# ANALYTICAL CALCULATION OF GRADIENTS FOR THE OPTIMIZATION OF H–PLANE FILTERS WITH THE FEM


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**Abstract – This paper introduces a method for the analytical calculation of gradients of a cost functions which is an attractive feature when optimizing microwave structures using field solvers. In contrast to utilizing finite differencing all gradients are computed from a single analysis of the structure regardless of its complexity. It is not even necessary to invert a large matrix; a linear system [A]{x}={b} is solved instead. No remeshing is required in the FEM and the gradient values are exact. The basic technique used in this new approach is applied to the optimization of H-Plane filters using the FEM.**


## I. INTRODUCTION

Evaluating the gradient of a cost function for the optimization of microwave circuits is usually based on the finite difference technique and can be a time consuming task. This is especially true when the circuit transfer function is calculated on the basis of a field-theory simulation tool since always two computations are necessary for one gradient. If, in addition, the number of independent variables is large, optimization can become an impossible task. In this contribution it will be shown that, under certain circumstances, the gradient of a cost function can be calculated analytically without using finite differences. The number of computations can be cut in half and well known disadvantages with finite differencing like inaccuracies at singularities in highly resonant structures are eliminated. The method has been applied successfully to the coupled integral equation technique (CIET) and the finite element method (FEM). Analytical calculation of the cost function is also possible with the adjoint network method (ANM) but requires a network representation of the structure to be optimized (and its adjoint). The mode matching technique (MMT) to calculate the fields is normally utilized to extract the network's representation in form of the admittance or scattering matrix. The ANM has been successfully applied to the optimization of filters and radiating structures.

Analytically calculating the gradient of a cost function directly in general numerical techniques without first deriving a network representation has not been published before. The possibility of doing so is of great interest as it offers a number of obvious advantages and also not so obvious ones, depending on the numerical method used. The new approach can not be applied to all numerical field computation methods since it requires a scattering problem representation of the whole micro-wave structure of the form which results directly from applying the FEM or a moment method (i.e. CIET), but is not necessarily limited to these methods. Here, [A] is a M x M matrix which depends on the independent variables and represents the structure to be optimized, {b} is the excitation and {x} is the response. For example, the vector {x} contains the

expansion coefficients in the MoM or the nodal values in FEM.
It will be shown, that as long as the partial derivatives of the matrix [A] and the excitation {b} are known analytically, all sensitivities can be determined analytically. Up to now this approach has been successfully tested with the MoM in the optimization of waveguide filters and was subsequently applied to the FEM. It can be extended to other methods for which the scattering problem can be formulated as above.
Gradient based optimization of microwave filters with the FEM using analytically calculated gradients of cost functions is a new technique which offers the following advantages:

1. The gradient of a cost function can be calculated analytically instead of using finite differences.
2. The generality of the FEM is maintained but the calculation time is reduced significantly.
3. Analytically calculated gradients (change of scattering parameters due to geometrical changes) are exact and singularities do not occur.
4. Combination of points 1 to 3 give a powerful and fast optimization tool for microwave filters and other structures.

In this paper the optimization of H–plane filters using 2D–FEM with analytically calculated gradients is reported. In comparison to the standard approach in which finite differences are needed to calculate the gradient of a cost function this new idea accelerates the FEM optimization by orders of magnitude. Furthermore, significant reduction of memory space is achieved and the accuracy is improved since analytically calculated gradients are exact and no singularities occur like in the finite difference approach. The method presented here can be extended to other structures and is not restricted to the 2D-FEM.

More advantages of this approach:

1. No network representation needed
2. Only one cost function evaluation instead of two
3. Higher accuracy compared to a finite difference scheme in particular in the vicinity of resonances
4. No remeshing of the structure required during gradient calculations
5. No matrix inversion necessary
6. Reduced memory requirements
7. Faster algorithms.

## II. THEORY

Assume that the scattering parameter $S_{11}(\omega)$ of a structure, which depends on the geometric parameters $a_i$, i = 1, ... N, is required to minimize a cost function of the form:

$$F(a_i) = \sum_n W_n \left[ \left| S_{11}(\omega_n, a_i) \right| - \left| S_{11}^{optimal}(\omega_n, a_i) \right| \right]^2$$

Here $W_n$ are constants and $\omega_n$ are specified frequencies in the desired band. The question we are trying to answer is: find the optimal set of parameters which minimize the function $F(a_i)$.
A major step in reaching the solution is the computation of the gradient of the function $F(a_i)$ with respect to the parameters $a_i$. Taking the partial derivative of the function against a generic parameter $a_i$, we get

$$\frac{\partial F}{\partial a_i} = 2 \sum_n W_n \left[ \left| S_{11}(\omega_n, a_i) \right| - \left| S_{11}^{optimal}(\omega_n, a_i) \right| \right] \frac{\partial \left| S_{11}(\omega_n, a_i) \right|}{\partial a_i}$$

It is easy to show that $\frac{\partial |S_{11}|}{\partial a_i} = \mathrm{Re}\left\{ \frac{|S_{11}|}{S_{11}} \frac{\partial S_{11}}{\partial a_i} \right\}$
In order not to cloud the main idea in cumbersome mathematical formulas, we simply assume that the scattering problem before us is put in the following matrix form

$$[A]\{x\}=\{b\}$$

Here, the matrix [A] represents the system, {b} is the excitation and {x} is the solution. The forms of these quantities depend on the method used and naturally the structure. Appropriate forms which result from the FEM will be given.

We assume that the scattering parameters are known in terms of the solution {x}. Consequently, the gradient of the scattering parameters with respect to the optimization variables are determined from the partial derivatives of the solution {x} with respect to the same optimization variables. This point can be easily seen using the chain rule. It is, therefore, sufficient to determine the following gradient $\nabla\{x\}$.

The gradient is with respect to the optimization variables. To compute this gradient, we take the partial derivative of the equation [A]{x} = {b} with respect to a generic independent variable u to get

$$\frac{\partial[A]}{\partial u}\{x\}+[A]\frac{\partial\{x\}}{\partial u}=\frac{\partial\{b\}}{\partial u}$$

which we rewrite as

$$[A]\frac{\partial\{x\}}{\partial u}=\frac{\partial\{b\}}{\partial u}-\frac{\partial[A]}{\partial u}\{x\}$$

From this equation, it is possible to determine the components of the gradient of the solution {x} by simply solving this linear set of equations instead of inverting the matrix [A]. Recall the solution {x} is already known.

Within the Finite Element Method (FEM), the gradient of the matrix [A] can be calculated analytically. This allows us to evaluate the gradient of the solution {x} without finite differences. If desired, it is still possible to use finite differences to compute the gradient of the matrix [A] and still compute the gradient of the solution from a single analysis of the structure thereby avoiding a potentially costly remeshing.

## III. RESULTS

The method described here is applied to the optimization of H-plane filters within the Finite Element Method. The structure is shown in Figure 1. All metallic surfaces are assumed lossless.

We assume that only the $TE_{10}$ mode is incident on the filter from the left side.

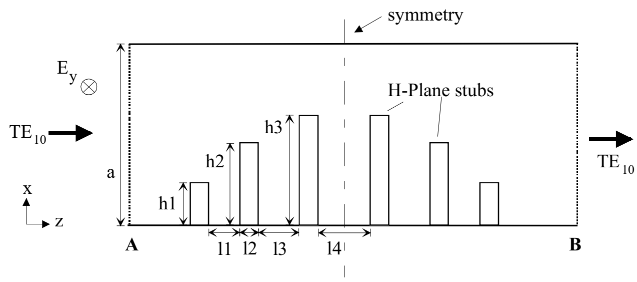


Fig1: H-Plane Stub Filter

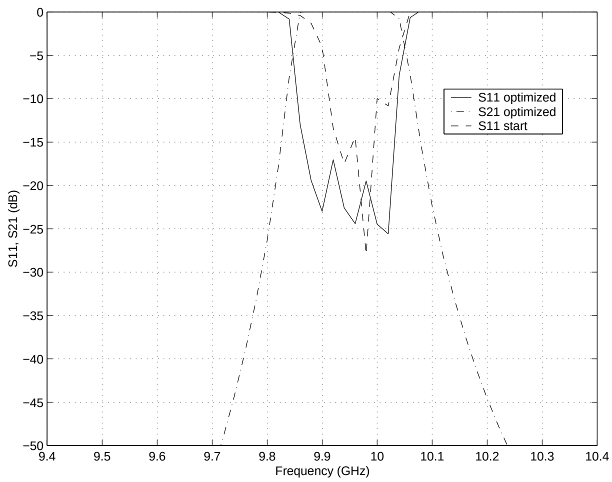


Fig. 2: Optimization results of the H-Plane Iris Filter (Fig. 1)

Fig. 2 shows the results of the optimization of the H-Plane Iris Filter (Fig. 1). The size of the standard waveguide (WR75) is a x b= 19.05 mm x 9.525 mm. The distances between the irises (thickness $l_2$=1 mm) have been kept fixed: $l_1$= 21.6580 mm, $l_3$=23.6964 mm, $l_4$=23.8727 mm. The starting values for the variable heights have been chosen as: $h_1$=6.8 mm, $h_2$=10.2 mm, $h_3$=11.3 mm. For the gradient optimization process we have utilized the Matlab routine "constr" and a triangular mesh with about 5500 triangles, 3000 nodes and 50 frequency points. The routine converged after 19 iterations and delivered the optimal heights: $h_1$=7.17485 mm, $h_2$=10.4285 mm, $h_3$=11.0367 mm. The results for $S_{11}$ for both start and optimized parameters are shown in Fig. 2. $S_{21}$ for the optimized values is also displayed. The optimized return loss in the pass band is better than 15 dB.

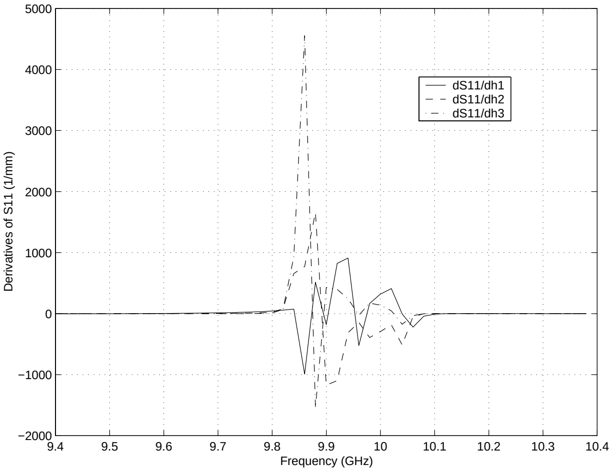


Fig. 3: Derivatives of $S_{11}$ of the H-Plane Iris Filter (Fig. 1)

In Fig. 3 the derivatives of $S_{11}$ with respect to the iris heights are shown. It can be seen that the investigated filter is very sensitive to structural changes and large values for the derivatives can occur. It is obvious that evaluating the gradients with finite differences can give errors especially near the sharp peaks.

## IV. CONCLUSION

We have developed a method for analytically calculating the gradients of a cost function in conjunction with a general field solver. The method only requires that the problem can be formulated in terms of a general inhomogeneous matrix equation such as generated by the Finite Element Method (FEM). All partial derivatives are determined from a single analysis of the structure, no matrix inversion is required, no remeshing of the FEM grid is needed. Results of the optimization show the validity of the whole process.

## V. REFERENCES


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