ANALYSIS OF PROPAGATION IN CORRUGATED WAVEGUIDES OF ARBITRARY CORRUGATION PROFILE

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I. INTRODUCTION

Corrugated circular waveguides can be found in antenna feeds where a high degree of symmetry in the radiation pattern or a very low crosspolarisation is required [1]. Dual-depth corrugated waveguides have also been used when the feed is operated at two separate frequency bands [2]. In order to reduces losses and breakdown in high power microwave applications, the corrugation profile is continuous [3] rather than involving step discontinuities.
A state-variable approach was used by Bromborsky and Ruth to determine the propagation constants of $\mathrm{TM}_{0(n)}$ modes in a sinusoidally corrugated circular waveguide [3]. This approach, however, requires basis functions which are both continuous and differentiable at least once over the period of the structure. Its applicability to multiple abrupt discontinuities has not been demonstrated. On the other hand, the expansion in space harmonics as described in [1], is extremely complicated when the unit cell contains more than one depth [2]. In addition to leading to complex matrices, it requires solving a non-linear determinant equation for the propagation constants. The disadvantage of a determinant formulation of periodic structures are well described by Davies [4].
An alternative approach, which leads to a generalized matrix eigenvalue problem when the unit cell of the periodic structure contains multiple discontinuities, was recently presented in [5]. The approach is exact for sharp discontinuities, but continuous corrugation profiles can be modelled by stair-case approximations which is demonstrated here at the example of sinusoidally oscillating waveguide walls. The dispersion curve of the accelerating modes $\mathrm{TM}_{0(n)}$ as well as hybrid modes with unit angular dependence are presented and compared with available data.

II. THEORY

We consider a structure consisting of a lossless corrugated circular waveguide whose internal radius is a continuous function $f(z)$ of z. The period of the structure is p. Note that modes of different angular dependencies are not coupled and, therefore, can be analyzed separately.
The continuous corrugation profile is approximated by n steps where n is increased until convergence is reached. Following the formulation presented in [5], a set of coupled integral equations for the transverse electric field at the discontinuities in the unit cell are derived. The Floquet condition is

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automatically satisfied by requiring the following relationship between the electric fields at discontinuities separated by one period

$$X_{n+1} = e^{-\gamma p} X_1 \tag{1}$$

Here, γ is the propagation constant of the Floquet mode.
The unknown electric fields are then expanded in series of basis functions

$$\mathbf{X}_i = \sum_{q=1}^{M} c_q^{(i)} \mathbf{Q}_q^{(i)}, \quad i = 1, \dots n \tag{2}$$

Applying the Method of Moments (MoM) to the coupled integral equations, we get a set of linear matrix equations in the expansion coefficients, namely

$$\begin{aligned}
[A][c^{(1)}] + [B][c^{(2)}] + e^{\gamma p}[C][c^{(1)}] &= 0 \\
[D][c^{(1)}] + [E][c^{(2)}] + [F][c^{(3)}] &= 0 \\
\dots + \dots + \dots &= 0 \\
[R][c^{(n-2)}] + [S][c^{(n-1)}] + [T][c^{(n)}] &= 0 \\
[U][c^{(n-1)}] + [V][c^{(n)}] + e^{-\gamma p}[W][c^{(n)}] &= 0 \qquad (3)
\end{aligned}$$

The entries of the matrices in these equations involve sums over the modes of the uniform sections approximating the profile of the unit cell. The modes of the sections are used as basis functions.
The set of matrix equations can be transformed into a generalized matrix eigenvalue problem of the form $[A][X] + \lambda[B][X] = 0$ [5].

III. RESULTS

Figure 1 shows the dispersion diagram of accelerating modes $\mathrm{TM}_{0(n)}$ in a sinusoidally corrugated circular waveguide. The internal radius of the waveguide is assumed of the form $R(z) = R_0(1 + \delta \cos(2\pi \frac{z}{p})$. Our numerical results are presented for $R_0 = p$ and $\delta = 0.1$ (solid lines) and are in good agreement with data taken from [3] (circles). Our results were obtained using 4 basis functions and 15 steps per period. More basis functions and more steps were used and led to neglegible changes.
One advantage of the present approach consists in its ability to determine not only propagating modes, but evanescent as well as complex modes from the same analysis. Figure 2 shows the real and imaginary parts of the propagation constant. No complex modes were encountered for these dimensions but they are present for larger values of δ.
The dispersion diagram of hybrid modes in this structure can also be determined by including both TE and TM modes in the analysis. Figure 3 shows the real and imaginary parts of the propagation constant. Complex modes are shown by the thick lines. These results were obtained using 4 TE and 4 TM basis functions and 12 steps per unit cell. Note that under the term complex modes are also included what is at times referred to as filtering modes with $Im[\gamma] = \frac{\pi}{p}$.

We finally mention that evanescent and complex modes can cause numerical difficulties when the period p of the structure is too large. This is due to calculations of terms $e^{\gamma p}$.

IV. CONCLUSIONS

The propagation constants of Floquet modes in corrugated circular waveguides with continuous corrugation profiles were determined from a formulation leading to a canonical matrix eigenvalue equation. The approach is validated by comparison to available data for the dispersion relation of $TM_{0(n)}$ modes in a sinusoidally corrugated waveguide. Results for hybrid modes were also presented.

REFERENCES

1. P. J. B. Clarricoats and A. D. Olver, Corrugated Horns for Microwave Antennas, Peregrinus, London, 1984.
2. A. D. Olver, K. K. Yang and P. J. B. Clarricoats, “Propagation and radiation behaviour of dual-depth corrugated horns”, Proc. Inst. Elec. Eng., vol. 131, pt. H, pp. 179-185, June 1984.
3. A. Bromborsky and B. Ruth, “Calculation of TM_{0n} dispersion relations in a corrugated circular waveguide”, IEEE Trans. Microwave Theory Tech., vol. 32, pp. 600-605, June 1984.
4. J. B. Davies,“Complete modes in uniform waveguides”, pp. 8-10 in Finite-Element Software for Microwave Engineering, T. Itoh, G, Pelosi and P. P. Silvester Ed. Wiley, New York, 1996.
5. S. Amari, R. Vahldieck and J. Bornemann, “Complete spectrum of multi-depth corrugated circular waveguides”, IEEE Microwave Guided Wave Lett., vol. 9, 1999 (in press).

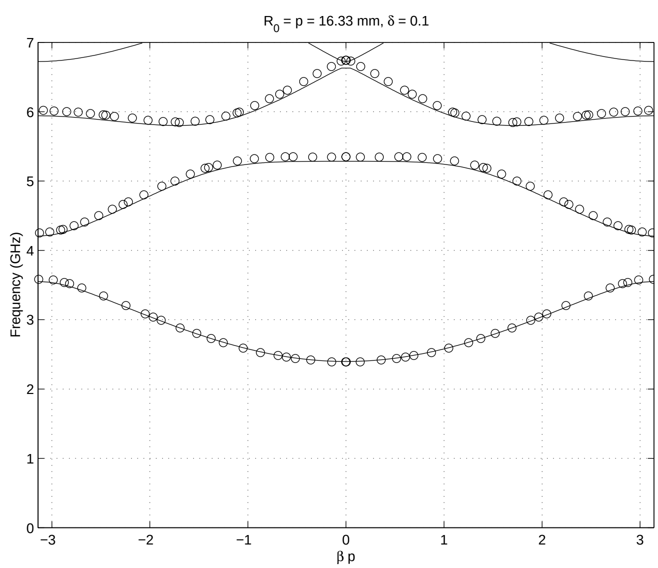

Figure 1: Dispersion characteristics of $TM_{0(n)}$ modes when R_0 = p = 16.33 mm and $\delta = 0.1$. The circles are from [3].

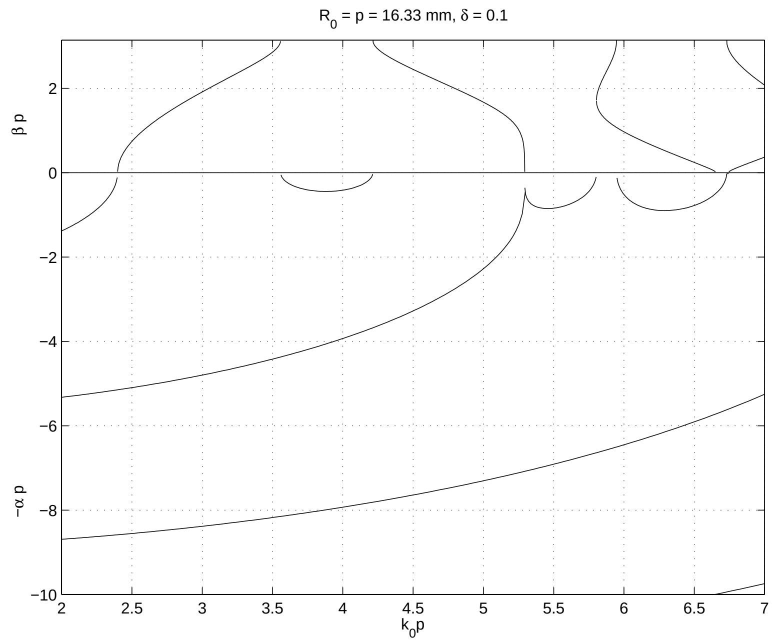

Figure 2: Real and imaginary parts of γp vs frequency ($\mathrm{TM}_{0(n)}$ modes). Same dimensions as Figure 1.

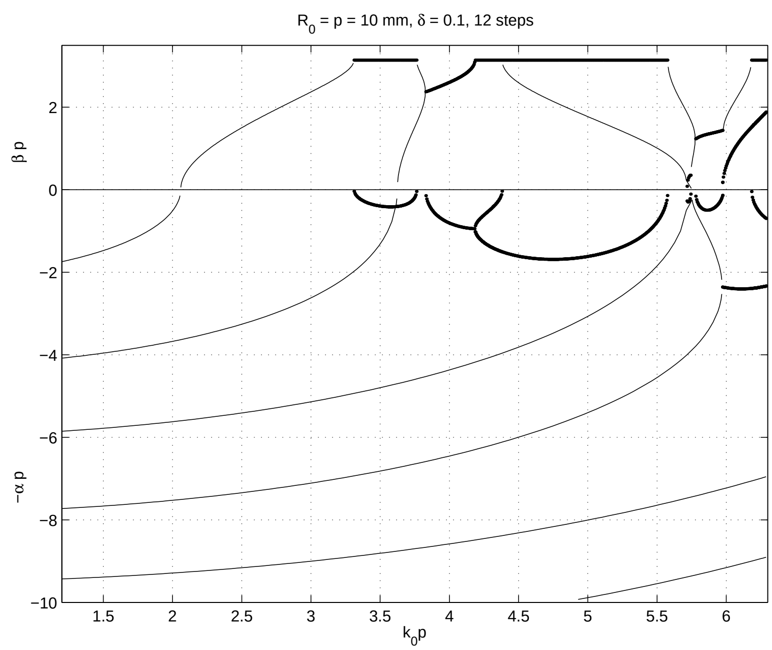

Figure 3: Real and imaginary parts of γp vs frequency of hybrid modes with unit azimuthal dependence ($\cos(\phi)$). Same dimensions as Figure 1.

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