F. Arndt, J. Bornemann, D. Grauerholz & R. Vahldieck
Microwave Department, University of Bremen, Germany

Material and Circuit Evaluation for Millimetre-Wave Applications: High-Q Fin-Line Filters for Millimetre Waves*

Abstract A new design theory for high-*Q* fin-line filters is described, including higher order mode propagation. These filters are well suited for use in millimetre-wave integrated circuits. The design data for two fin-line filters are given for mid-band frequencies of approximately 15 and 34 GHz. The measured insertion losses in the passband are only 0.25 and 0.5 dB, respectively, for these two frequencies.

Résumé Cet article expose une nouvelle théorie pour le calcul des filtres à ligne nageoire à coefficient Q élevé, prenant en compte la propagation des modes d'ordre supérieur. Ces filtres conviennent bien aux circuits intégrés en onde millimétrique. On présente les caractéristiques de conception de deux filtres pour des fréquences centrées aux environs de 15 et 34 GHz; les pertes par insertion respectives, mesurées dans la bande passante, ne dépassent pas 0,25 et 0,5 dB aux deux fréquences en question.

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Introduction

In a fin-line structure[1], metal inserts ('fins') are printed on a dielectric substrate which bridges the broad walls of a rectangular waveguide (Fig. 1). This permits circuit elements to be photoetched by low-cost techniques. Because the dimensions of practical fin-line circuits remain compatible with chip and beam-lead devices, fin-line structures are very attractive when designing millimetre-wave integrated circuits.

Very many components, both active and passive, have already been made using this technique[2–5], but there still exists a paucity of suitable theoretical research on fin-line structures. So far only experimental design data[2–5] and first-order design theories[6–7] are available, based on the assumption of fundamental modes in the fin-lines only. For discontinuities in the slot width g (Fig. 1), as is the case in filter structures, however, this first-order theory leads to incorrect results because, as has been shown by our own measurements, higher order modes have an important influence on the transmission and reflection coefficients.

The measured Q (including the effect of losses within the irises) of a grounded fin-line increases with the width g of the gap between the fins. The optimal value is reached[2] when the normalised gap width g/b is 1. High-Q fin-line filters can therefore be designed (Fig. 2) consisting of a grounded fin-line structure with alternating metallic bridges and gaps over the total height of the rectangular waveguide. No design theory has been available until now for such high-Q filters.

The purpose of this paper is therefore to introduce such a theory, which will take into account the higher order mode propagation. The measured frequency response of the fin-line filters that have been designed and operated at about 15 and 34 GHz shows good agreement between theory and practical results. The high-Q design leads to losses of typically only 0.25 and 0.5 dB, respectively, at these two frequencies.

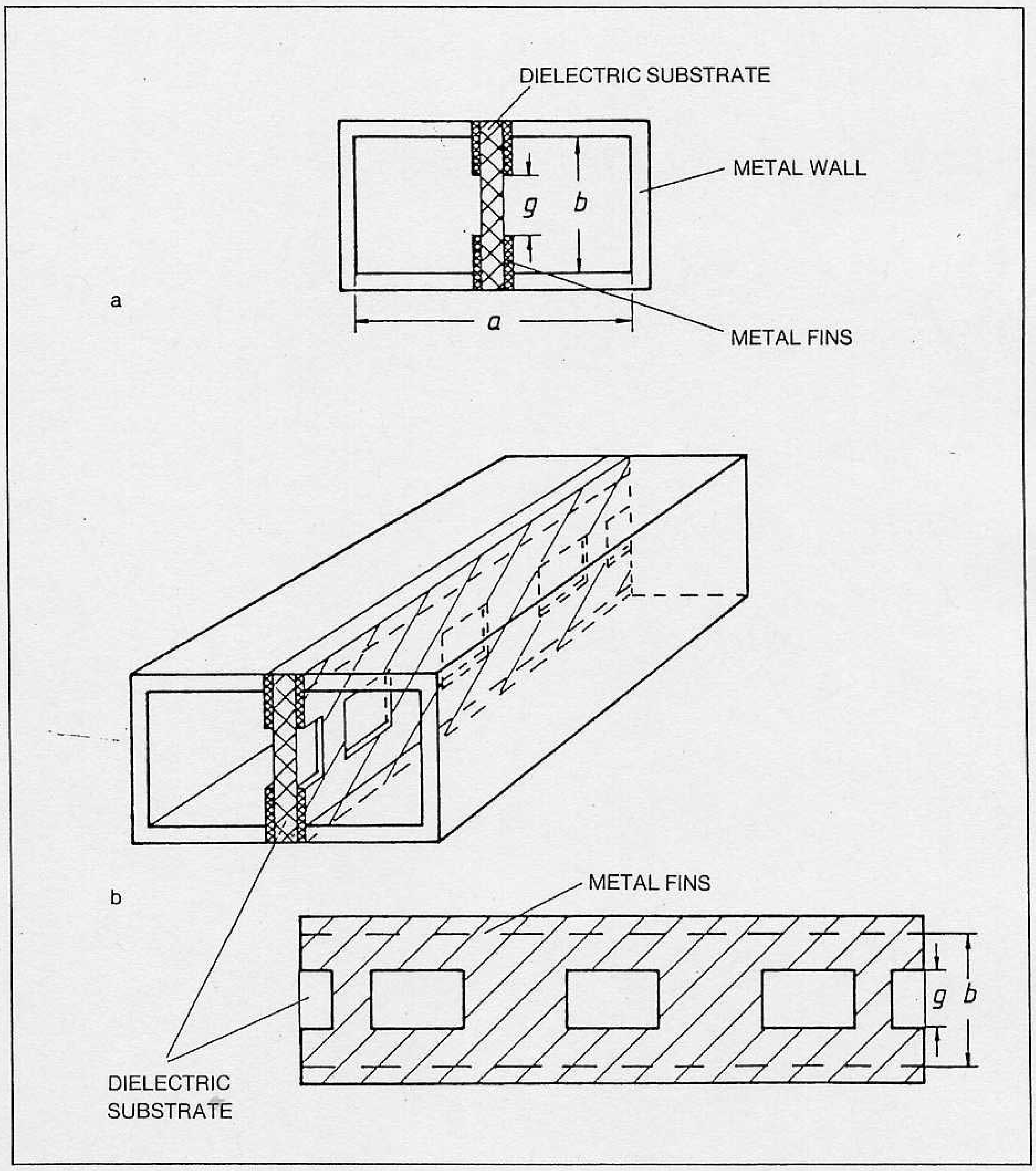


Figure 1. General fin-line filter configuration
a. grounded fins
b. bandpass structure

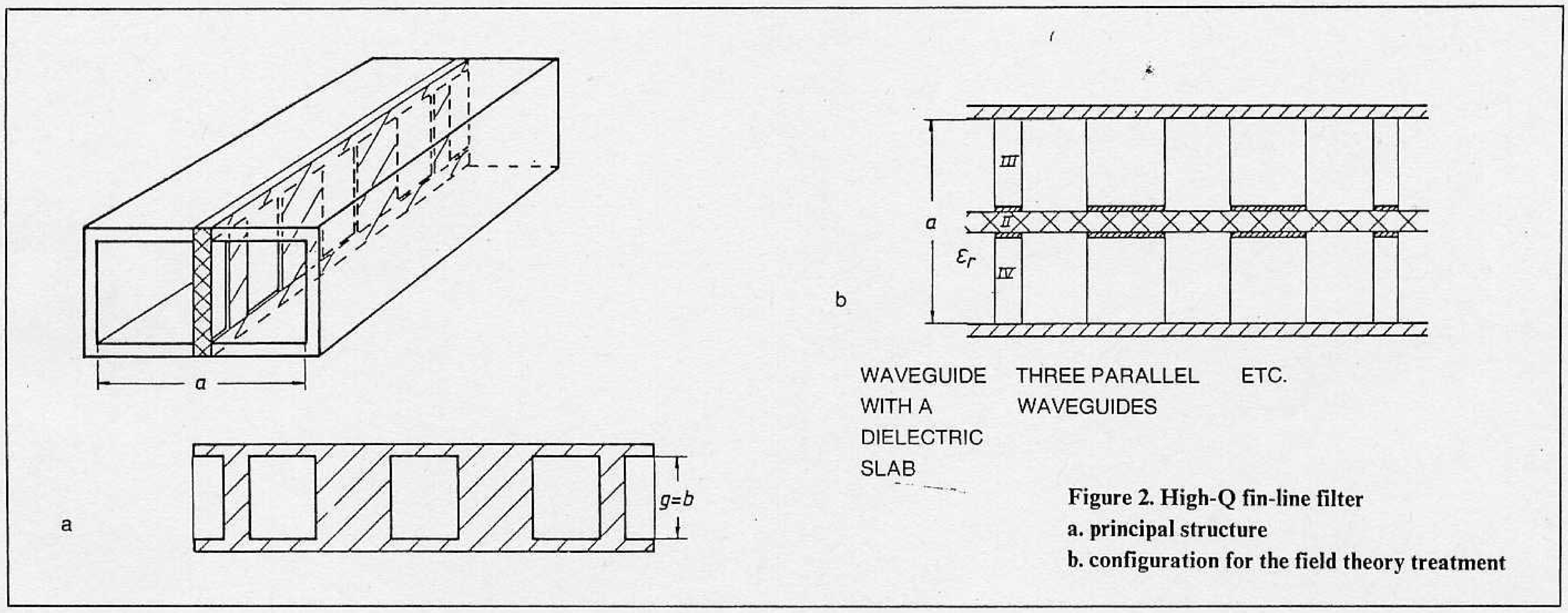


Figure 2. High-Q fin-line filter
a. principal structure
b. configuration for the field theory treatment

Design theory

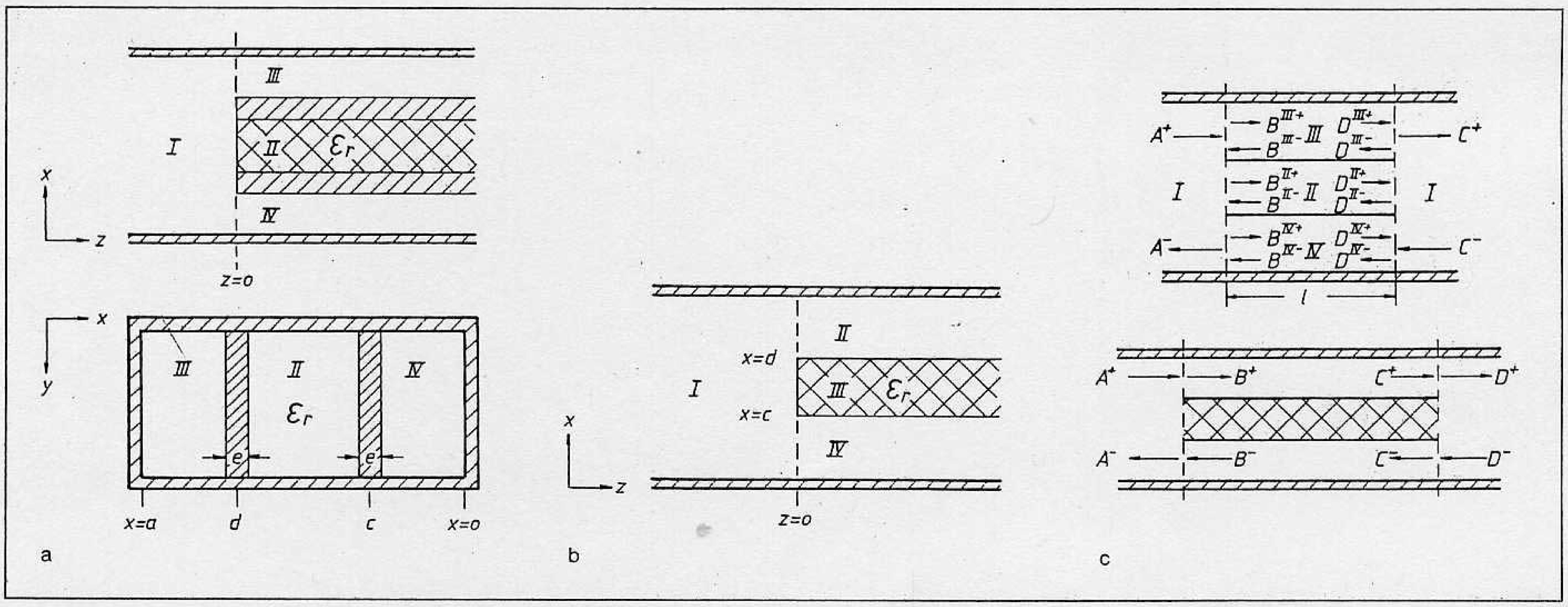
Since the theory is based on the exact-field treatment of the structure shown in Figure 2, the high-Q fin-line filter is regarded as consisting of alternating waveguide structures (Fig. 2b): a waveguide with a dielectric slab perpendicular to the ground wall and three parallel waveguides, the middle one of which is filled with dielectric. This implies the calculation of the scattering coefficients of the two waveguide transitions shown in Figure 3. The scattering matrix of the total fin-line filter structure can then be obtained by suitably combining the transitions, the length of waveguide I being reduced to zero if structures a and b (or inverse) are joined together directly.

Waveguide transition to three waveguides

The wave incident in I (Fig. 3a) is assumed to consist of h_{mo}-modes; there is no discontinuity in the y-direction, so that the only field components in the v regions ($v=$ I, II, III, IV) are E_y^v, H_x^v, H_z^v. The waves are calculated by the method of field expansion into eigenmodes:

$$\begin{bmatrix} E_y^v \\ \\ H_x^v \\ \\ H_z^v \end{bmatrix} = \begin{bmatrix} -\omega\mu \sum\limits_m k_{zm}^v A_m^v \sin \dfrac{m\pi}{p^v} \cdot f^v \\ \\ \sum\limits_m k_{zm}^{v\,2} A_m^v \sin \dfrac{m\pi}{p^v} \cdot f^v \\ \\ -j \sum\limits_m k_{zm}^{v\,2} A_m^v \cos \dfrac{m\pi}{p^v} \cdot f^v \end{bmatrix} \cdot e^{-jk_{zm}^v \cdot z} \tag{1}$$

Figure 3. Waveguide transitions
a. transition to three parallel waveguides
b. transition to a waveguide with a dielectric slab
c. wave amplitude vectors of the corresponding structures of finite length



with the abbreviations (Fig. 3a)

$$\begin{bmatrix} f^{\mathrm{I}} \\ f^{\mathrm{II}} \\ f^{\mathrm{III}} \\ f^{\mathrm{IV}} \end{bmatrix} = \begin{bmatrix} x \\ d - \frac{e}{2} - x \\ a - x \\ c - \frac{e}{2} - x \end{bmatrix}$$

$$\begin{bmatrix} p^{\mathrm{I}} \\ p^{\mathrm{II}} \\ p^{\mathrm{III}} \\ p^{\mathrm{IV}} \end{bmatrix} = \begin{bmatrix} a \\ d - e - c \\ a - d + \frac{e}{2} \\ c - \frac{e}{2} \end{bmatrix}$$

$$\begin{bmatrix} k_{zm}^{\mathrm{I}^2} \\ k_{zm}^{\mathrm{II}^2} \\ k_{zm}^{\mathrm{III}^2} \\ k_{zm}^{\mathrm{IV}^2} \end{bmatrix} = \begin{bmatrix} k_0^2 & - \left(\frac{m\pi}{a}\right)^2 \\ k_0^2 \varepsilon_r & - \left(\frac{m\pi}{p^{\mathrm{II}}}\right)^2 \\ k_0^2 & - \left(\frac{m\pi}{p^{\mathrm{III}}}\right)^2 \\ k_0^2 & - \left(\frac{m\pi}{p^{\mathrm{IV}}}\right)^2 \end{bmatrix}$$

$$k_0^2 = \mu_0 \varepsilon_0$$

At $z = 0$ the waves are coupled by matching the various tangential field components at the interface. This leads to two equations for the amplitudes $(A)^+$, $(B)^+$ of the forward waves and $(A)^-$, $(B)^-$ of the backward waves (Fig. 3c):

$$(A)^+ + (A)^- = (N)^v ((B)^{v+} + (B)^{v-}) \qquad v = \mathrm{II, III, IV}$$
$$(A)^+ - (A)^- = \sum_{v=\mathrm{II}}^{\mathrm{IV}} (L)^v ((B)^{v+} - (B)^{v-}) \tag{2}$$

The terms $(N)^v$ and $(L)^v$ are elucidated in Appendix 1.

The scattering matrix (S) of the structure with finite length l (Fig. 3c) is given by

$$\begin{pmatrix} (A)^- \\ (C)^+ \end{pmatrix} = \begin{pmatrix} (S_{11}) & (S_{12}) \\ (S_{21}) & (S_{22}) \end{pmatrix} \begin{pmatrix} (A)^+ \\ (C)^- \end{pmatrix} \tag{3}$$

with

$$(S_{11}) = (S_{22}) = (W)^{-1} \{[(\pi) + (E)]^{-1}(\phi)[(\pi)+(E)]^{-1}(\phi) - [(\pi)+(E)]^{-1}[(\pi)-(E)]\}$$
$$(S_{12}) = (S_{21}) = (W)^{-1} \{[(\pi)+(E)]^{-1}(\phi) - [(\pi)+(E)]^{-1}(\phi)[(\pi)+(E)]^{-1}[(\pi)-(E)]\} \tag{4}$$

The abbreviations are explained in Appendix 2.

Transition to a waveguide with a dielectric slab

For this structure (Fig. 3b), the waves in regions II, III and IV propagate with a common propagation factor k_{zm} which is determined by the boundary conditions along the dielectric slab:

$$\left.\begin{aligned} E_y^{\mathrm{II}} &= E_y^{\mathrm{IV}} \\ H_z^{\mathrm{II}} &= H_z^{\mathrm{IV}} \end{aligned}\right|_{x=c} \qquad \left.\begin{aligned} E_y^{\mathrm{II}} &= E_y^{\mathrm{III}} \\ H_z^{\mathrm{II}} &= H_z^{\mathrm{III}} \end{aligned}\right|_{x=d} \tag{5}$$

With Equation 1, a system of linear equations is obtained in which the determinant is required to be zero. This leads to a transcendent equation for the propagation factor k_{zm} which is solved numerically.

The scattering matrix (S) of the structure of finite length l (Fig. 3c) is given by

$$\begin{pmatrix} (A)^- \\ (D)^+ \end{pmatrix} = \begin{pmatrix} (S_{11}) & (S_{12}) \\ (S_{21}) & (S_{22}) \end{pmatrix} \begin{pmatrix} (A)^+ \\ (D)^- \end{pmatrix} \tag{6}$$

with

$$(S_{11}) = (S_{22}) = (Q_{11}) + (Q_{12})(R)(W)(Q_{22})(R)(Q_{21})$$

$$(S_{12}) = (S_{21}) = (Q_{12})(R)(W)(Q_{21})$$

The abbreviations are explained in Appendix 3.

For the computer calculation of the total scattering matrix, the expansion into twelve eigenmodes in each region (see Equation 1) has proved to be sufficient.

Results

Figure 4 shows the calculated insertion loss (scattering coefficient S_{21}) in dB as a function of frequency for a nine-step fin-line filter (four metal inserts), designed for a mid-band frequency of 14.26 GHz, mounted into a P-band waveguide ($a=15.699$ mm, $b=7.899$ mm). The dielectric substrate is RT/duroid 5880 with an $\varepsilon_r=2.22$.

The design process is started with filter dimensions given by Saad & Schünemann[6,8] for the narrow slot filter ($g \ll b$, cf. Fig. 1), which exhibits a very poor measured insertion loss of more than about 2 dB. The total scattering matrix of the fin-line filter can be calculated for given parameters. An optimising computer program is used[9], which varies the input parameters until the desired

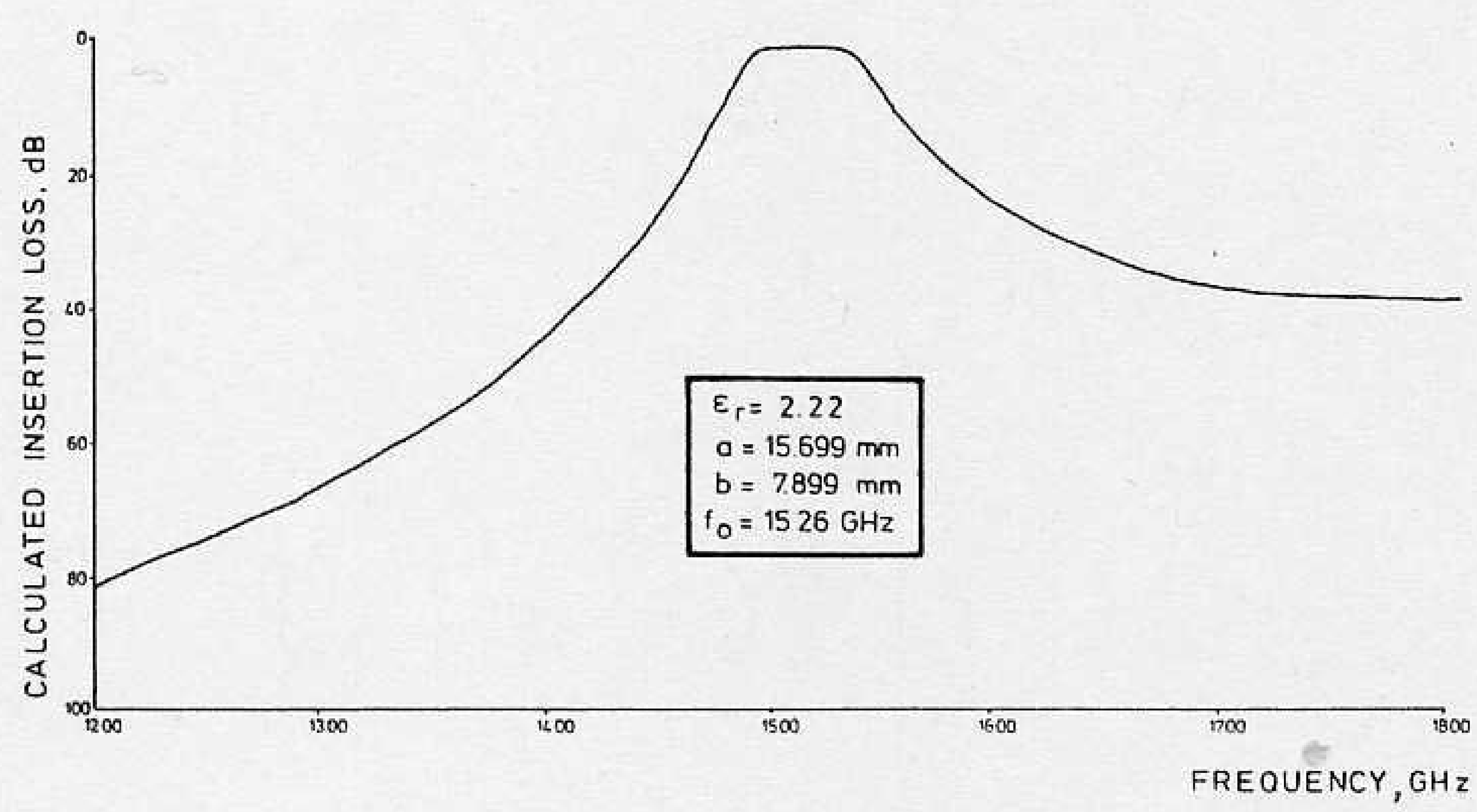


Figure 4. P-band fin-line filter. Calculated insertion loss as a function of frequency

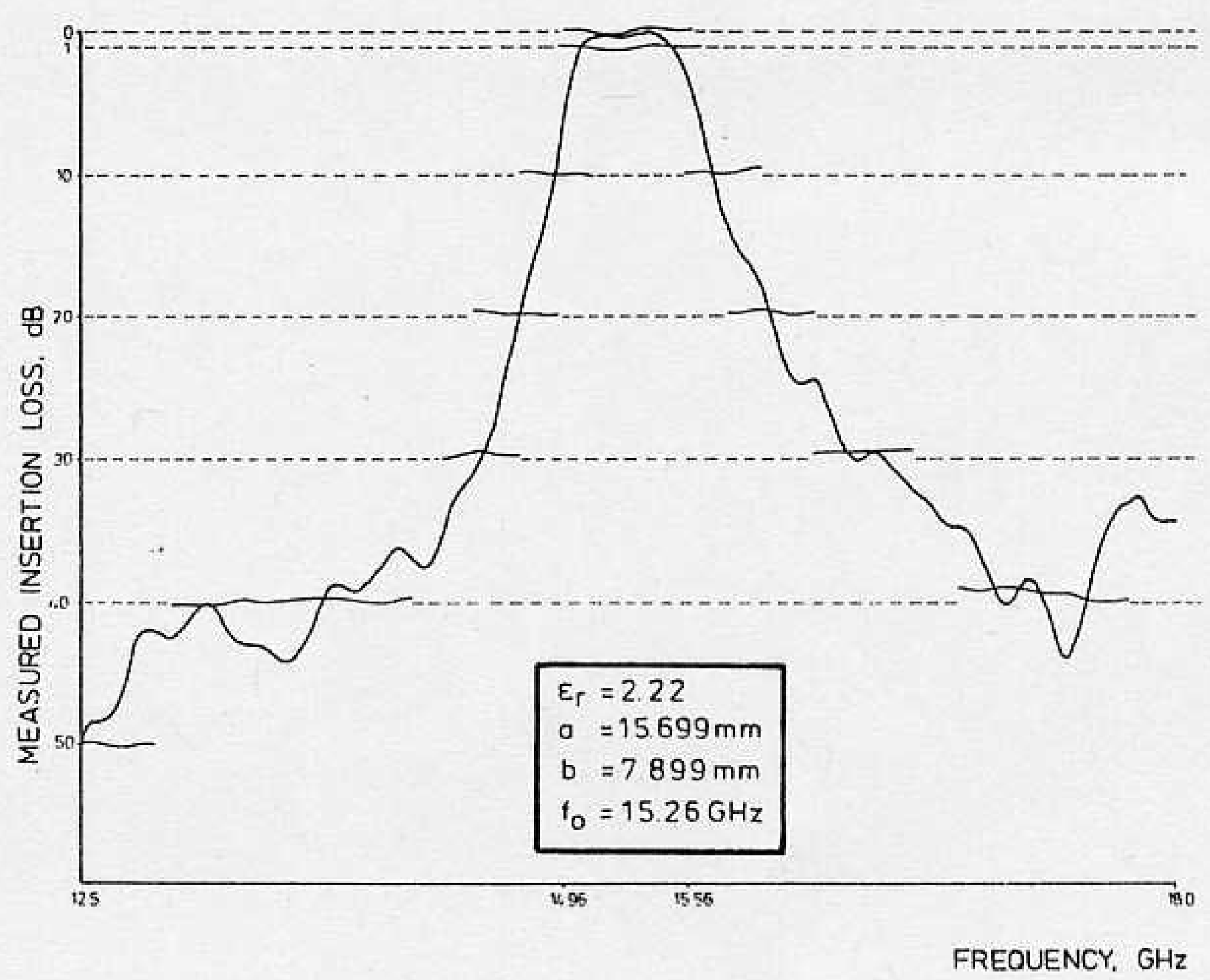
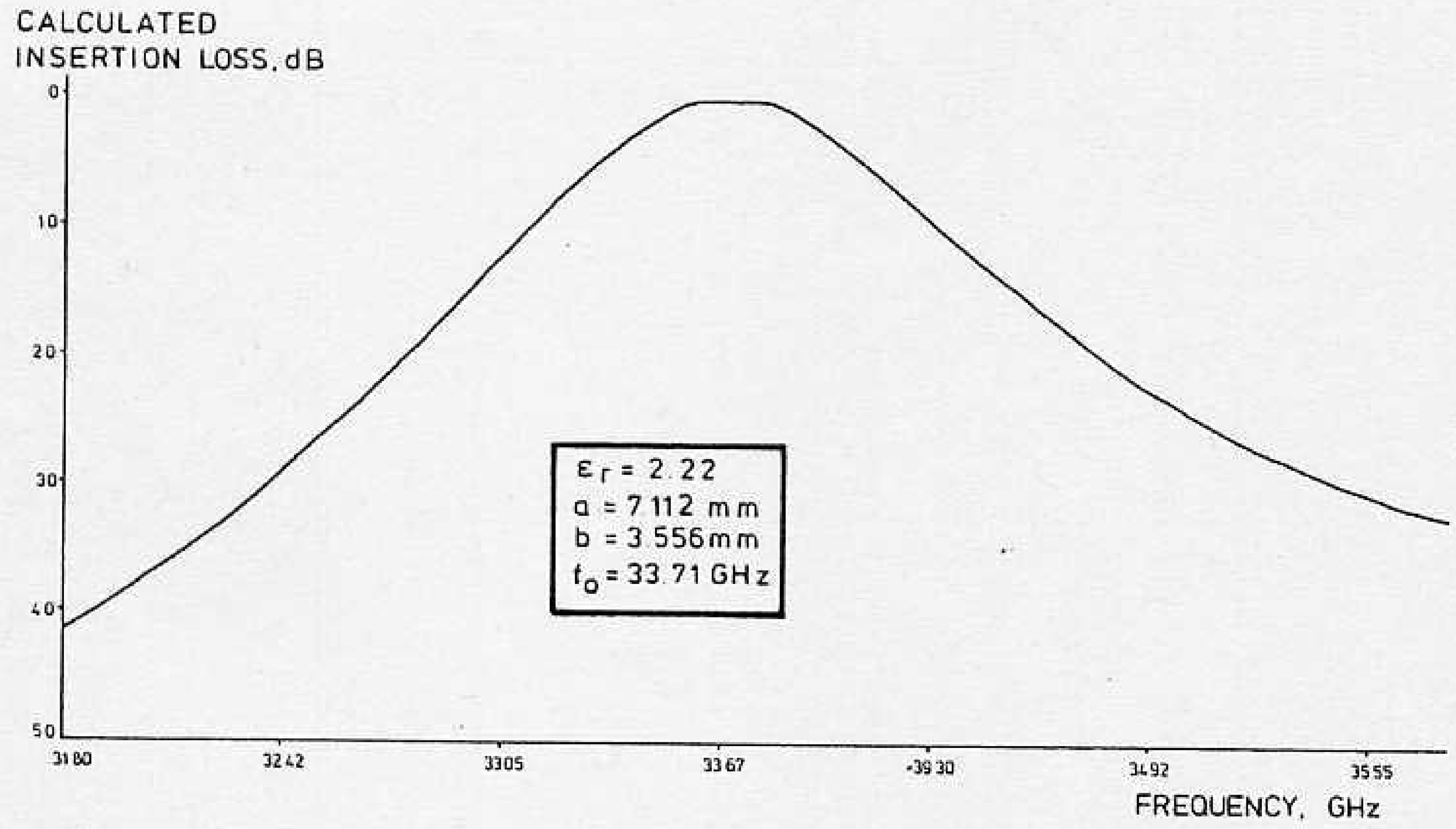


Figure 5. P-band fin-line filter. Measured insertion loss as a function of frequency



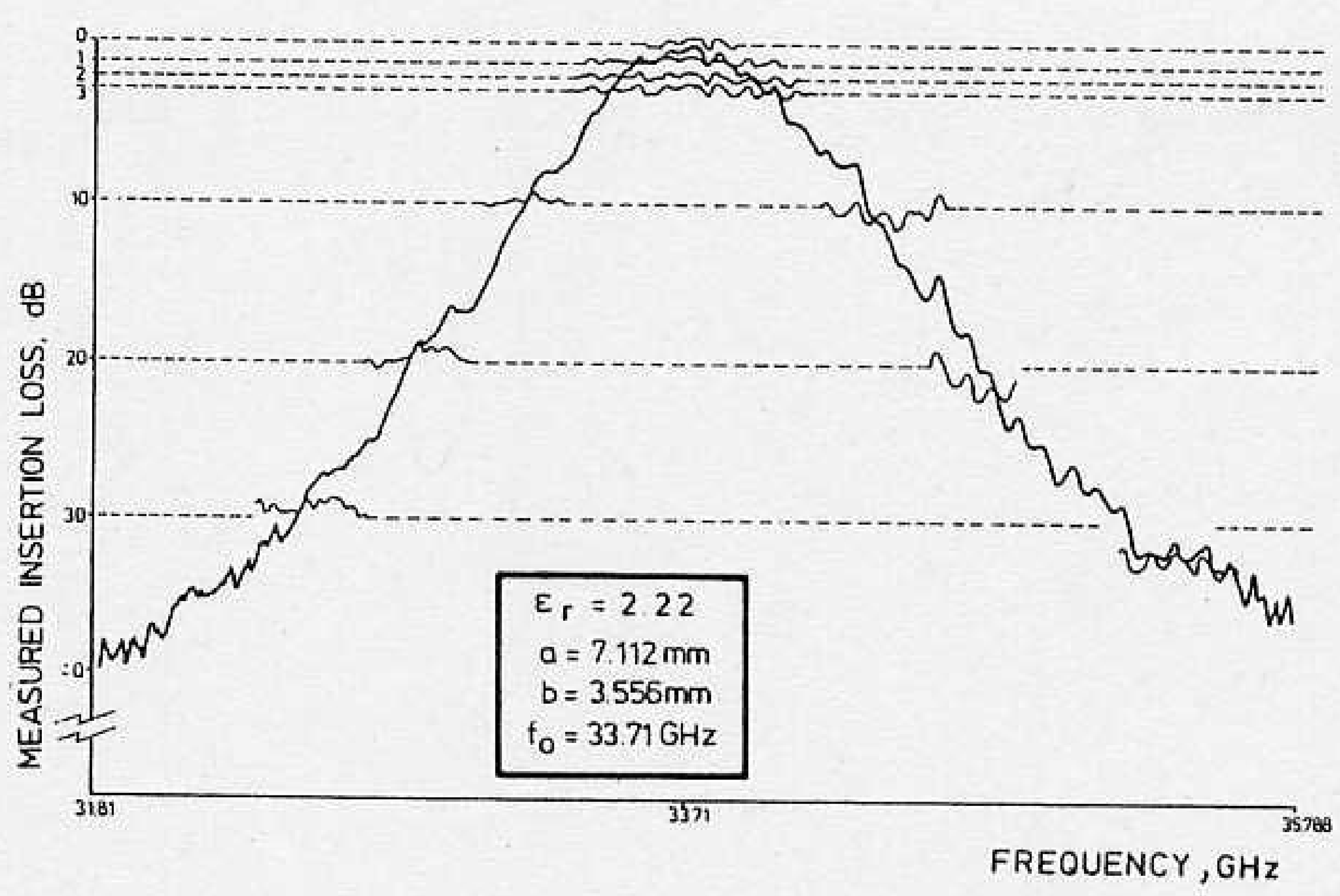


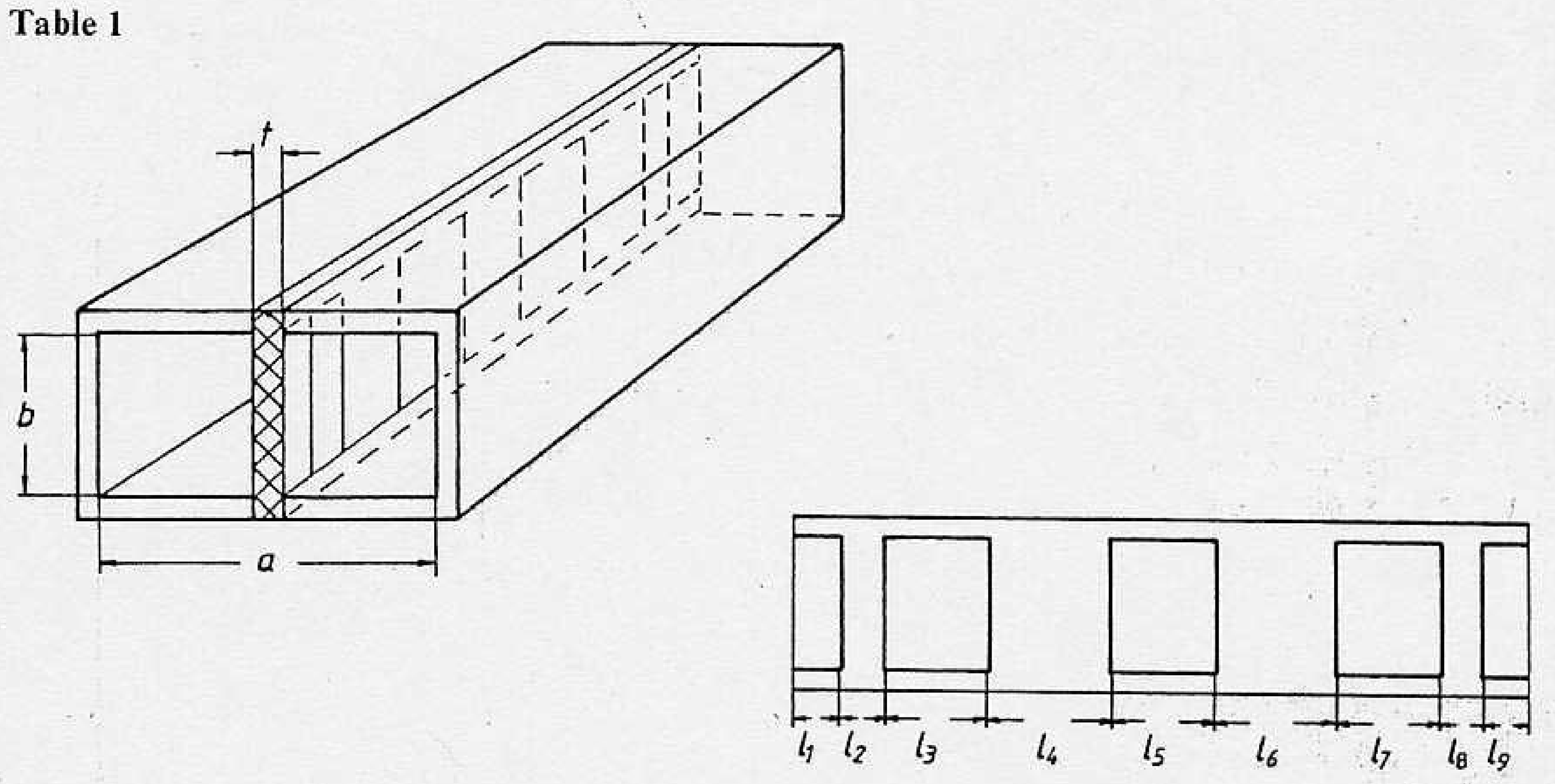
Figure 6. R-band fin-line filter. Calculated insertion loss as a function of frequency

Figure 7. R-band fin-line filter. Measured insertion loss as a function of frequency

insertion loss is obtained for a given bandwidth. The theoretical insertion losses for the 15.26 GHz and 33.71 GHz fin-line filters are 0.05 dB, and 0.10 dB, respectively, at mid-band frequency. The dimensions of the fin-line steps are given in Table 1. Figure 5 shows the measured insertion loss to be in good agreement with the calculated curve. The typical measured insertion loss in the passband is only about 0.25 dB.

Figure 6 shows the calculated insertion loss for a corresponding fin-line filter designed for 33.71 GHz (R-band waveguide, $a = 7.112$ mm, $b = 3.556$ mm). The dimensions of the fin-line steps are given in Table 1. The calculation also agrees well with the measured curve (Fig. 7). The typical measured insertion loss in the passband is only about 0.5 dB. The two photoetched fin-line structures are shown in Figure 8.

Table 1



Fin-Line Filter	Dimensions* in mm a	b	$l_1 = l_9$	$l_2 = l_8$	$l_3 = l_7$	$l_4 = l_6$	l_5
P-Band f_0 = 15.26 GHz	15.699	7.899	27.432	2.1	8.376	7.8	8.384
R-Band f_0 = 33.71 GHz	7.112	3.556	39.26	0.72	3.734	3.918	3.736

Dielectric Material: RT/Duroid 5880 $\varepsilon_r = 2.22$
Thickness t = 0.254 mm

* The l-dimensions do not include the etching error. After etching l_2, l_4, l_6 and l_8 are reduced by 0.06 mm.

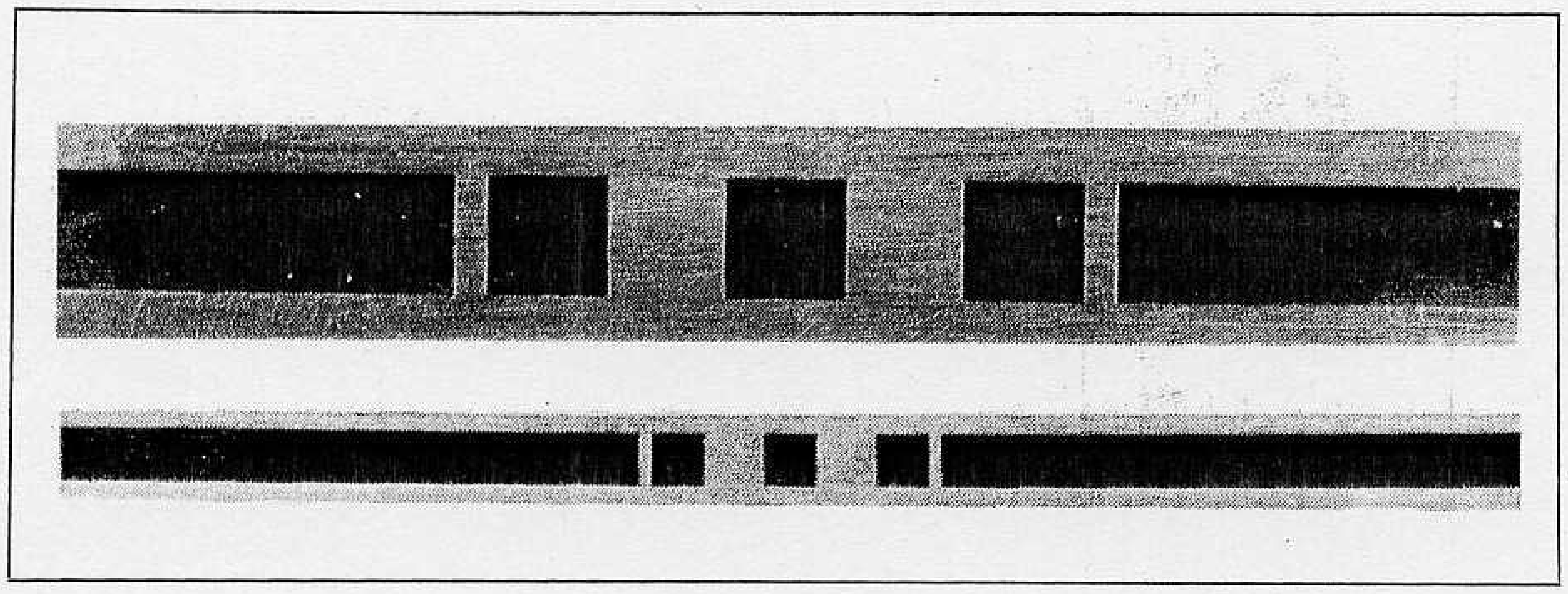


Figure 8. Two photoetched fin-line structures (for 15 GHz and 33.7 GHz)

Conclusion

A novel design theory has been shown to lead to high-Q fin-line filters suitable for millimetre waves. They consist of alternating metallic bridges and gaps on a dielectric slab mounted between the broad walls of a rectangular waveguide. The theory is based on field expansion into eigenmodes within the subregion. By matching the various tangential field components, linear-equation systems for the scattering coefficients have been derived which can be solved numerically. The calculated and measured insertion-loss curves for the two nine-step filters that have been constructed agree very well. The theoretical insertion losses for the filter designs are 0.05 dB (for 15.26 GHz) and 0.10 dB (for 33.71 GHz). The typical measured insertion losses, which are only about 0.25 dB and 0.5 dB, respectively, show the excellent suitability of these fin-line filters for use in the design of high-Q millimetre-wave integrated circuits.

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Appendix 1

Abbreviations in the scattering matrix of the waveguide transition to three waveguides (Equation 2)

$(N)^{\nu}$:

$$(D_{IImn})^{-1}(D_{IImn}) = (N)^{\mathrm{II}}$$
$$(D_{Hmk})^{-1}(D_{Hkk}) = (N)^{\mathrm{III}}$$
$$(D_{Hml})^{-1}(D_{Hll}) = (N)^{\mathrm{IV}}$$

with the matrix coefficients

$$D_{Hmn} = k_{zm}^{\mathrm{I}^2} A_m^{\mathrm{I}} H_{mn} \qquad D_{Hnn} = \frac{d-c}{2} k_{zn}^{\mathrm{II}^2} A_n^{\mathrm{II}}$$

$$D_{Hmk} = k_{zm}^{I^2} A_m^I H_{mk} \qquad D_{Hkk} = \frac{a-d}{2} k_{zk}^{III^2} A_k^{III}$$

$$D_{Hml} = k_{zm}^{I^2} A_m^I \mathrm{H}_{ml} \qquad D_{Hll} = \frac{c}{2} k_{zl}^{IV^2} A_l^{IV}$$

and coupling integrals

$$H_{mn} = \int_{x=c}^{d} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{d-c} (d-x) \mathrm{d}x$$

$$H_{mk} = \int_{x=d}^{a} \sin \frac{m\pi}{a} x \sin \frac{k\pi}{a-d} (a-x) \mathrm{d}x$$

$$H_{ml} = \int_{x=0}^{c} \sin \frac{m\pi}{a} x \sin \frac{l\pi}{c} x \, \mathrm{d}x$$

$(L)^v$: $\quad (D_{Emm})^{-1} (D_{Emn}) = (L)^{II}$

$$(D_{Emm})^{-1} (D_{Emk}) = (L)^{III}$$

$$(D_{Emm})^{-1} (D_{Eml}) = (L)^{IV}$$

with the matrix coefficients

$$D_{Emm} = \frac{a}{2} k_{zm}^{I} A_m^I \qquad D_{Emk} = k_{zk}^{III} A_k^{III} H_{mk}^T$$

$$D_{Emn} = k_{zn}^{II} A_n^{II} H_{mn}^T \qquad D_{Eml} = k_{zl}^{IV} A_l^{IV} H_{ml}^T$$

and coupling integrals

$$H_{nm} = \int_{x=c}^{d} \sin \frac{n\pi}{d-c} (d-x) \sin \frac{m\pi}{a} x \, \mathrm{d}x$$

$$H_{km} = \int_{x=d}^{a} \sin \frac{k\pi}{a-d} (a-x) \sin \frac{m\pi}{a} x \, \mathrm{d}x$$

$$H_{lm} = \int_{x=0}^{c} \sin \frac{l\pi}{c} x \sin \frac{m\pi}{a} x \, \mathrm{d}x$$

$$H_{nm}^T = H_{mn}$$

Appendix 2 **Abbreviations in the scattering matrix of the waveguide transition to three waveguide structures of finite length (Equation 4)**

$$(W) = (E) - ((\pi) + (E))^{-1} (\phi)((\pi) + (E))^{-1} (\phi)$$

(E) = unit matrix

$$(\pi) = \sum_v \left[2(L)^v(\delta)^v - (L)^v(N)^{v-1}\right] \qquad (\delta)^v = ((E)-(R)^v(R)^v)^{-1}(N)^{v-1}$$

$$(\phi) = \sum_v 2(L)^v(\varepsilon)^v \qquad (\varepsilon)^v = ((E)-(R)^v(R)^v)^{-1}(R)^v(N)^{v-1}$$

with the diagonal matrix of the waveguides with finite length l

$$(R)^v = \begin{bmatrix} e^{-jk^v_{z1}.l} & \cdots & 0 \\ \vdots & \ddots & \\ 0 & & e^{-jk^v_{zm}.l} \end{bmatrix}$$

Appendix 3

Abbreviations in the scattering matrix of the waveguide transition to a waveguide with a dielectric slab of finite width and length (Equation 6)

$$(W) = ((E)-Q_{22})(R)(Q_{22})(R))^{-1} \qquad R_{mm} = e^{-jk_{zm}.l}$$

The scattering coefficients of the single step are:

$$(Q_{11}) = 2(D_H)^{-1}(L_H)(M)-(E) \qquad (Q_{21}) = 2(M)$$

$$(Q_{12}) = 2(D_H)^{-1}(L_H)(M)(D_E)^{-1}(L_E) \qquad (Q_{22}) = 2(M)(D_E)^{-1}(L_E)-(E)$$

$$(M) = ((D_E)^{-1}(L_E) + (D_H)^{-1}(L_M))^{-1}$$

$$D_{Emm} = \frac{a}{2} k^{\mathrm{I}}_{zm} A^{\mathrm{I}}_m \qquad D_{Hmm} = \frac{a}{2} k^{\mathrm{I}^2}_{zm} A^{\mathrm{I}}_m$$

The coefficients of the matrices (L_E) and (L_H) are given by:

$$k_{zk}(A^{\mathrm{II}}_k I^{\mathrm{II}}_{mk} + B^{\mathrm{II}'}_k I^{\mathrm{II}'}_{mk} + A^{\mathrm{III}}_k(-T_{1k} I^{\mathrm{III}}_{mk} + I^{\mathrm{III}}_{mk}) + A^{\mathrm{IV}}_k I^{\mathrm{IV}}_{mk}) = L_{Emk}$$

$$k^2_{zk}(A^{\mathrm{II}}_k I^{\mathrm{II}}_{mk} + B^{\mathrm{II}'}_k I^{\mathrm{II}}_{mk} + A^{\mathrm{III}}_k(-T_{1k} I^{\mathrm{III}}_{mk} + I^{\mathrm{III}}_{mk}) + A^{\mathrm{IV}}_k I^{\mathrm{IV}}_{mk}) = L_{Hmk}$$

with the abbreviation $T_{1k} = \tan(k^{\mathrm{III}}_{xk} \cdot a)$.

Coupling integrals:

$$I^{\mathrm{II}}_{mk} = \int_{x=c}^{d} \cos(k^{\mathrm{II}}_{xk} x) \sin\left(\frac{m\pi}{a} x\right) \mathrm{d}x$$

$$I^{\mathrm{II}'}_{mk} = \int_{x=c}^{d} \sin(k^{\mathrm{II}}_{xk} x) \sin\left(\frac{m\pi}{a} x\right) \mathrm{d}x$$

$$I^{\mathrm{III}}_{mk} = \int_{x=d}^{a} \cos(k^{\mathrm{III}}_{xk} x) \sin\left(\frac{m\pi}{a} x\right) \mathrm{d}x$$

$$I_{mk}^{III'} = \int_{x=d}^{a} \sin(k_{xk}^{III} x) \sin\left(\frac{m\pi}{a} x\right) dx$$

$$I_{mk}^{IV} = \int_{x=0}^{c} \sin(k_{xk}^{IV} x) \sin\left(\frac{m\pi}{a} x\right) dx$$

The common propagation factor k_{zm} of the regions II, III, IV is determined by:

$$\begin{bmatrix} k_{xm}^{II^2} \\ k_{xm}^{III^2} \\ k_{xm}^{IV^2} \end{bmatrix} = \begin{bmatrix} k_0^2 \cdot \varepsilon_r \\ k_0^2 \\ k_0^2 \end{bmatrix} - k_{zm}^2$$

$$\frac{1}{k_{xm}^{II}} \cdot \tan(k_{xm}^{II}(d-c)) + \frac{1}{k_{xm}^{III}} \cdot \tan(k_{xm}^{III}(a-d)) + \frac{1}{k_{xm}^{IV}} \cdot \tan(k_{xm}^{IV} \cdot c)$$

$$- \frac{k_{xm}^{II}}{k_{xm}^{III} \cdot k_{xm}^{IV}} \cdot \tan(k_{xm}^{II}(d-c)) \cdot \tan(k_{xm}^{III}(a-d)) \cdot \tan(k_{xm}^{IV} \cdot c) = 0$$

The waves in the regions I, II, III, IV are expanded into the eigenmodes:

$$E_y^{I} = -\omega\mu \sum_{m=1} k_{zm}^{I} A_m^{I} \sin \frac{m\pi}{a} x \cdot e^{-jk_{zm}^{I} \cdot z}$$

$$H_z^{I} = -j \sum_{m=1} \frac{m\pi}{a} k_{zm}^{I} A_m^{I} \cos \frac{m\pi}{a} x \cdot e^{-jk_{zm}^{I} \cdot z}$$

$$H_x^{I} = \sum_{m=1} \left(k_0^2 - \left(\frac{m\pi}{a}\right)^2\right) A_m^{I} \sin \frac{m\pi}{a} x \cdot e^{-jk_{zm}^{I} \cdot z}$$

$$E_y^{II} = -\omega\mu \sum_{m=1} k_{zm} (A_m^{II} \cos(k_{xm}^{II} \cdot x) + B_m^{II} \sin(k_{xm}^{II} \cdot x)) \cdot e^{-jk_{zm} \cdot z}$$

$$H_z^{II} = -j \sum_{m=1} k_{zm} k_{xm}^{II} (-A_m^{II} \sin(k_{xm}^{II} \cdot x) + B_m^{II} \cos(k_{xm}^{II} \cdot x)) \cdot e^{-jk_{zm} \cdot z}$$

$$H_x^{II} = \sum_{m=1} (\varepsilon_r k_0^2 - k_{xm}^{II^2}) (A_m^{II} \cos(k_{xm}^{II} \cdot x) + B_m^{II} \sin(k_{xm}^{II} \cdot x)) \cdot e^{-jk_{zm} \cdot z}$$

$$E_y^{III} = -\omega\mu \sum_{m=1} k_{zm} A_m^{III} (-\tan(k_{xm}^{III} \cdot a) \cos(k_{xm}^{III} \cdot x) + \sin(k_{xm}^{III} \cdot x)) \cdot e^{-jk_{zm} \cdot z}$$

$$H_z^{III} = -j \sum_{m=1} k_{zm} k_{xm}^{III} A_m^{III} (\tan(k_{xm}^{III} \cdot a) \sin(k_{xm}^{III} \cdot x) + \cos(k_{xm}^{III} \cdot x) e^{-jk_{zm} \cdot z}$$

$$H_x^{III} = \sum_{m=1} (k_0^2 - k_{xm}^{III^2}) A_m^{III} (-\tan(k_{xm}^{III} \cdot a) \cos(k_{xm}^{III} \cdot x) + \sin(k_{xm}^{III} \cdot x)) \cdot e^{-jk_{zm} \cdot z}$$

$$E_y^{IV} = -\omega\mu \sum_{m=1} k_{zm} A_m^{IV} \sin(k_{xm} \cdot x) \cdot e^{-jk_{zm} \cdot z}$$

$$H_z^{IV} = -j \sum_{m=1} k_{zm} k_{xm}^{IV} A_m \cos(k_{xm}^{IV} \cdot x) \cdot e^{-jk_{zm} \cdot z}$$

$$H_x^{IV} = \sum_{m=1} (k_0^2 - k_{xm}^{IV^2}) A_m^{IV} \sin(k_{xm}^{IV} \cdot x) \cdot e^{-jk_{zm} \cdot z}$$

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