

Characterization of a Class of Waveguide Discontinuities Using a Modified TE_{mn}^x Mode Approach

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Abstract—This paper presents a modified TE_{mn}^x wave approach which is used in conjunction with the mode matching method for the field-theory modeling of a class of waveguide discontinuities. In particular, for characterizing waveguide discontinuities in which resonant effects occur, this method resolves conflicting results which have been observed using the conventional TE_{mn}^x mode matching technique, commonly known from the literature, and the generalized analysis based on a linear superposition of TE_{mn}^z and TM_{mn}^z modes. It is found that results from the modified TE_{mn}^x mode approach are consistent with the generalized analysis and agree well with measurements on iris filters and corrugated waveguide polarizers. In comparison with the generalized TE–TM mode analysis, the modified TE_{mn}^x mode procedure consumes less memory and CPU time and provides improved convergence behavior without sacrificing design accuracy.

I. INTRODUCTION

MODERN microwave and millimeter-wave computer-aided design packages are based on numerically efficient yet accurate computation techniques for the field-theory modeling of circuit discontinuities. In particular, computer-aided design of waveguide components at millimeter wavelengths requires accurate computation of circuit S parameters to avoid or minimize cut-and-try manufacturing cycles.

In this paper, we will investigate different approaches for characterizing single- or double-plane discontinuities which are integral design elements for a wide range of waveguide components such as transformers [1], [3], [4], iris and corrugated waveguide filters [1]–[3], [5], [6], evanescent-mode band-pass filters [7]–[10], horn antennas [11], [12], and polarizers [11]–[16].

Numerical characterization of the type of waveguide junction discontinuities shown in Fig. 1 is usually carried out by a generalized analysis utilizing a linear superposition of TE_{mn} – TM_{mn} modes. Based on the assumption that an incident TE_{10} wave would excite TE_{mn} and TM_{mn} waves, two vector potential functions are necessary to describe the four field components (E_x, E_y, H_x, H_y) to be matched at $z = 0$ [3]–[5]. However, the double-plane step discontinuity can be viewed as a combination of two single-plane steps in which only the E -plane discontinuity

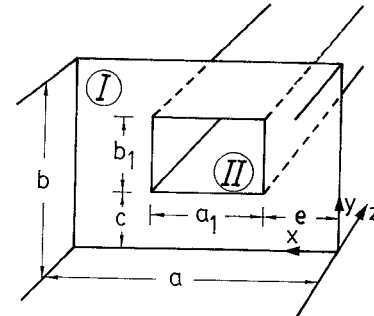


Fig. 1. Double-step discontinuity in rectangular waveguide.

introduces an E_x component through, e.g., the $TE_{1,2}$ and the $TM_{1,2}$ modes. On the other hand, field computations using the TE_{mn} – TM_{mn} approach have indicated that this E_x component is fairly small, and by neglecting this component no significant error is introduced. This assumption has already been used in [18] and [19] to characterize E -plane discontinuities for diplexer applications. In these papers, a TE_{mn}^x approach has been utilized which automatically excludes the existence of an E_x component. The advantage of this approach over the TE_{mn} – TM_{mn} approach is that in the latter case many higher order terms in the continuity equations are necessary to approximate a small or even zero E_x -field component. In the TE_{mn}^x approach, the E_x field is assumed to be zero, allowing much faster converging results. In other words, the number of modes required to model the field at the discontinuity is reduced to N_{TE} , compared with $M_{TE} + P_{TM}$ for the TE–TM mode case [17] (in general, $M > N \approx P$). This significantly speeds up the calculations and, at the same time, reduces the storage requirements.

This procedure has been successfully applied in some special cases [8], [17]–[19]. In cases, however, where resonant effects occur in the discontinuity plane, conflicting results with the TE_{mn} – TM_{mn} mode approach have been observed. This discrepancy is due to the fact that in the TE_{mn}^x mode approach, the resulting equation system contains two unknowns but, rigorously, requires three field components (E_y, H_x, H_y) to be matched. In many applications, neglecting the H_y component in the matching conditions does not lead to wrong results. In other cases, however, it does. Therefore, this paper focuses on a

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modified TE_{mn}^x mode approach and its comparison with results obtained by the conventional TE-to- x and the generalized TE-TM mode procedure. In particular, the three different analysis methods are:

Method 1: The conventional TE_{mn}^x mode method, where only E_y and H_x are matched.

Method 2: The modified TE_{mn}^x mode procedure matching E_y and H_x or H_y alternatively.

Method 3: The generalized TE-TM mode analysis matching the field components E_x, E_y, H_x, H_y .

The excellent agreement between the modified TE_{mn}^x routine (method 2), the TE-TM mode analysis (method 3), and measurements is demonstrated for the examples of resonant iris filters and corrugated waveguide polarizers.

II. THEORY

In this section, only the basic steps of the modified TE_{mn}^x mode formulation are presented. For details on the generalized TE_{mn} - TM_{mn} mode field description and the corresponding modal scattering matrix calculation, the reader is referred to [20].

In the case of the TE_{mn}^x mode representation, the transverse field components in region $i = I, II$ (c.f. Fig. 1),

$$\begin{aligned} E_y^i &= \frac{\partial}{\partial z} A_{hx}^i \\ H_x^i &= \frac{j}{\omega\mu_0} \left[k_0^2 + \frac{\partial^2}{\partial x^2} \right] A_{hx}^i \\ H_y^i &= \frac{j}{\omega\mu_0} \frac{\partial^2}{\partial x \partial y} A_{hx}^i \end{aligned} \quad (1)$$

are derived from the x component of a vector potential

$$\begin{aligned} A_{hx}^i &= 2 \sum_{q=1}^{\infty} \sqrt{\frac{\omega\mu_0/k_{zq}^i}{F_{\square}^i [k_0^2 - (k_{xm}^i)^2]}} T_{mn}^i(x, y) \\ &\quad \cdot (V_q^i e^{-jk_{zq}^i z} - R_q^i e^{+jk_{zq}^i z}) \end{aligned} \quad (2)$$

where V_q^i and R_q^i are the wave amplitudes traveling in the positive and negative z direction, respectively, and F_{\square}^i equals $a \cdot b$ (region I) or $a_1 \cdot b_1$ (region II). The index q is related to combinations (m, n) of the waveguide cross-section functions

$$\begin{aligned} T_{mn}^I(x, y) &= \sin \left\{ (2m-1) \frac{\pi}{a} x \right\} \cos \left\{ \frac{2n\pi}{b} y \right\} / \sqrt{1 + \delta_{on}} \\ T_{mn}^{II}(x, y) &= \sin \left\{ (2m-1) \frac{\pi}{a_1} (x-e) \right\} \\ &\quad \cdot \cos \left\{ \frac{2n\pi}{b_1} (y-c) \right\} / \sqrt{1 + \delta_{on}} \end{aligned} \quad (3)$$

(δ_{on} = Kronecker delta)

by rearranging them with respect to increasing cutoff frequencies, e.g.,

$$k_{zq}^i = k_{zmn}^i = \sqrt{k_0^2 - (k_{xm}^i)^2 - (k_{yn}^i)^2}. \quad (4)$$

The square root function in (2) is determined by a power normalization for each mode:

$$P_q^i = \begin{cases} 1W & \text{propagating mode} & [k_0^2 > (k_{xm}^i)^2 + (k_{yn}^i)^2] \\ -jW & \text{evanescent mode} & [k_0^2 > (k_{xm}^i)^2] \\ +jW & \text{evanescent mode} & [k_0^2 < (k_{xm}^i)^2] \end{cases} \quad (5)$$

which shows that the TE_{mn}^x formulation presented here satisfies the same conditions in terms of real and reactive power as required for the TE_{mn} - TM_{mn} mode description (c.f. [20]).

Matching the field components of (1) at the common interface of regions I and II and truncating the infinite sums in (2) yields three matrix equations:

$$E_y: \quad V^I + R^I = L_E(V^{II} + R^{II}) \quad (6)$$

$$H_x: \quad L_{H_x}(V^I - R^I) = V^{II} - R^{II} \quad (7)$$

$$H_y: \quad L_{H_y}(V^I - R^I) = V^{II} - R^{II} \quad (8)$$

with the matrix elements given by

$$(L_E)_{pq} = \frac{4}{\sqrt{F_{\square}^I F_{\square}^{II}}} \sqrt{\frac{k_{zq}^{II} [k_0^2 - (k_{xp}^I)^2]}{k_{zp}^I [k_0^2 - (k_{xq}^{II})^2]}} \int_{F_{\square}^{II}} T_p^I T_q^{II} dF \quad (9)$$

$$(L_{H_x}) = (L_E)^T \quad T = \text{transposed} \quad (10)$$

$$\begin{aligned} (L_{H_y})_{qp} &= \frac{4}{\sqrt{F_{\square}^I F_{\square}^{II}}} \sqrt{\frac{k_{zq}^{II} [k_0^2 - (k_{xq}^{II})^2]}{k_{zp}^I [k_0^2 - (k_{xp}^I)^2]}} \frac{1}{(k_{xq}^{II})^2 (k_{xp}^I)^2} \\ &\quad \cdot \int_{F_{\square}^{II}} \frac{\partial}{\partial x \partial y} T_p^I \frac{\partial}{\partial x \partial y} T_q^{II} dF. \end{aligned} \quad (11)$$

To calculate the modal scattering matrix of the double-step discontinuity,

$$\begin{bmatrix} R^I \\ V^{II} \end{bmatrix} = (S) \begin{bmatrix} V^I \\ R^{II} \end{bmatrix} \quad (12)$$

only two equations of the set (6)–(8) are needed to separate the two unknowns R^I and V^{II} . Matching E_y and H_x and ignoring H_y leads to excellent results in some special cases, e.g. [18] and [19], but leads to wrong results for resonating double-step discontinuities. On the other hand, merely considering E_y and H_y for the matching conditions or, alternatively, H_x and H_y lacks information in the presence of TE_{m0}^x modes.

Therefore, the following modified TE_{mn}^x procedure is proposed here. First, E_y is matched using (6) and (9), and matrices L_{H_x} and L_{H_y} are calculated according to (10)

and (11), respectively. A new matrix (L_H) is then formed by copying elements from either L_{Hx} or L_{Hy} into L_H where

$$\begin{aligned}
 (L_H)_{qp} &= (L_{Hx})_{qp} && \text{if mode } q \text{ or mode } p \\
 &&& \text{is a TE}_{m0}^x \text{ type.} \\
 (L_H)_{qp} &= (L_{Hy})_{qp} && \text{if neither mode } q \text{ nor mode } p \\
 &&& \text{is a TE}_{m0}^x \text{ type.} \quad (13)
 \end{aligned}$$

Consequently, L_H contains data which are derived from matching conditions of either H_x or H_y . Hence this procedure merges (7) and (8) into one equation and allows the resulting matrix system (6)–(8) to be solved for its unknowns R^I and V^{II} .

The modal scattering matrix of the double-step discontinuity

$$\begin{aligned}
 S_{11} &= [L_E L_H + U]^{-1} [L_E L_H - U] \\
 S_{12} &= 2[L_E L_H + U]^{-1} L_E \\
 S_{21} &= L_H [U - S_{11}] \\
 S_{22} &= U - L_H S_{12}, \quad U = \text{unit matrix} \quad (14)
 \end{aligned}$$

calculated under the condition (13) shows excellent agreement with measured data. Compared with the TE–TM analysis, this new procedure considerably reduces the matrix sizes to be processed by the computer. Moreover, the fact that a smaller number of modes interact between discontinuities than in the discontinuity plane itself can be easily implemented in this approach. By these measures, the processing time of the algorithm can be made up to five times faster. (For further derivations leading to the modal scattering matrix of cascaded discontinuities, the reader is referred to [20] and [21].)

III. RESULTS

Waveguide iris filters can operate in two different modes. In the first category, half-wave resonators are connected by irises which act as coupling elements only. The transmission behavior of a single iris is shown in Fig. 2. The three methods investigated yield identical results within the plotting accuracy. Therefore, excellent agreement is usually obtained by all three methods when compared with measured data of an iris-coupled half-wave resonator filter. This is demonstrated in Fig. 3 using measurements carried out in [5]. However, the TE_{mn}^x procedures (methods 1 and 2) require only 30% of the CPU time needed in the generalized TE–TM analysis (method 3).

In the second category, the iris filter utilizes the resonances of the irises itself rather than waveguide resonators. In this mode, the irises are required to operate above cutoff, which occurs at 37.5 GHz in the example of Fig. 4. Exactly at this frequency, the first method reveals some instabilities due to the lack of H_y field matching, whereas the other two methods are in good agreement in modeling the resonance effect of the iris. Fig. 5 shows a comparison with measured data on this type of iris filter

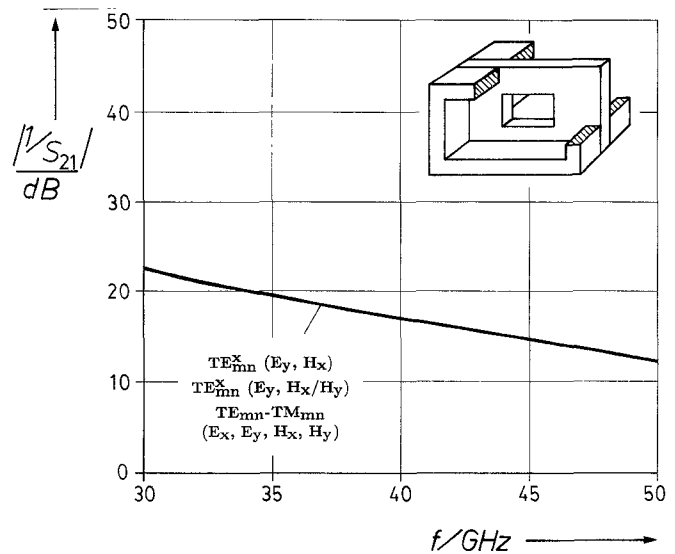


Fig. 2. Insertion loss of a nonresonant iris in rectangular waveguide ($a = 2b = 7.112$ mm, $a_1 = b_1 = 2$ mm, iris thickness = 0.5 mm). Results of all methods investigated within plotting accuracy.

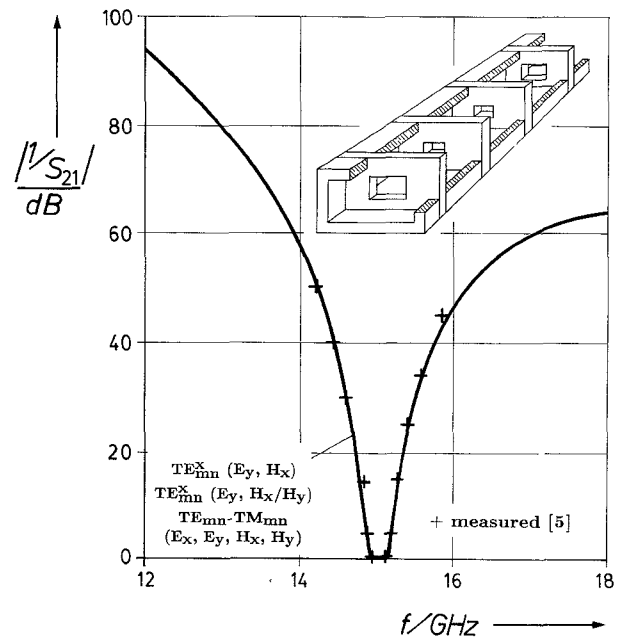


Fig. 3. Insertion loss of three-resonator iris-coupled *Ku*-band waveguide filter according to Tucholke [5] (— theoretical results obtained by all three methods; ++ measured [5]).

presented by Chen [2]. As expected from the results of Fig. 4, the conventional TE_{mn}^x analysis matching E_y and H_x (method 1) leads to incorrect results for the filter insertion (and return) loss, while, surprisingly enough, the bandwidth and the stopband attenuation are almost identical to the other two methods. Methods 2 and 3 show identical results, which closely agree with measurements. In terms of the CPU time required, the ratio of 1:5 in favor of the new TE_{mn}^x analysis (method 2) clearly demonstrates its advantage over the generalized TE–TM procedure.

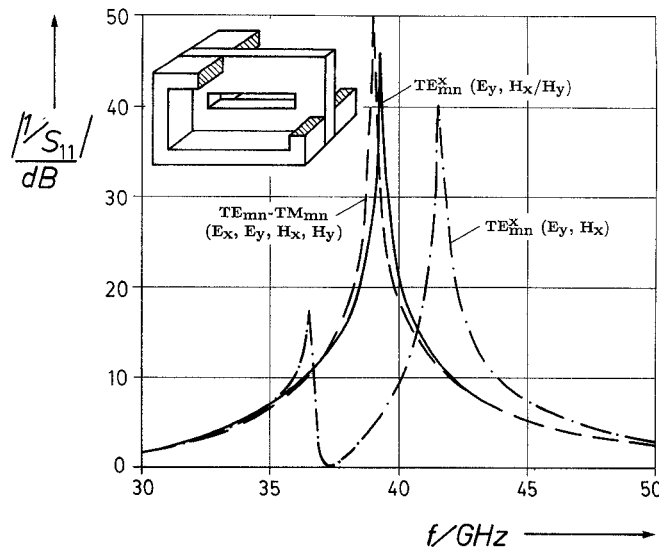


Fig. 4. Return loss of resonant iris in rectangular waveguide ($a = 2b = 7.112$ mm, $a_1 = 4$ mm, $b_1 = 1$ mm, iris thickness = 0.5 mm). Methods of analysis using \cdots — TE_{mn}^x modes matching E_y, H_x (method 1); \cdots — TE_{mn}^x modes matching $E_y, H_x/H_y$ (method 2); \cdots — $TE-TM$ modes matching E_x, E_y, H_x, H_y (method 3).

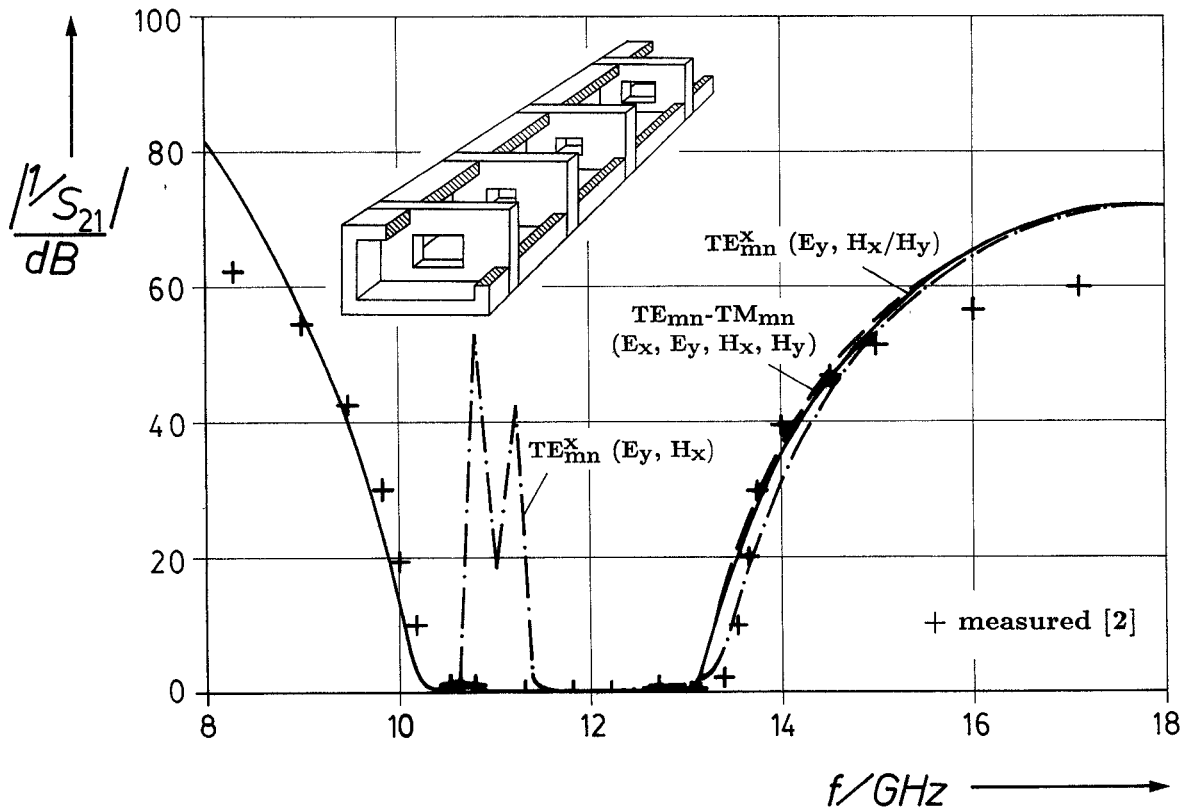


Fig. 5. Comparison between theories and measurements for the example of an X-band seven resonant-iris waveguide filter according to Chen [2] (+ + measured [2]; sets of modes as in Fig. 4).

Finally, the new approach is tested against measured data of E -plane corrugated waveguide square polarizers. Although the discontinuities involved are not double-plane steps in general, both discontinuity planes are involved when the transmission phase difference between a TE_{01} and the orthogonal TE_{10} mode excitation is calculated.

Since the structure consists of 22 capacitive (TE_{10}) or inductive (TE_{01}) irises, phase relations are very critical. Fig. 6(a) shows a comparison with phase difference measurements presented by Dewey [11]. The agreement is extremely close, thereby verifying the phase accuracy of this new method. Moreover, the calculated input return

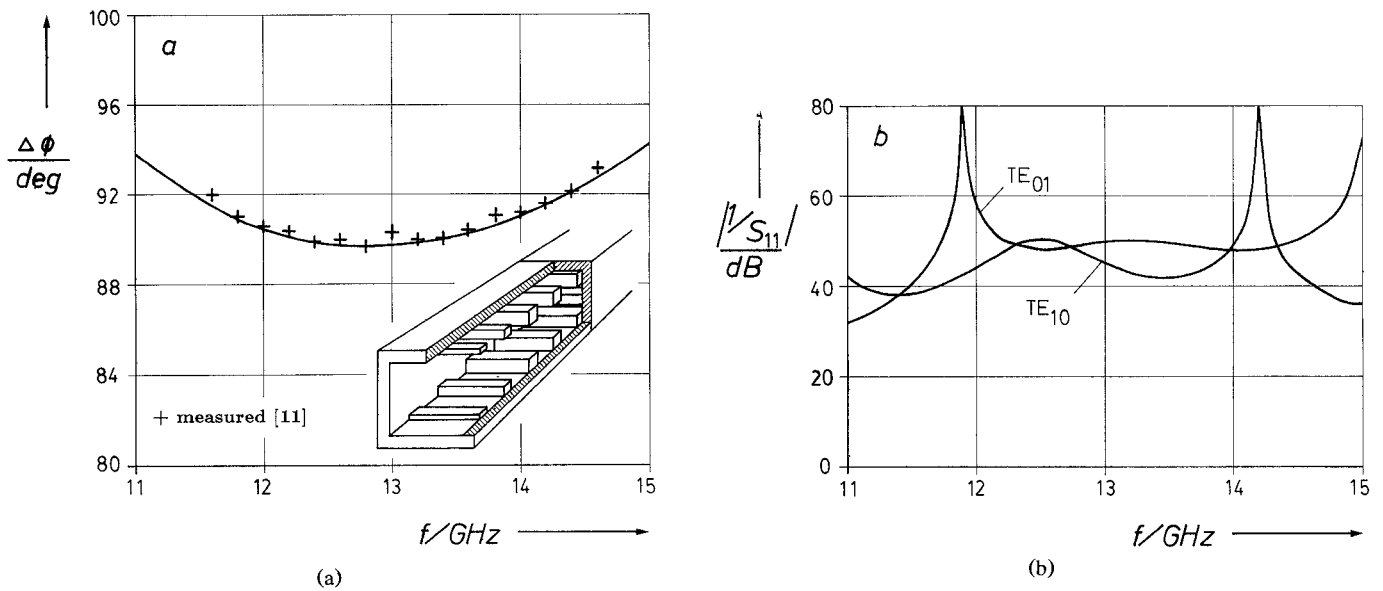


Fig. 6. E -plane corrugated waveguide polarizer according to Dewey [11] (— TE_{mn}^x mode theory with $E_y, H_x/H_y$ matching (method 2)). (a) Differential phase shift $\Delta\phi = \phi(TE_{01}) - \phi(TE_{10})$ (+ + measured [11]). (b) Calculated input return loss for TE_{01} and TE_{10} mode excitation.

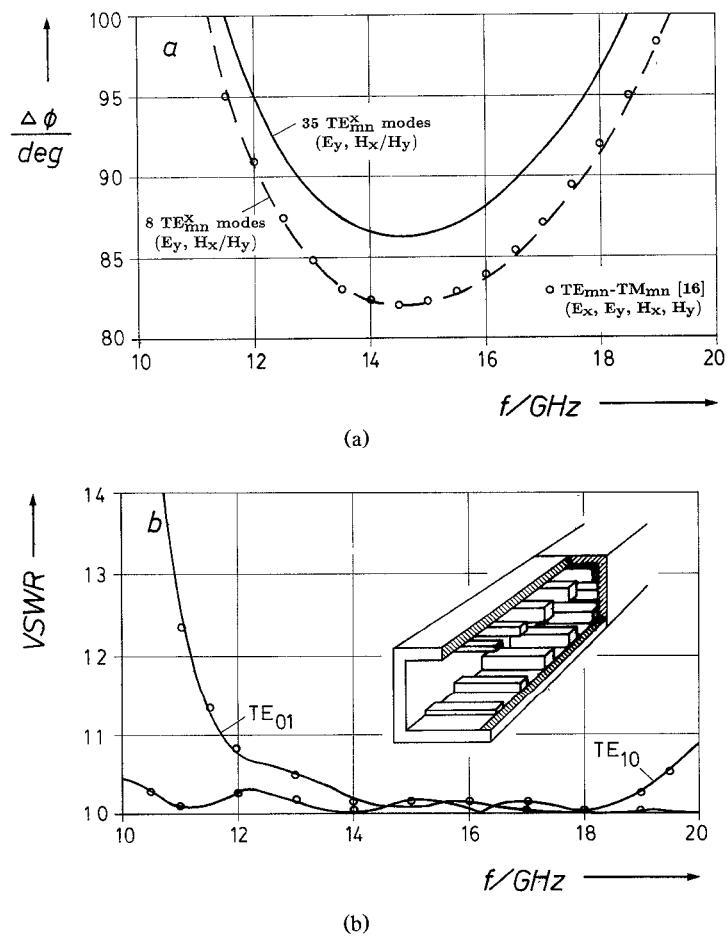


Fig. 7. Polarizer design with linearly tapered corrugation according to [16]; methods of analysis using 8 (----) and 35 (—) TE_{mn}^x modes matching $E_y, H_x/H_y$ (method 2); $\circ\circ\circ$ $TE_{mn}-TM_{mn}$ modes matching E_x, E_y, H_x, H_y (method 3) [16]. (a) Differential phase shift. (b) VSWR.

loss values of TE_{10} and TE_{01} mode excitation (Fig. 6(b)) are within the measured margins specified in [11]: return loss (TE_{10}) better than 36.5 and 40 dB for 12–12.5 GHz and 14–14.5 GHz, respectively; return loss (TE_{01}) better than 34 and 40 dB in the same frequency ranges. Nearly identical results are obtained with a TE–TM mode analysis, as presented in [16].

Fig. 7(a) shows the differential phase shift of a linearly tapered corrugated polarizer which has been designed using the TE–TM mode analysis [16]. Although calculations with eight TE_{mn}^x modes are in good agreement with the performance given in [16], computations using 35 modes as a result of a convergence analysis reveal deviations of more than 4° . (Design tolerances for this component are usually limited to $\pm 1^\circ$ [11], [14].) It is believed that this discrepancy is caused by relative convergence phenomena of the generalized TE–TM mode analysis (method 3). Particularly in the case of a TE_{10} mode excitation, this method matches also the (vanishing) E_x field component in the discontinuity plane [16] and, consequently, requires a considerably larger number of modes than the TE_{mn}^x procedure where the $E_x = 0$ condition is directly incorporated. Therefore, the results of Fig. 7(a) also imply a convergence improvement in favor of the TE_{mn}^x procedure compared with the generalized TE–TM mode analysis. The calculation of the VSWR behavior (Fig. 7(b)) is relatively insensitive to the number of modes. The results of an eight and a 35 mode analysis are within the plotting accuracy and agree closely with the data presented in [16].

IV. CONCLUSIONS

A modified TE_{mn}^x mode approach for waveguide double-plane discontinuity modeling is presented and compared with alternative mode matching techniques. The applicability of the new approach matching E_y and H_x or H_y field components alternatively is demonstrated, whereas mode matching utilizing only E_y and H_x field components turned out to be unsuitable for general applications. With storage savings of more than 50% and CPU time reductions down to 20% with maintained accuracy and improved phase convergence compared with the generalized TE–TM procedure, the new TE_{mn}^x mode method offers an attractive solution for the numerically efficient yet accurate computer-aided design of waveguide components involving double-step discontinuities.

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