these structures presents an additional numerical challenge. A more efficient approach using the Mode-Matching Technique was proposed by Esteban and Rebollar although the propagation constants are still determined iteratively from a non-linear determinant equation [2].

In this paper we present a formulation in which the Floquet condition is taken into account from the outset. The propagation constants are determined from the *classical* eigenvalues of a matrix instead of a determinant. A large number of modes can be rapidly and accurately investigated.

In this work we focus attention on modes with unit angular dependence since the fundamental mode of a circular waveguide (TE_{11}) has the same angular distribution.

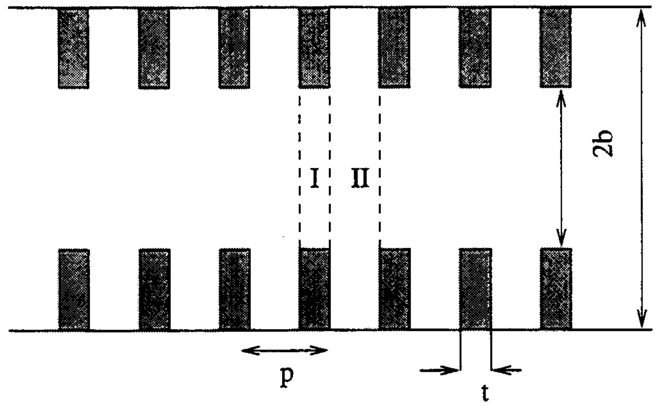

Figure 1: Cross section of a corrugate circular waveguide

2 Theory

The structure under consideration is depicted in Figure 1. It consists of a lossless corrugated circular waveguide of radius a. The inner radius of the corrugations of thickness t is b. The periodicity of the corrugations is p.
We focus attention on modes with angular dependence $\cos(\phi)$, other modes can be analyzed similarly. The modes of regions I and II are derived from electric and magnetic potentials which we denote by $\Phi_n^{Ite,tm}$ and $\Phi_n^{IIte,tm}$ and assume normalized. The propagation constants and the wave admittances are denoted by $k_{zn}^{jte,tm}$ and $Y_n^{jte,tm}$ where $j = I, II$.
Let us assume that the exact distribution of the electric field at $z = 0$, $z = t$ and $z = p$ be denoted by three unknown vector functions $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$, respectively. The Floquet condition is satisfied through the choice

$$\mathbf{X}_3 = e^{-\gamma p}\mathbf{X}_1 \tag{2}$$

Given the symmetry of structure, we expand the unknown functions $\mathbf{X}_1$ and $\mathbf{X}_2$ in series of TE_{n1} and TM_{n1} modes, i.e

$$\mathbf{X}_1 = \sum_{i=1}^{M} c_i^{te}\mathbf{a}_z \times \nabla\Phi_i^{Ite} + \sum_{i=1}^{M} c_i^{tm}\nabla\Phi_i^{Itm} \tag{3}$$

and

$$\mathbf{X}_2 = \sum_{i=1}^{M} d_i^{te}\mathbf{a}_z \times \nabla\Phi_i^{Ite} + \sum_{i=1}^{M} d_i^{tm}\nabla\Phi_i^{Itm} \tag{4}$$

To determine the expansion coefficients, we derive two coupled vector integral equations for the functions $\mathbf{X}_1$ and $\mathbf{X}_2$, which are then solved by the moment method [3]. It can be shown that the expansion coefficients satisfy the following generalized eigenvalue equation

$$\begin{bmatrix} A & 0 & B & C \\ 0 & D & C^t & E \\ B & C & A & 0 \\ C^t & E & 0 & D \end{bmatrix} \begin{bmatrix} c^{te} \\ c^{tm} \\ d^{te} \\ d^{tm} \end{bmatrix} + \begin{bmatrix} Fe^{-\gamma p} & Ge^{-\gamma p} & 0 & 0 \\ G^t e^{-\gamma p} & Je^{-\gamma p} & 0 & 0 \\ 0 & 0 & Fe^{\gamma p} & Ge^{\gamma p} \\ 0 & 0 & G^t e^{\gamma p} & Je^{\gamma p} \end{bmatrix} \begin{bmatrix} c^{te} \\ c^{tm} \\ d^{te} \\ d^{tm} \end{bmatrix} = 0. \tag{5}$$

The entries of the submatrices in this equation involve sums over the modes of the two regions and will be given during the presentation.

The generalized eigenvalue equation, equation (5), is not in a convenient form because of the appearance of two different functions of γ ($e^{\gamma p}$ and $e^{-\gamma p}$). It can be, however, easily transformed into the the more convenient form.

$$[K]\,[v] + e^{-\gamma p}\,[L]\,[v] = 0 \tag{6}$$

which can be straightforwardly solved using standard software packages.

3 Results

To establish the validity of the approach, we determine the dispersion characteristics of a corrugated waveguide analyzed by Clarricoats and Olver [1]

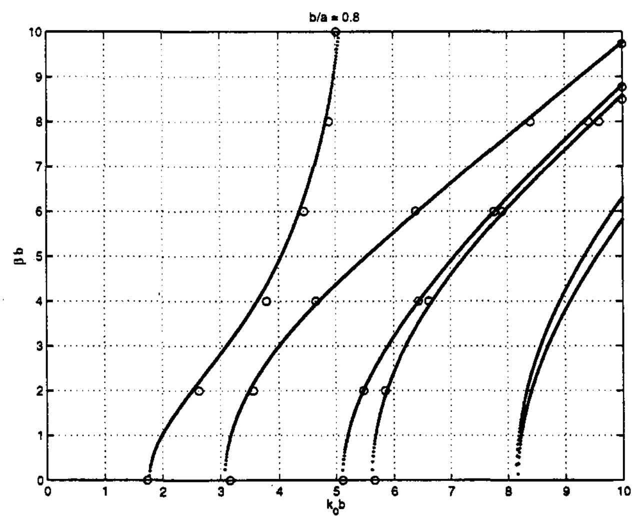

Figure 2: Dispersion characteristics of a corrugated waveguide. b/a=0.8, t=0, p=a/5.

Figure 2 shows the dispersion characteristics of the first 4 modes with the angular dependence $\cos(\phi)$ or $\sin(\phi)$. The circles are from reference [1] and the dots are from the present work using four basis functions and summing 50 terms in the entries of the matrices in equations (13)-(20).
The speed of the present method allows the calculation of the entire dispersion diagram ($k_0 - \beta$ diagram). Figure 3 shows the $k_0 - \beta$ diagram when $t = 0.5a$ $p = a$ and $b = 0.5a$. The presence of the band-gaps in the diagram is clearly visible as is expected from a periodic structure [4]. The data for Figure 3 was generated using four basis functions and 50 terms. A convergence analysis showed negligible changes when more basis functions or more terms are used. As an indiction of the speed of the method, it takes less than 3 milliseconds per frequency point on a Sparc 10 workstation. The fact that the propagation constants are determined from the classical matrix eigenvalues instead of a determinant allows us to investigate not only the propagating modes but also the evanescent and complex modes as well. Figure 4 shows the real and imaginary parts of the propagation constant γ as a function of frequency. As it can be seen, our results agree well with those presented in [2] (stars).

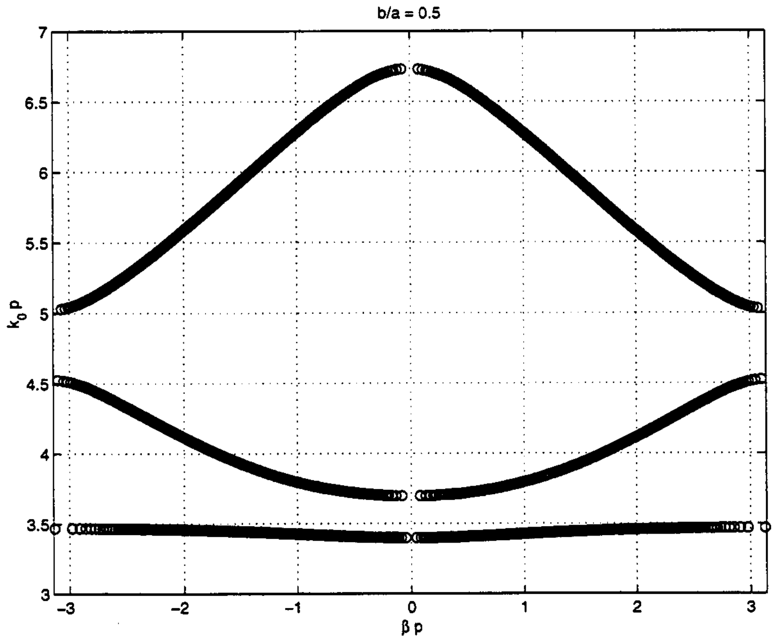

Figure 3: $k_0 - \beta$ diagram of the first 3 branches when $t = 0.2a$, $p = a$ and $b = 0.5a$. The band-gaps are clearly visible.

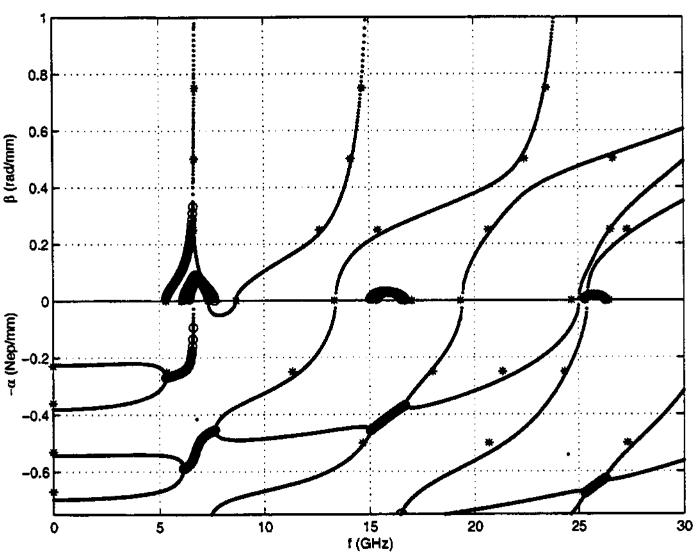

Figure 4: γ versus frequency for a circular corrugated waveguide when $b/a = 0.4$, $b = 10$ mm, $t = 0.01$ mm and $p = 1.01$ mm. The stars are from [2].

4 Conclusions

The propagation properties of corrugated circular waveguides were accurately determined from a fullwave analysis. The Floquet condition is enforced without recourse to space harmonics, instead it is viewed as *a priori* information which is directly included in the formulation form the outset. The propagation constant are determined from the classical eigenvalues of a generalized matrix eigenvalue problem instead of a determinant equation. The speed of the method allows the analysis of the entire dispersion diagram with minimal numerical effort. Results obtained form the present work were compared with available data and excellent agreement was documented.

References

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