AN INTEGRAL EQUATION TECHNIQUE FOR APPLICATIONS IN NARROWBAND RESONANT WAVEGUIDE STRUCTURES

Jens Bornemann
Dept. of Elec. & Comp. Engr, University of Victoria, Victoria, B.C., V8W 3P6 Canada
Smain Amari
Dept. of Elec. & Comp. Engr, Royal Military College, Kingston, ON, K7K 7B4 Canada
Uwe Rosenberg
Marconi Communications GmbH, D-71522 Backnang, Germany

Abstract: An integral equation technique for applications in narrowband resonant waveguide structures is presented. Owing to the incorporation of edge conditions at all discontinuities, the system matrix is kept small. Moreover, its block-diagonal structure lends itself to fast processing techniques. Single- and dual-mode applications in circular and rectangular waveguides demonstrate the advantage of the method.

INTRODUCTION

Over the past decade, computational techniques for the analysis of electromagnetic field problems have experienced a significant degree of sophistication [1]. Resonant structures with their characteristically sharp peaks in the frequency response, however, still pose problems for numerical techniques due to a substantial demand for computing resourses [2]. In general, field solvers for resonant systems require extremely fine discretization, e.g. [3], while in series expansion techniques, only a very large number of expansion terms can achieve a reasonable convergence behaviour of the circuit response [4].

Therefore, this paper focuses on a fast, yet accurate integral equation technique for the analysis of narrowband resonant waveguide structures. The salient features of this method can be summarized as follows: Edge conditions are readily incorporated in the basis functions at rectangular and/or circular discontinuities. The edge conditions are simultaneously taken into account in order to accurately capture the interactions between closely spaced field singularities and the different polarizations in multimode resonators. Expansion terms enter the system matrix only as sums in the system matrix's elements, thus allowing every element to be tested individually for convergence. The size of the system matrix depends exclusively on the low number of edge-conditioned basis functions. Interactions between discontinuities are formulated such that the system matrix becomes highly sparse. Therefore, the matrix scheme is easily adaptable to sparse-matrix LU decompositions. In order to determine the scattering behaviour of the resonant structures, the evaluation of coefficients is limited to very specific locations. However, coefficients within the analyzed structure can easily be extracted, if required, for the computation of the entire electromagnetic field. Scattering parameters in the vicinity of the respective ports of the resonant structure usually require the system to be solved with only a single excitation vector.

THEORY

In this section, we will highlight the choice of basis functions in the two different coordinate systems. For a detailled description of the procedure leading to the block-diagonal matrix structure, the reader is referred to [4], [5]. At each discontinuity formed by a junction of two waveguides, the electric field is expanded in a series of basis functions which vanish everywhere except for the common aperture region. The basis functions involve the modal functions of the common aperture, denoted by $T_{h,e}$, and coefficients $c_{r,s}$ for TE and TM modes, respectively.

In the cartesian coordinate system, edge conditions are applied at x_l, x_u, y_l, y_u. If one or more walls of two connected guides are aligned, then the edge conditions of the respective mirror images are included. This leads to coupling integrals which can be integrated analytically. The resulting expressions involve bessel functions of the first kind and of order 1/6, which can be computed straightforwardly, e.g. [6]. Thus for rectangular structures, the aperture functions are

$$\overrightarrow{E_\square}(x,y) = \sum_r \frac{[\nabla T_{\square hr} \times \vec{e}_z] c_r}{\left[\left(1-\frac{x}{x_l}\right)\left(\frac{x}{x_u}-1\right)\left(1-\frac{y}{y_l}\right)\left(\frac{y}{y_u}-1\right)\right]^{1/3}} + \sum_s \frac{[-\nabla T_{\square es}] c_s}{\left[\left(1-\frac{x}{x_l}\right)\left(\frac{x}{x_u}-1\right)\left(1-\frac{y}{y_l}\right)\left(\frac{y}{y_u}-1\right)\right]^{1/3}}$$

In polar coordinates, the electric field in the aperture is expressed as

$$\overrightarrow{E_O}(\rho,\varphi) = \sum_r \frac{[\nabla T_{Ohr}(\rho,\varphi) \times \vec{e}_z] c_r}{[1-(\rho/r_0)^2]^{1/3}} + \sum_s \frac{[-\nabla T_{Oes}(\rho,\varphi)] c_s}{[1-(\rho/r_0)^2]^{1/3}}$$

which requires coupling integrals to be evaluated numerically.

The procedure then continues by relating the basis function coefficient vectors $\underline{c}^{i}$ at the ith discontinuity to those of their immediate neighbours $\underline{c}^{i-1}$ and $\underline{c}^{i+1}$. The resulting coupled integral equations system is solved using Galerkin's method.

RESULTS AND DISCUSSION

Fig. 1 shows the structure and performance of a four-cavity cylindrical iris filter. Up to 100 expansion terms and 19 edge-conditioned basis functions are used in the calculation. The entire design process including synthesis, fine-optimization and analysis is done in less than 20 minutes on an ordinary PC. In order to improve the stop-band towards higher frequencies, the diameter of the cavities is chosen slightly larger than those of the input/output guides. Excellent agreement with the mode-matching technique (MMT) is observed.

A single cavity operating TE_{201} and TM_{210} modes is shown in Fig. 2. The input guide couples simultaneously to both resonant modes which is utilized to create a transmission zero (attenuation pole) in the upper stopband at 13.45 GHz. However, due to the relatively large cavity, two spurious-mode resonances appear between 11.5 and 12 GHz. They can be eliminated by selecting smaller cavities supporting lower-order modes. This is shown in Fig. 3 at the example of a two-cavity arrangement supporting both TE_{101} and TM_{110} modes. Input bypass coupling is used to create two transmission zeros, one above and one below the passband of the filter.

Whereas the structure in Fig. 1 is a special case which can be solved with TE_{1n} modes and related basis functions, the configurations in Figs. 2 and 3 require the entire mode spectrum. These designs are carried out with up to 1900 expansion terms (modes) and 23 edge-conditioned basis functions. Very good agreement with the MMT is demonstrated. However, the results using our technique have been obtained with CPU-time savings of at least one order of magnitude compared to those using the MMT. Up to 285 modes have been considered in the MMT, and many more will be required in Fig. 3 to achieve convergence toward the equiripple return loss.

In conclusion, the technique presented here is especially useful in narrowband systems as it allows the inclusion of a large number of expansion terms within the framework of a relatively small number of edge-conditioned basis functions and a block-diagonal system matrix.

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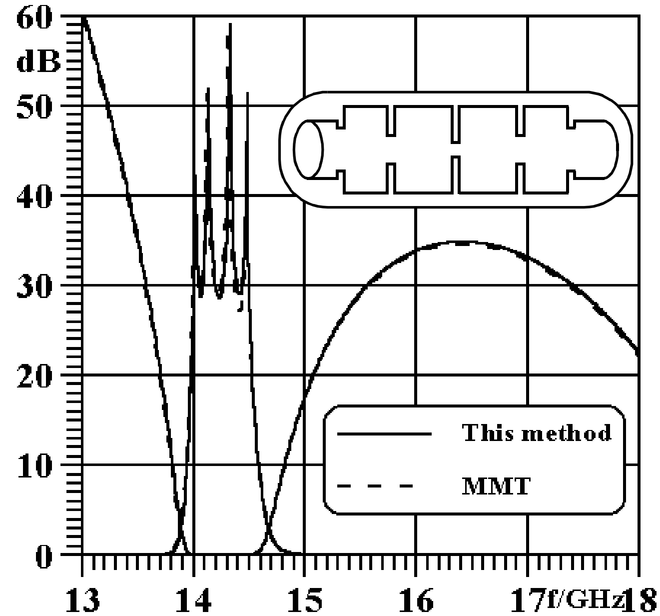

Fig. 1 Scattering parameters of a circular-iris structure and comparison with the mode-matching method (MMT).

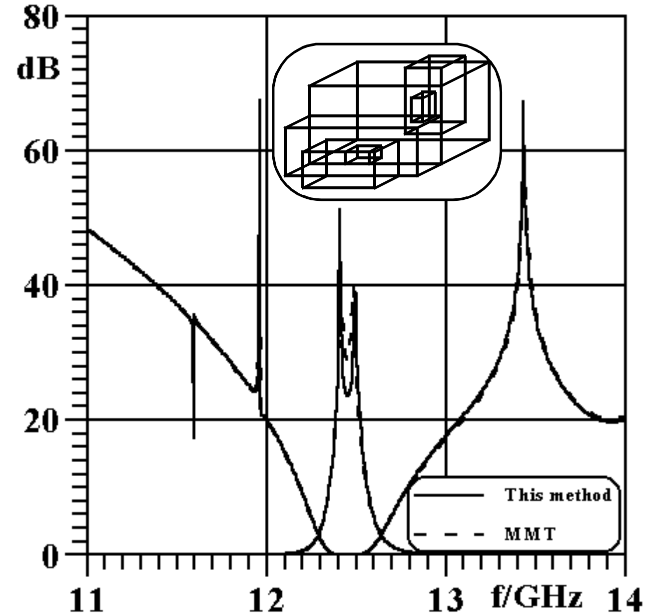

Fig. 2 Two-pole filter (based on TE201 and TM210 modes) with input bypass coupling to create a transmission zero in upper stopband.

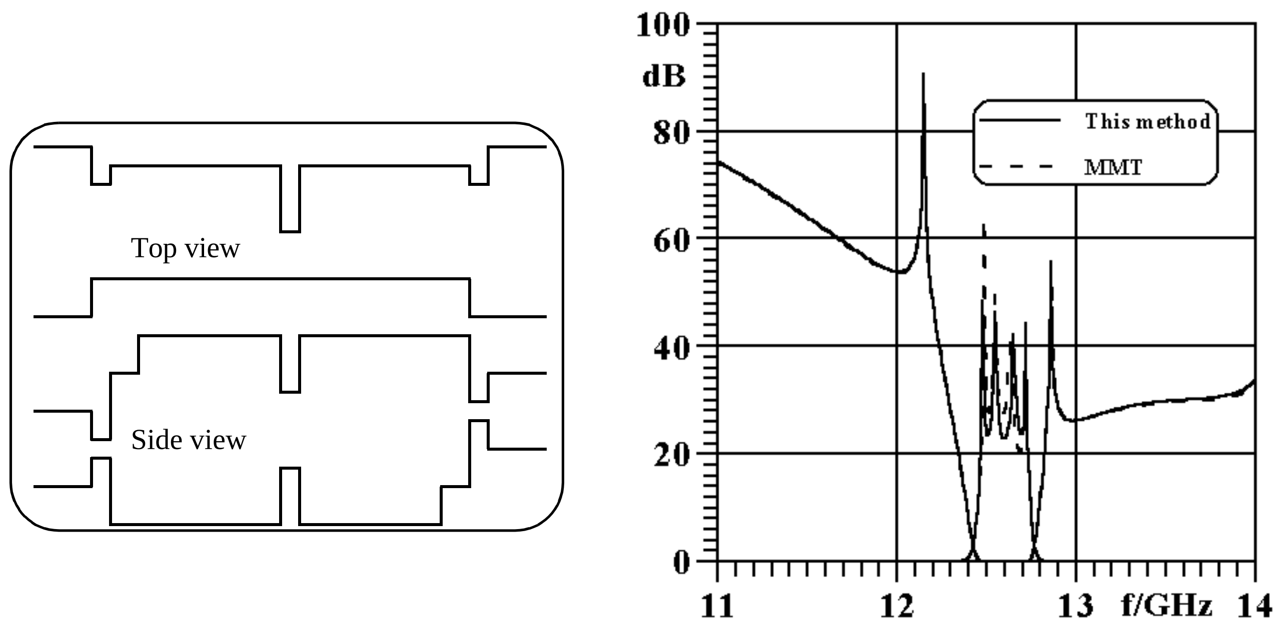

Fig. 3 Basic layout and performance (including comparison with the mode-matching technique) of a four-pole filter (based on TE101 and TM110 modes) with input bypass couplings to create transmission zeros to the right and left of passband.