Singular Value Decomposition Improves Accuracy and Reliability of T-Septum Waveguide Field-Matching Analysis

Vladimir A. Labay and Jens Bornemann

Laboratory for Lightwave Electronics, Microwaves and Communications (LLiMic), Department of Electrical and Computer Engineering, University of Victoria, Victoria, British Columbia, Canada

Received June 10, 1991; revised August 26, 1991.

ABSTRACT

A singular value decomposition (SVD) is used to solve the characteristic matrix equation of 2 different field-matching analyses as applied to T-septum waveguide structures. It is demonstrated that SVD avoids the need for complicated numerical zero-detection algorithms, eliminates the major disadvantages of the classical field-matching technique, and hence, improves the accuracy and reliability of the computed results. The values obtained are in close agreement with available theoretical data and measurements.

I. INTRODUCTION

Numerical techniques, as applied to the 2-dimensional problem of passive microwave and millimeter-wave structures [1, 2] require the solution of the related homogeneous matrix equation. The accuracy and reliability of most of the currently used techniques suffer from the presence of poles and extremely steep-gradient zeros in the determinant function [3–5]. Problems are associated with overlooking zeros (solutions of the characteristic equation), with inaccuracy in the vicinity of zeros, and with poles located extremely close to zeros.

For configurations with a rectangular waveguide enclosure, 2 types of mode-matching approaches are commonly applied to solve for propagation constants, cutoff frequencies, and characteristic impedances: first, the classical formulation which assumes standing wave patterns in the cross-section's subregions and, therefore, introduces boundary conditions prior to subregion interface relations [4,6] (we will call this method standing-wave formulation); second, a method based on transversely propagating waves where the boundaries are incorporated as resonance conditions in the very last step of the formulation [1,7] (transverse-resonance method). The standing wave formulation has the advantage of independently selecting the number of expansion terms in the separate subregions but suffers from poles in the determinant function [8]. Whereas, in comparison, the transverse resonance method has a determinant free of poles, but is restricted to equal numbers of expansion terms in each subregion which causes numerical instabilities [8].

Therefore, this article focuses on a singular value decomposition technique [9] in order to eliminate the problems related to the behavior of the determinant function. For the above-mentioned mode-matching procedures, 2 methods to numerically solve the characteristic matrix equation are investigated: first, the conventional method which searches for the zeros of the system

CCC 1050-1827/92/020082-08$04.00

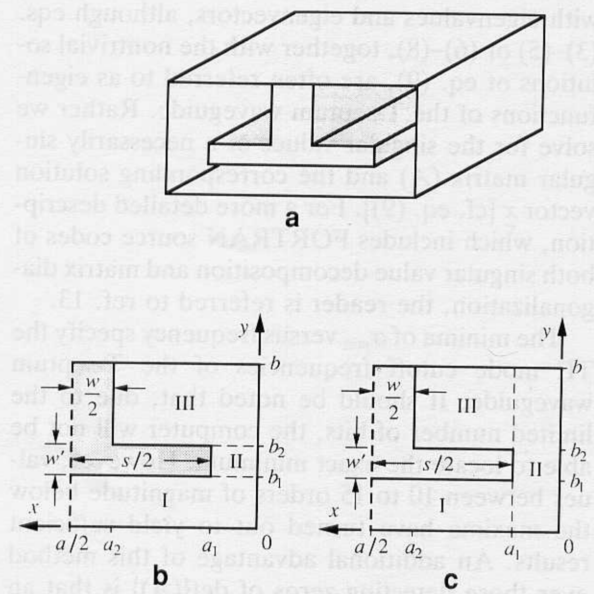


Figure 1. T-septum waveguide: (a) geometry; (b) dimensions and subregions for transverse-resonance method; (c) dimensions and subregions for standing-wave formulation.

determinant; and second, a method looking for the minima of the lowest singular value of the characteristic matrix. It is found that by using the singular value decomposition technique on the T-septum waveguide eigenvalue problem [3,10–12], the necessity of handling poles and extreme gradients as observed in the determinant functions can be completely avoided.

In terms of accuracy and reliability, this contributes to a significant improvement of the computer-aided analysis. Moreover, it makes the standing-wave formulation the superior technique of the 2 field-matching procedures. By circumventing the need for the detection of poles and zeros in the determinant function, this method's flexibility of independently selecting the numbers of expansion terms in different subregions of the T-septum waveguide (cf. Fig. 1) offers a clear advantage, particularly if small gap widths are involved.

II. THEORY

The T-septum waveguide structure is shown in Figure 1a. Without sacrificing the generality of the singular value decomposition technique, we will restrict the demonstration of the method to the computation of TE mode characteristics. The electromagnetic field in regions $i = \mathrm{I, II}$, and III (cf Fig. 1b and c)

$$\vec{E}^i = -\nabla \times \left\{ \vec{e}_z \sqrt{\frac{\omega\mu_o}{k_z}}\, \Psi^i(x, y) e^{-jk_z z} \right\} \quad (1)$$

$$\vec{H}^i = \frac{1}{j\omega\mu_o} \nabla \times \nabla \times \left\{ \vec{e}_z \sqrt{\frac{\omega\mu_o}{k_z}}\, \Psi^i(x, y) e^{-jk_z z} \right\} \quad (2)$$

is derived from cross-section functions (Fig. 1b)

$$\Psi^I(x, y) = \sum_{n=1}^{N} \cos\left\{(2n-1)\frac{\pi}{a}x\right\} \cdot \left(A_n^I e^{-jk_{yn}^I y} + B_n^I e^{+jk_{yn}^I y}\right) \quad (3)$$

$$\Psi^{II}(x, y) = \sum_{n=0}^{N-1} \frac{\cos\left(\frac{n\pi}{a_1}x\right)}{\sqrt{1+\delta_{on}}} \cdot \left(A_n^{II} e^{-jk_{yn}^{II} y} + B_n^{II} e^{+jk_{yn}^{II} y}\right) \quad (4)$$

$$\Psi^{III}(x, y) = \sum_{n=0}^{N-1} \frac{\cos\left(\frac{n\pi}{a_2}x\right)}{\sqrt{1+\delta_{on}}} \cdot \left(A_n^{III} e^{-jk_{yn}^{III} y} + B_n^{III} e^{+jk_{yn}^{III} y}\right) \quad (5)$$

or alternatively (Fig. 1c)

$$\Psi^I(x, y) = \sum_{m=0}^{M-1} A_m^I \frac{\sin\left\{k_{xm}^I\left(x - \frac{a}{2}\right)\right\}}{k_{xm}^I} \frac{\cos\left(\frac{m\pi}{b_1}y\right)}{\sqrt{1+\delta_{om}}} \quad (6)$$

$$\Psi^{II}(x, y) = \sum_{n=0}^{N-1} A_n^{II} \cos(k_{xn}^{II}x) \frac{\cos\left(\frac{n\pi}{b}y\right)}{\sqrt{1+\delta_{on}}} \quad (7)$$

$$\Psi^{III}(x, y) = \sum_{p=0}^{P-1} A_p^{III} \cos\{k_{xp}^{III}(x - a_2)\} \frac{\cos\left\{\frac{p\pi}{b-b_2}(y - b_2)\right\}}{\sqrt{1+\delta_{op}}} \quad (8)$$

where δ_{on} is the Kronecker delta. Eqs. (3)–(5) correspond to the transverse resonance method in $\vec{e}_y$ direction, whereas the classical technique using standing waves in both $\vec{e}_x$ and $\vec{e}_y$ directions is described by eqs. (6)–(8). It should be noted that the transverse-resonance method cannot be applied in $\vec{e}_x$ direction, since the left-hand side boundaries of subregions I and III (Fig. 1b and c) have different coordinates with respect to x. In comparison, the standing wave formulation is extremely flexible with respect to different patterns of subregion selection, since boundary conditions are always immediately incorporated in the cross-section functions [cf eqs. (6)–(8) and Fig. 1c].

By matching the tangential field components at the subregion interfaces and, in the case of eqs. (3)–(5), inserting the electric walls at $y = 0, b$ as resonance conditions [7], the characteristic equations of both formulations are of the form

$$(\underline{A})\underline{x} = 0 \tag{9}$$

The zeros of the determinant of $(\underline{A})$ specify the TE mode cutoff frequencies, and vector $\underline{x}$ holds elements related to the amplitude coefficients in eqs. (3)–(5) or (6)–(8).

As will be shown in the next section, the reliability of detecting a given number of successive zeros of $\det\{(\underline{A})\}$, which is absolutely required for, e.g., bandwidth calculations and 3-dimensional modal analysis [1,2,8], might be questionable owing to the presence of poles and extremely steep-gradient zeros in the determinant function. In order to avoid the numerical problems associated with the detection of the zeros, the utilization of singular value decomposition (SVD) [9] is proposed. Matrix $(\underline{A})$ is decomposed into three matrices

$$(\underline{A}) = (\underline{W})(\underline{S})(\underline{V})^T \tag{10}$$

where T means transposed, $(\underline{S})$ is a diagonal matrix formed by the singular values in decreasing order, and the columns of $(\underline{W})$ and $(\underline{V})$ are the left and right singular vectors of $(\underline{A})$, respectively [9].

$$\sigma_{\min} = 0 \text{ if and only if } \det\{(\underline{A})\} = 0 \tag{11}$$

$\sigma_{\min}$ is the last element of $(\underline{S})$, and the last column of $(\underline{V})$ contains the elements of the corresponding solution vector $\underline{x}$. It should be mentioned that this procedure differs from matrix diagonalization techniques, e.g., $(\underline{A}) = (\underline{P})^T \operatorname{diag}\{\lambda_i\}(\underline{P})$, which are commonly used to find the eigenvalues λ of a nonsingular matrix in the form $\det\{(\underline{A}) - \lambda(\underline{I})\} = 0$. In this article, we are not concerned with eigenvalues and eigenvectors, although eqs. (3)–(5) or (6)–(8), together with the nontrivial solutions of eq. (9), are often referred to as eigenfunctions of the T-septum waveguide. Rather we solve for the singular values of a necessarily singular matrix $(\underline{A})$ and the corresponding solution vector $\underline{x}$ [cf. eq. (9)]. For a more detailed description, which includes FORTRAN source codes of both singular value decomposition and matrix diagonalization, the reader is referred to ref. 13.

The minima of $\sigma_{\min}$ versus frequency specify the TE mode cutoff frequencies of the T-septum waveguide. It should be noted that, due to the limited number of bits, the computer will not be able to locate the exact minimum. However, values between 10 to 15 orders of magnitude below the maxima have turned out to yield sufficient results. An additional advantage of this method over those detecting zeros of $\det\{(\underline{A})\}$ is that an increase of the minima of $\sigma_{\min}$ indicates a reduced accuracy in the computation. Therefore, the determination of an optimum number of expansion terms can easily be incorporated (cf. ref. 14).

Up to now, no shortcomings of the SVD procedure have been experienced. Although the CPU time required for one singular value decomposition is slightly higher than that of one determinant calculation, the overall time to find a given number of successive cutoff frequencies of the T-septum waveguide [solutions of eq. (9)] is reduced by a factor of 3 to 4 on the average. This is due to the absence of poles and the generally moderate shape of the singular value function, as will be demonstrated in the next section.

III. RESULTS

Figure 2a and b show the typical behavior of the system determinant in eq. (9) for the transverse-resonance method and the standing-wave formulation, respectively. A logarithmic scale is chosen for the ordinate, in order to account for the many orders of magnitude in determinant value variation. The linear-to-logarithmic conversion used is

$$\det(\cdot)/\log \text{ scale} = a \operatorname{sgn}\{\det(\cdot)\} \log_{10}\{1 + |\det(\cdot)|\} \tag{12}$$

where a is a scaling factor, sgn is the signum function, and $\det(\cdot)$ denotes the determinant of $(\underline{A})$ obtained by the corresponding numerical model. In both cases, the zeros, which correspond to the

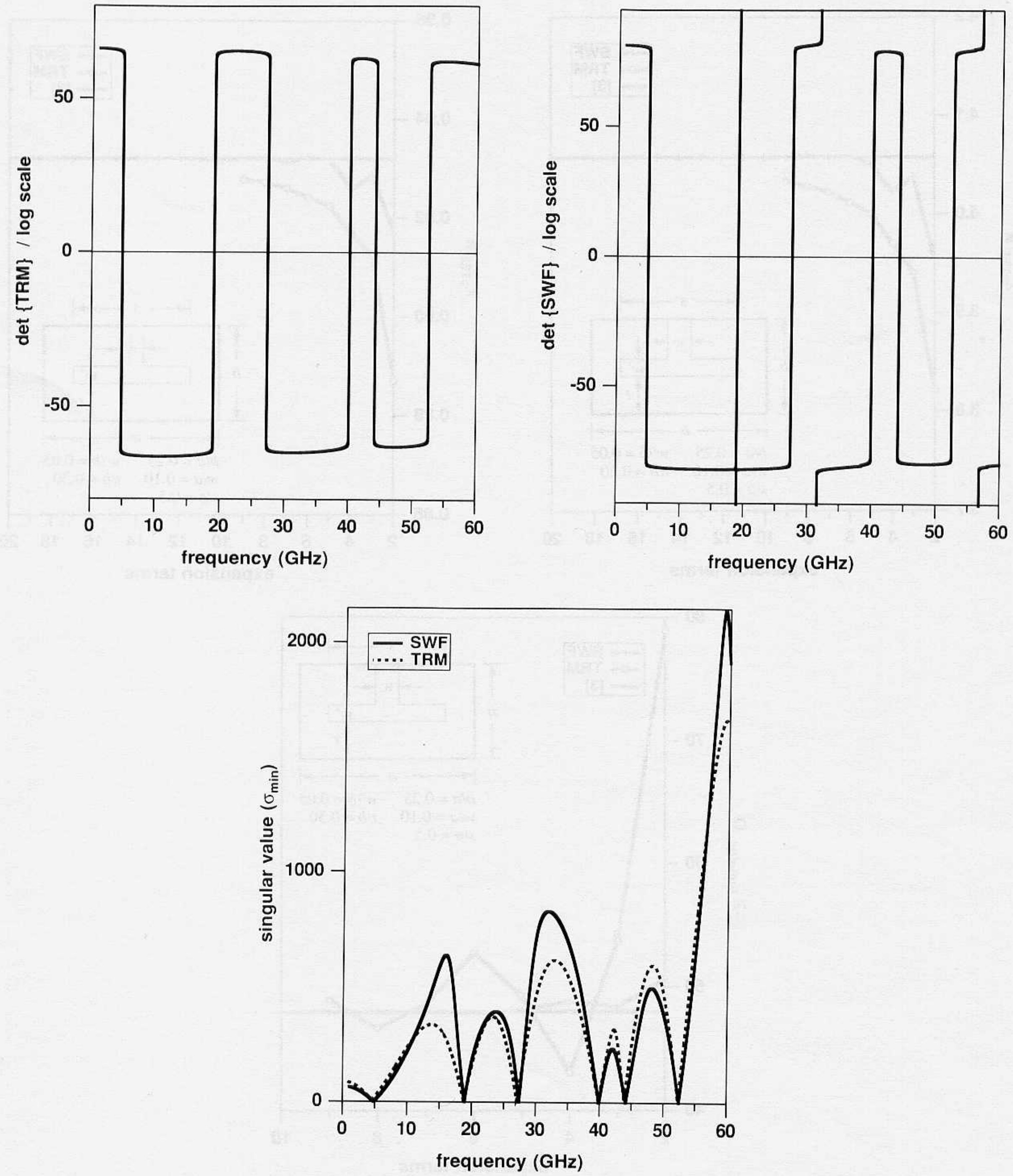


Figure 2. Typical behavior of system determinant versus frequency: (a) transverse-resonance method (TRM); (b) standing-wave formulation (SWF); (c) minimum singular value for the transverse resonance method and the standing wave formulation. Dimensions (cf. Fig. 1): a = 15.799 mm, $b = a_1$ = 0.25a, a_2 = 0.45a, b_1 = 0.35b, b_2 = 0.4b.

TE mode cutoff frequencies of a T-septum waveguide structure, are extremely difficult to detect. This is even more true for the standing-wave formulation, where poles can be extremely close to zeros (cf. Fig. 2b at 18.8 GHz). Figure 2c demonstrates that by applying the singular value decomposition technique, these problems can be completely avoided. Here, the minima of the smallest singular value correspond to the zeros in Figure 2a and b, and can easily be located by a

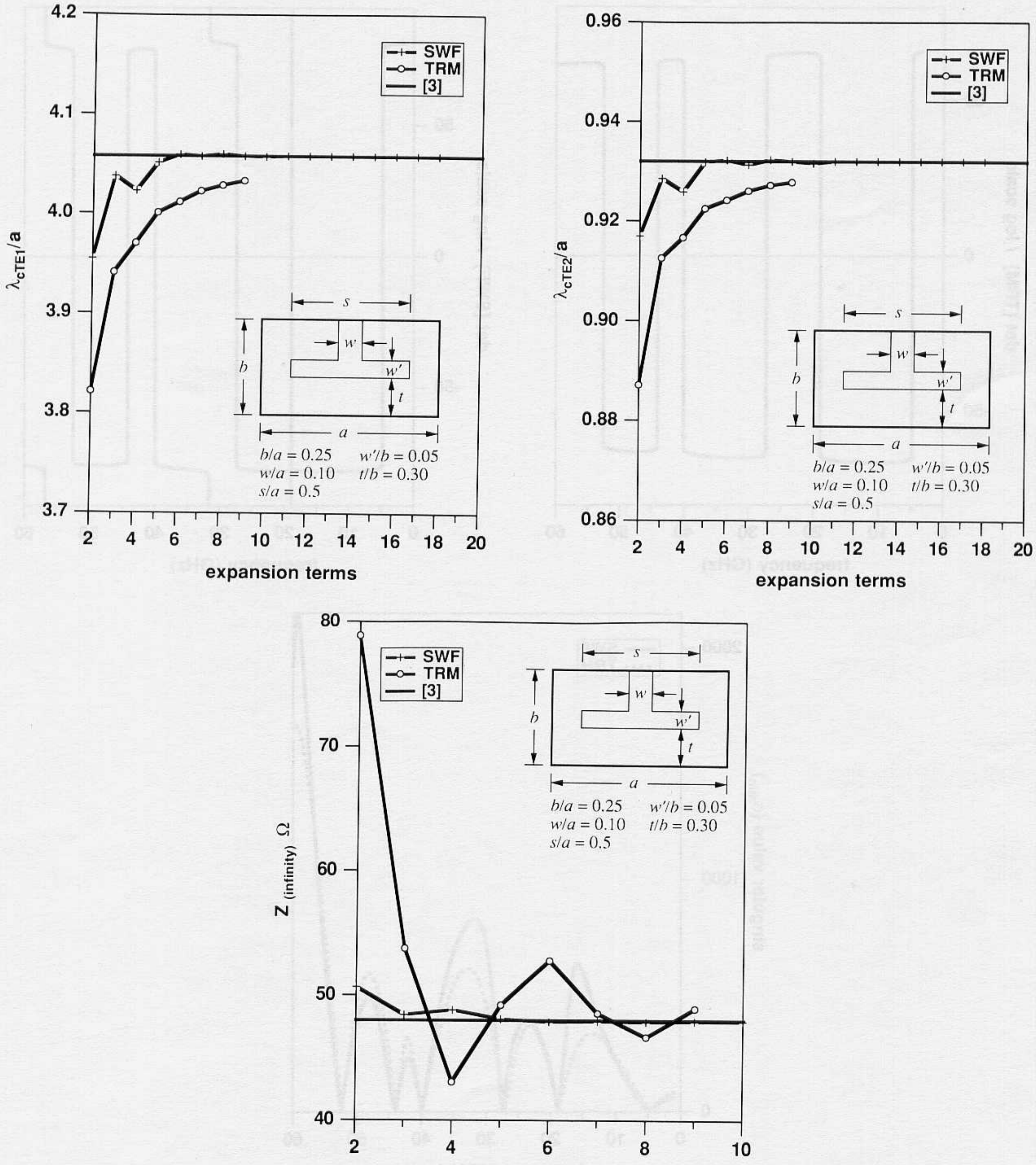


Figure 3. Convergence analysis and comparison with ref. 3: (a) normalized cutoff wavelength of fundamental mode; (b) normalized cutoff wavelength of first higher-order mode; (c) characteristic impedance (identification of numerical methods as in Fig. 2).

minimum search algorithm. The method is also capable of detecting numerical inaccuracies, in which cases the minima would no longer be close to the abscissa but would move upward with increasing inaccuracy.

Figure 3 shows a convergence analysis for the 2 different mode-matching techniques using singular value decomposition for the computation of both the cutoff wavelength and the characteristic impedance. Due to the restriction of identical number of expansion terms in all subregions, the transverse resonance method can only be used up to $N = 9$ expansion terms per subregion [cf. eqs. (3)–(5)] for this structure. For the standing-wave formulation, the comparable number N of expansion terms is that in region II [cf. eqs. (6)–(8) and Fig. 1c]. The values for M and P in regions I and II, respectively, are then chosen according to the dimensions of the subregions (cf. ref. 1, ch. 9). Convergence of the first (Fig. 3a) and second (Fig. 3b) normalized cutoff wavelength is obtained for more than 11 expansion terms. Fig. 3c shows the related plot for the characteristic impedance as defined in ref. 3. Excellent convergence and agreement with data in ref. 3 is obtained with the standing-wave formulation, whereas the transverse-resonance method performs rather poorly. This is mainly due to the different subregion divisions in Fig. 1b and c. As pointed out earlier, however, the transverse-resonance method is not applicable in $\vec{e}_x$ direction. It should also be noted that beyond 6 expansion terms, the transverse-resonance method in Figure 3c requires a mainframe extended precision compiler as opposed to the standing-wave formulation, which is operational on modern double precision workstations.

When using the SVD algorithm for both field-matching techniques, the main advantage of the standing-wave formulation over the transverse resonance method, namely, the flexibility of selecting different subregion expansion term ratios, becomes obvious, particularly for small gap width. This is demonstrated in Figure 4. As the T-septum width, s, is increased, the transverse resonance method (Fig. 4a) experiences numerical problems as a result of the many terms that have to be considered in subregion II (cf. Fig. 1b). Reducing the number of expansion terms down to 4 slightly alleviates the problem, but leads to increased deviations from the reference values. With the flexible expansion term ratio and the singular value decomposition technique to circumvent the poles in the system determinant, the standing-wave formulation (Fig. 4b) is found to produce results in extremely close agreement with previously published data and confirms measurements, as demonstrated in Table I. Although the second cutoff frequency by Mansour and MacPhie [12] is slightly closer to the measured values, this data can neither be confirmed by this theory nor by the one presented by Zhang and Joines [3]. Deviations in characteristic impedance are due to measurement procedures described in ref. 3.

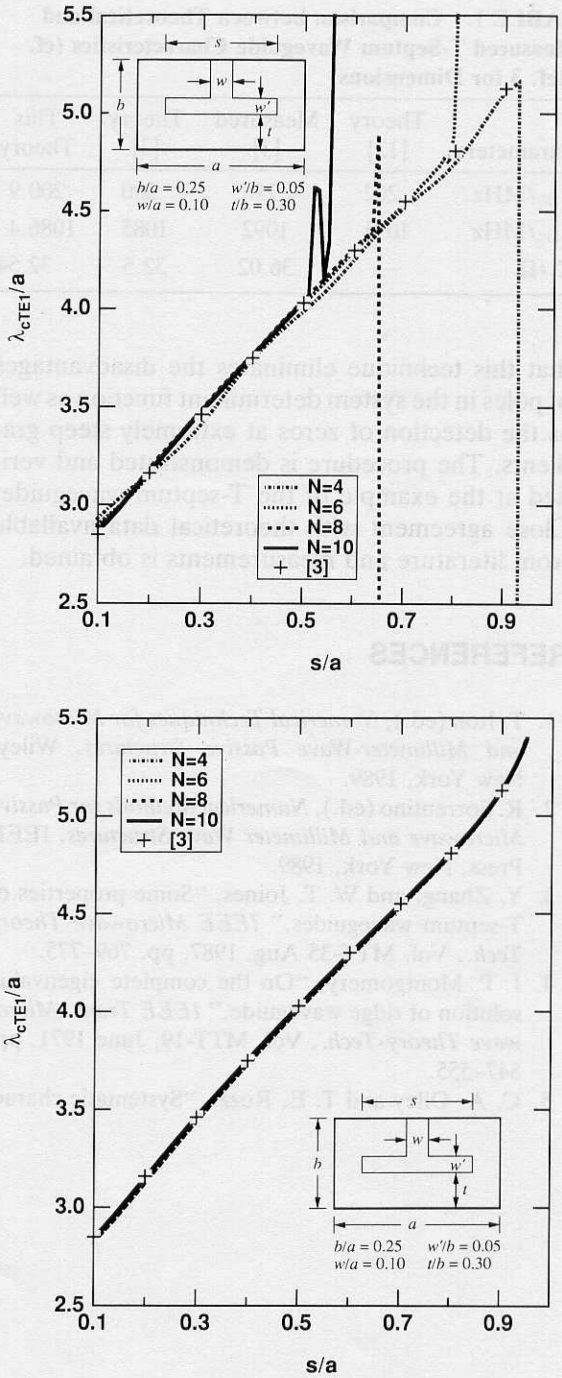


Figure 4. Normalized cutoff wavelength of T-septum waveguide as a function of normalized septum width: (a) transverse-resonance method; (b) standing-wave formulation.

IV. CONCLUSION

A singular value decomposition method is applied to improve the accuracy and reliability of T-septum waveguide field-matching analysis. It is found

TABLE I. Comparison between Theoretical and Measured T-Septum Waveguide Characteristics (cf. Ref. 3 for Dimensions)

Parameters	Theory [12]	Measured [3]	Theory [3]	This Theory
f_{cTE_1}/MHz	202	216	200	200.9
f_{cTE_2}/MHz	1091	1092	1085	1086.4
Z_π/Ω	—	36.02	32.5	32.54

that this technique eliminates the disadvantages of poles in the system determinant function as well as the detection of zeros at extremely steep gradients. The procedure is demonstrated and verified at the example of the T-septum waveguide. Close agreement with theoretical data available from literature and measurements is obtained.

REFERENCES

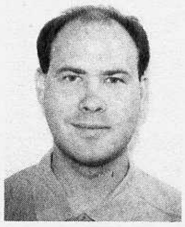
1. T. Itoh (ed.), *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*. Wiley, New York, 1989.
2. R. Sorrentino (ed.), *Numerical Methods for Passive Microwave and Millimeter Wave Structures*. IEEE Press, New York, 1989.
3. Y. Zhang, and W. T. Joines, "Some properties of T-septum waveguides," *IEEE Microwave Theory Tech.*, Vol. MTT-35 Aug. 1987, pp. 769–775.
4. J. P. Montgomery, "On the complete eigenvalue solution of ridge waveguide," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-19, June 1971, pp. 547–555.
5. C. A. Olley and T. E. Rozzi, "Systematic characterization of the spectrum of unilateral finline," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-34, Nov. 1986, pp. 1147–1156.
6. A. Wexler, "Solution of waveguide discontinuities by modal analysis," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-15, Sept. 1967, pp. 508–517.
7. R. Vahldieck and J. Bornemann, "A modified mode-matching technique and its application to a class of quasi-planar transmission lines," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-33, Oct. 1985, pp. 916–926.
8. J. Bornemann and F. Arndt, "Transverse resonance, standing wave and resonator formulation of the ridge waveguide eigenvalue problem and its application to the design of E-plane finned waveguide filters," *IEEE Trans. Microwave Theory Tech.*, Vol. 38, Aug. 1990, pp. 1104–1113.
9. R. A. Horn and C. A. Johnson, *Matrix Analysis*. Cambridge University Press, New York, 1985.
10. R. S. Elliot, "Two-mode waveguide for equal mode velocities," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-16, May 1968, pp. 282–286.
11. P. K. Saha and G. G. Mazumder, "Bandwidth characteristics of inhomogeneous T-septum waveguides," *IEEE Trans. Microwave Theory Tech.*, Vol. 37, June 1989, pp. 1021–1026.
12. R. R. Mansour and R. H. MacPhie, "Properties of dielectric-loaded T-septum waveguides," *IEEE Trans. Microwave Theory Tech.*, Vol. 37, Oct. 1989, pp. 1654–1657.
13. W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes—The Art of Scientific Computing (Fortran Version)*. Cambridge University Press, New York, 1989.
14. S. W. Lee, W. R. Jones, and J.J. Campbell, "Convergence of numerical solutions of iris-type discontinuity problems," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-19, June 1971, pp. 528–536.

BIOGRAPHY

Jens Bornemann was born in Hamburg, Germany on May 26, 1952. He received the Ing. (grad) degree in electrical engineering from the Fachhochschule Hamburg, Germany, in 1977 and the Dipl.-Ing. and Dr.-Ing. degrees, both in electrical engineering, from the University of Bremen, Germany, in 1980 and 1984, respectively.

From 1980–1983, he was a Research and Teaching Assistant in the Microwave Department at the University of Bremen, working on quasi-planar waveguide configurations and computer-aided E-plane filter design. After a 2-year period as a consulting engineer, he joined the University of Bremen again, in 1985, where he was Assistant Professor. Since April 1988, he has been an Associate Professor at the University of Victoria, Victoria, British Columbia, Canada. His current research activities include microwave system design and problems of electromagnetic field theory in integrated circuits and radiating structures.

Dr. Bornemann was one of the recipients of the A. F. Bulgin Premium of the Institution of Electronic and Radio Engineers in 1983. He is a senior member of the IEEE, serves on the Editorial Board of the *IEEE Transactions on Microwave Theory and Techniques*, and has authored and coauthored more than 50 technical papers.



Vladimir A. Labay was born May 11, 1965 in Winnipeg, Manitoba, Canada. He received the BSc (E.E.) and MSc (E.E.) from the University of Manitoba in 1987 and 1990, respectively. Presently, he is a PhD candidate in electrical engineering at the University of Victoria in British Columbia. His current research interests include numerical field modeling of microwave and millimeter-wave components with emphasis on the characterization of discontinuities within arbitrarily shaped structures.