provement obtained when the 5-ft. high inverted L (IL57) is used, compared to that when the 5-ft. whip (VW5) or even the 8-ft. whip (VW8) is used. Also shown is the improvement gained when the 8-ft.-high inverted-L (IL87) is used, as compared to the results when 8-ft. whip is used. Thus, for a given height, it is preferable to use an inverted L rather than a whip (for example, the IL57 gives about a 4–9-dB improvement over the VW5). Examination of Figure 6(b) discloses that the CC2T condenser antenna (2-ft. height, 6-ft. diameter) gives a field strength across almost the entire band, that is higher than that of the 8-ft. whip (i.e., gives a height reduction of at least 4:1). It also shows that the 7-in.-high (3-ft. diameter) CP73 antenna's signal level is, at most, only about 8 dB lower than that of the 5-ft. whip (VW5). Figure 6(c) shows that height reductions of about 2:1, were measured for the inverted L compared to the corresponding whips, because the IL87's field was virtually the same as that of the 15-ft. whips and the IL27's field was about equal to or greater than that of the 5-ft. whip. The inverted L also produces radiation directly above the antenna and hence both NVIS and ground-wave jamming can occur at the same time. Such NVIS propagation has been shown to provide jamming over hundreds of miles where the ground wave is too weak [11].

It is also noted that successful two-way communication over the entire 1.80–30.0-MHz bands was established over both ground waves and sky waves (the latter including long distances up to 5,000 miles for frequencies above about 7 MHz) with a 100-W input to all the antennas in Table 1 (except the PPC antenna, which was not tried).

During these measurements it was noted that the SGC-230 automatic coupler retunes to any frequency (when original tuning, which takes a few seconds, is first done) in about 10 ms for all of the above antennas except the whips of heights of about 5 ft. or less, and the condenser antennas of heights 2 ft. or less, where hovering occurs and well over 10 ms lapse before final tuning is established. However, the U.S. Army is now sponsoring R/D on automatic couplers that are superior (with much faster tuning time and the capability to handle higher-Q antennas) to those employed in these tests, to avoid this hovering problem.

V. CONCLUSIONS

Based on the above it is concluded that

1. Predicted ground-wave performance based on the computed current moments of (6) with measured base currents can be used to approximately judge one antenna compared to another.
2. For equal ground-wave performance, approximate height reductions of 4:1 and 2:1 can be achieved by using either condenser or inverted-L antennas, respectively, as compared to using whip antennas.
3. The inverted L is especially attractive because it occupies less space, can provide an enhanced ground wave compared to a whip of the same height, and also provides a degree of NVIS jamming.
4. Use of a superconducting coil in the coupler can reduce the heights even further [12], as can be precisely quantified using the circuit of Figure 1(c), as is presently being studied.

REFERENCES

1. K. Fujimato, A. Henderson, K. Hirasawa, and J. R. James, *Small Antennas*, Wiley, New York, 1987.
2. T. S. M. Maclean and Z. Wu, *Radiowave Propagation Over Ground*, Chapman and Hall, New York, 1993, pp. 105–106.
3. E. C. Jordan and K. Balmain, *Electromagnetic Waves and Radiating Systems* (2nd ed.), Prentice-Hall, Englewood Cliffs, NJ, 1968, pp. 628–657.
4. R. J. King, "Electromagnetic Wave Propagation Over a Constant Impedance Plane," *Radio Sci.*, Vol. 4, No. 2, 1969, pp. 255–268.
5. R. W. P. King, M. Owens, and T. T. Wu, *Lateral Electro-magnetic Waves*, Springer, New York, 1992, Chap. 4, pp. 88–121.
6. F. E. Terman, *Electronic and Radio Engineering* (4th ed.), McGraw-Hill, New York, 1955, Chap. 22, pp. 803–808.
7. J. R. Wait, *Electromagnetic Wave Theory*, Harper and Row, New York, 1985, Chap. 6.
8. Model SGC-230, "SGC Smarttuner," SGC, Inc. SGC Building, 13737 S. E. 26th St., P.O. Box 326, Bellvue, WA, 98009.
9. DIGI-FIELD Field Strength Meter, I. C. Engineering, 16350 Ventura Blvd., Suite 125, Encino, CA 91436.
10. Model FT-890 H. F. Transeiver, Yaseu Co., C.P.O. Box 1500, Tokyo, Japan.
11. M. A. Wallace, "HF Radio in Southwest Asia," *IEEE Commun. Mag.*, Vol. 30, No. 1, 1992, pp. 58–61.
12. R. C. Hansen, "Superconducting Antennas," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. AES-26, No. 2, 1990, pp. 345–355.

Received 6-27-96

Microwave and Optical Technology Letters, 13/5, 267–274

CCC 0895-2477/96

ACCURATE SOLUTION OF WAVEGUIDE MODEL OF MICROSTRIP DISCONTINUITY WITH THE USE OF BASIS FUNCTIONS WITH EDGE CONDITIONS

Smain Amari, Thierry Lenadan, Jens Bornemann, and Rüdiger Vahldieck
Laboratory for Lightwave Electronics, Microwaves and Communications
(LLiMiC)
Department of Electrical and Computer Engineering
University of Victoria, Victoria, British Columbia, Canada V8W 3P6

KEY TERMS
Microstrip discontinuity, step discontinuity, scattering analysis

ABSTRACT
The problem of scattering from a microstrip step discontinuity is analyzed by an integral equation technique with the use of basis functions that include the edge conditions. The presence of magnetic walls in the waveguide model of the microstrip line makes it more convenient to expand the tangential magnetic field at the interface, instead of the tangential electric field as in the case of metallic waveguides. The problem of relative convergence is not encountered as the modes of the two waveguides enter only in the computation of the inner products that are tested for convergence. Results for the reflection coefficient and the magnetic field at the interface are presented and compared with available data to demonstrate the accuracy and efficiency of the approach. © 1996 John Wiley & Sons, Inc.

I. INTRODUCTION

Step discontinuities are frequently encountered in microstrip line circuits. In many applications, the waveguide model, which was originally proposed by Wolff, Kompa, and Mehran et al. [1] and subsequently investigated by other researchers [2, 3], has been found useful in the determination of the scattering at the discontinuity.

In this model, the original microstrip line is replaced by a parallel-plate waveguide, with magnetic side walls, filled with a fictitious material whose dielectric constant is given by the effective dielectric constant of the original microstrip line. The width of the waveguide is given by the so-called effective width of the original microstrip line.

The investigation of microstrip line discontinuities within this model has been carried out with the use of the mode-matching technique [3]. The method is, however, known to suffer from the phenomenon of relative convergence [3]. In addition, its slow convergence, when sharp metallic edges are present, is attributed to its failure to include the edge conditions.

In this article, we propose to start from an integral-equation formulation of the microstrip discontinuity problem in such a way that the edge condition is systematically included. *At the discontinuity*, a change of basis is performed; that is, instead of representing the fields as a superposition of the normal modes of the waveguides, a different set of basis functions is used. The efficiency of such a technique depends on the availability of basis functions that represent the local behavior of the electromagnetic field, and on the simplicity of implementing such a change of basis. Because of the orthogonality of the normal modes over the cross section of the waveguide, the transformation from a representation of the transverse magnetic field at the discontinuity, which uses basis functions with the edge conditions, to the modal representation of the MMT is diagonal. This does not hold for the transverse electric field at the discontinuity because of the presence of the magnetic side walls. The situation is dual to that of a waveguide with metallic walls.

This article is organized as follows. The next section presents a brief discussion of the theoretical formulation of the problem. The choice of basis functions is presented in Section IV and the numerical results in Section V.

II. THEORETICAL FORMULATION

The waveguide model of a microstrip line discontinuity is shown in Figure 1. It is assumed that the walls of the waveguides as well as the dielectrics, with dielectric constants ϵ_1 and ϵ_2 respectively, are lossless. We are only interested in the reflection and transmission properties of the structure when the fundamental mode is incident from Waveguide 1.

Following the mode-matching technique, we expand the transverse electromagnetic fields in terms of the normal modes of the two waveguides. In Waveguide 1, there are reflected modes in addition to the incident fundamental mode (with amplitude equal to unity), whereas only forward-traveling modes are present in Waveguide 2. Because of the symmetry of the structure only TE_{1n} modes are present [3].

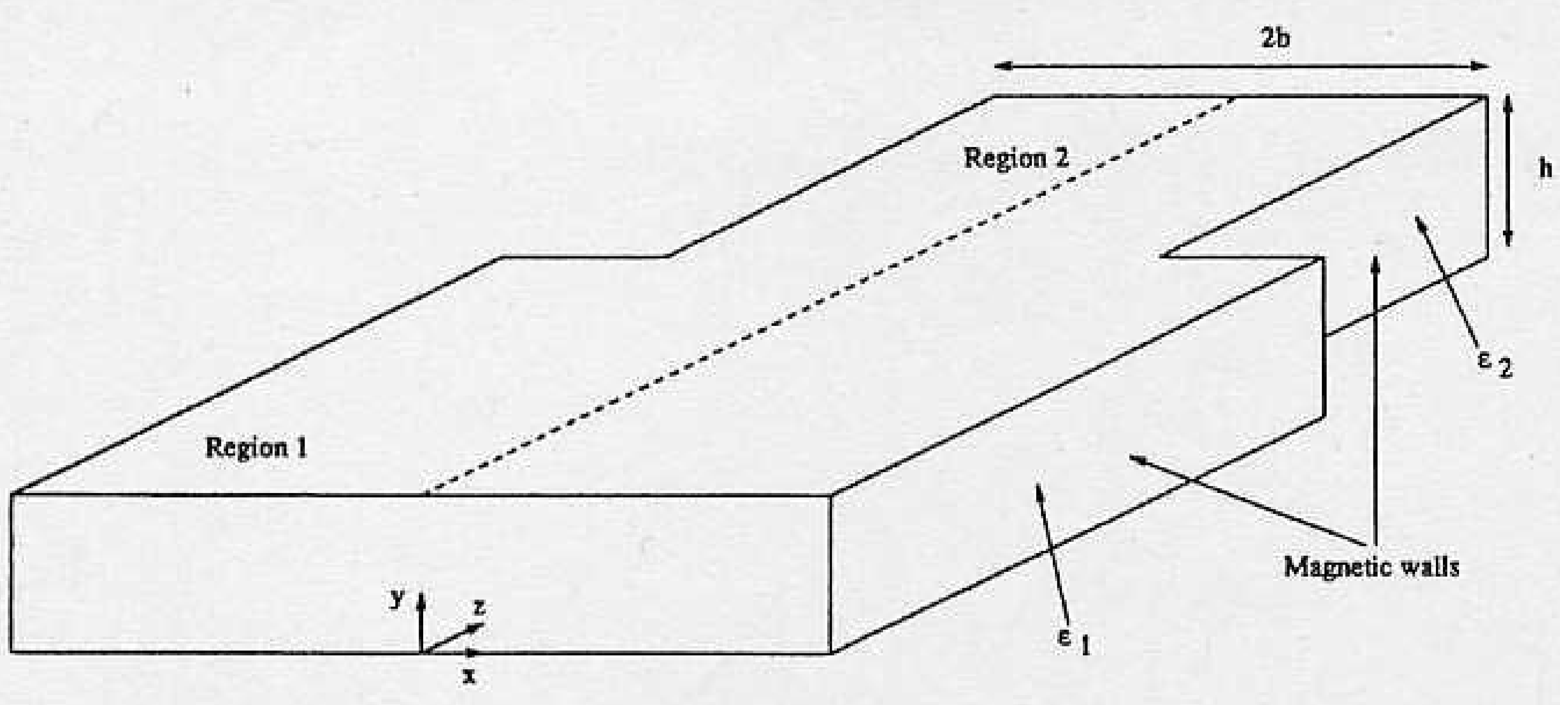

Figure 1 Geometry of a microstrip discontinuity with the waveguide model

In Region 1, we have

$$H_x^1(x, z) = \Phi_{11}(x)e^{-jk_{11}^z z} + \sum_{n=1}^{\infty} B_n\Phi_{1n}(x)e^{jk_{1n}^z z}, \tag{1a}$$

$$E_y^1(x, z) = -Y_{11}\Phi_{11}(x)e^{-jk_{11}^z z} + \sum_{n=1}^{\infty} B_n Y_{1n}\Phi_{1n}(x)e^{jk_{1n}^z z}. \tag{1b}$$

Similarly, in region 2, we have

$$H_x^2(x, z) = \sum_{n=1}^{\infty} F_n\Phi_{2n}(x)e^{-jk_{1n}^z z} \tag{2a}$$

and

$$E_y^2(x, z) = -\sum_{n=1}^{\infty} F_n Y_{2n}\Phi_{2n}(x)e^{-jk_{1n}^z z}, \tag{2b}$$

where

$$\Phi_{1n}(x) = \cos\left[(n-1)\pi\frac{x}{a}\right], \qquad n = 1, 2, \ldots, \tag{3a}$$

$$\Phi_{2n}(x) = \cos\left[(n-1)\pi\frac{x}{b}\right], \qquad n = 1, 2, \ldots, \tag{3b}$$

$$k_{1n}^z = \begin{cases} +\sqrt{\omega^2\epsilon_1\epsilon_0\mu_0 - \left(\frac{(n-1)\pi}{a}\right)^2}, & \text{propagating mode} \\ -j\sqrt{\left(\frac{(n-1)\pi}{a}\right)^2 - \omega^2\epsilon_1\epsilon_0\mu_0}, & \text{evanescent mode} \end{cases} \tag{4}$$

and $Y_{1n} = \frac{k_{1n}^z}{\omega\mu_0}$. Similar expressions hold for waveguide 2.

The boundary conditions of the problem are given by

$$H_x^1(x, z=0) = H_x^2(x, z=0), \qquad 0 \le x \le b, \tag{5a}$$

$$E_y^1(x, z=0) = E_y^2(x, z=0), \qquad 0 \le x \le b, \tag{5b}$$

and

$$H_x^1(x, z=0) = 0, \qquad b \le x \le a. \tag{5c}$$

Instead of using the modal expansions (1), (2) in Eqs. (5), we perform a change of basis functions such that the boundary condition (5c) is automatically satisfied. Let $Q_i(x)$ denote a generic element of such a set such that

$$Q_i(x) = 0, \qquad b \le x \le a, \qquad \forall i \tag{6}$$

The magnetic field at the discontinuity is therefore expanded in a series of the form

$$H_{x\,\mathrm{gap}}(x) = \sum_{i=1}^{M} c_i Q_i(x) \qquad (7)$$

The number of basis functions M is increased until convergence is achieved. With the use of Eq. (7) and the modal expansions (1), (2) in the boundary condition (5a), we can express the modal expansion coefficients as follows:

$$B_n = \delta_{n1} + \frac{1}{Y_{1n}} \sum_{i=1}^{M} c_i \tilde{Q}_{1i}(n) = \delta_{1n} + \frac{1}{Y_{1n}} \sum_{i=1}^{M} c_i \frac{2}{a(1+\delta_{n1})} \int_0^b Q_i(x)\Phi_{1n}(x)\,dx \qquad (8a)$$

$$F_n = -\frac{1}{Y_{2n}} \sum_{i=1}^{M} c_i \tilde{Q}_{2i}(n) = \frac{1}{Y_{1n}} \times \sum_{i=1}^{M} c_i \frac{2}{b(1+\delta_{n1})} \int_0^b Q_i(x)\Phi_{2n}(x)\,dx. \qquad (8b)$$

To determine the expansion coefficients c_i, we use Eqs. (8) in the boundary condition of the electric field, Eq. (5b), and apply Galerkin's method to obtain a set of linear equations

$$[A][c] = [U], \qquad (9)$$

where

$$[A]_{ij} = \sum_{n=1}^{\infty} \left[\frac{\tilde{Q}_{1i}(n)\tilde{Q}_{1j}(n)}{Y_{1n}} + \frac{b}{a} \frac{\tilde{Q}_{2i}(n)\tilde{Q}_{2j}(n)}{Y_{2n}} \right] \qquad (10a)$$

and

$$[U]_i = -2\frac{\tilde{Q}_{1i}(1)}{Y_{11}}. \qquad (10b)$$

Once the expansion coefficients are determined from Eq. (9), the reflection coefficient is given by Eq. (8a) by setting $n = 1$.

Note also that the modes of the two waveguides appear only in computing the matrix elements in Eq. (9). The sums are tested for convergence, thereby leaving only one free parameter in the solution, that is, the number of basis functions. The problem of relative convergence is not present.

III. BASIS FUNCTIONS

To guarantee numerical efficiency, a set of basis functions that contain as much information about the behavior of the field at the discontinuity, especially its singular nature, must be used. In this specific situation, the nature of the singularity of the field at the edge depends on the difference between the dielectric constants of the two waveguides. Following the analysis presented in [4] we determined that when $\epsilon_1 = 2.2$ and $\epsilon_2 = 2.1$, the singularity is accurately given by $x^{2/3-1}$. When $\epsilon_1 = 10.2$ and $\epsilon_2 = 2.2$, the singularity is $x^{0.55-1}$. For the numerical example considered in this article ($\epsilon_1 = 2.2$, $\epsilon_2 = 2.1$), we assume that the H_x component has a singularity of the form $x^{2/3-1}$ as x approaches b. We take the symmetry of the field about the yz plane into account, and the following basis functions are used:

$$Q_i(x) = \frac{\cos\left[(i-1)\pi \dfrac{x}{b}\right]}{\left[1 - \left(\dfrac{x}{b}\right)^2\right]^{1/3}}, \qquad i = 1, 2, \ldots. \qquad (11)$$

The spectrum of these basis functions in each of the two waveguides can be expressed in terms of Bessel functions of the first kind of order $\frac{1}{6}$ [5], namely

$$\tilde{Q}_{1k}(n) = \frac{b}{a} \frac{\Gamma(\frac{1}{2})\Gamma(\frac{2}{3})}{2(1+\delta_{n1})} \left[\frac{J_{1/6}\left[\left|(k-1) + (n-1)\frac{b}{a}\right|\pi\right]}{\left[\left|(k-1) + (n-1)\frac{b}{a}\right|\pi/2\right]^{1/6}} + \frac{J_{1/6}\left[\left|(k-1) - (n-1)\frac{b}{a}\right|\pi\right]}{\left[\left|(k-1) - (n-1)\frac{b}{a}\right|\pi/2\right]^{1/6}} \right] \qquad (12a)$$

and

$$\tilde{Q}_{2k}(n) = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{2}{3})}{2(1+\delta_{n1})} \left[\frac{J_{1/6}[|k+n-2|\pi]}{[|k+n-2|\pi/2]^{1/6}} + \frac{J_{1/6}[|k-n|\pi]}{[|k-n|\pi/2]^{1/6}} \right]. \qquad (12b)$$

IV. NUMERICAL RESULTS

The present technique is applied to determine the reflection properties of a typical discontinuity. More specifically, we assume that the effective dielectric constants are 2.2 and 2.1 in Regions 1 and 2, respectively, and take $a = 100$ mils and $b = 26.1$ mils. The operating frequency is 2 GHz.

Figure 2 is a plot of the magnetic field H_x at the interface as computed from the basis functions. That is, from

$$H_x(x) = \sum_{i=1}^{M} c_i Q_i(x) \qquad (13)$$

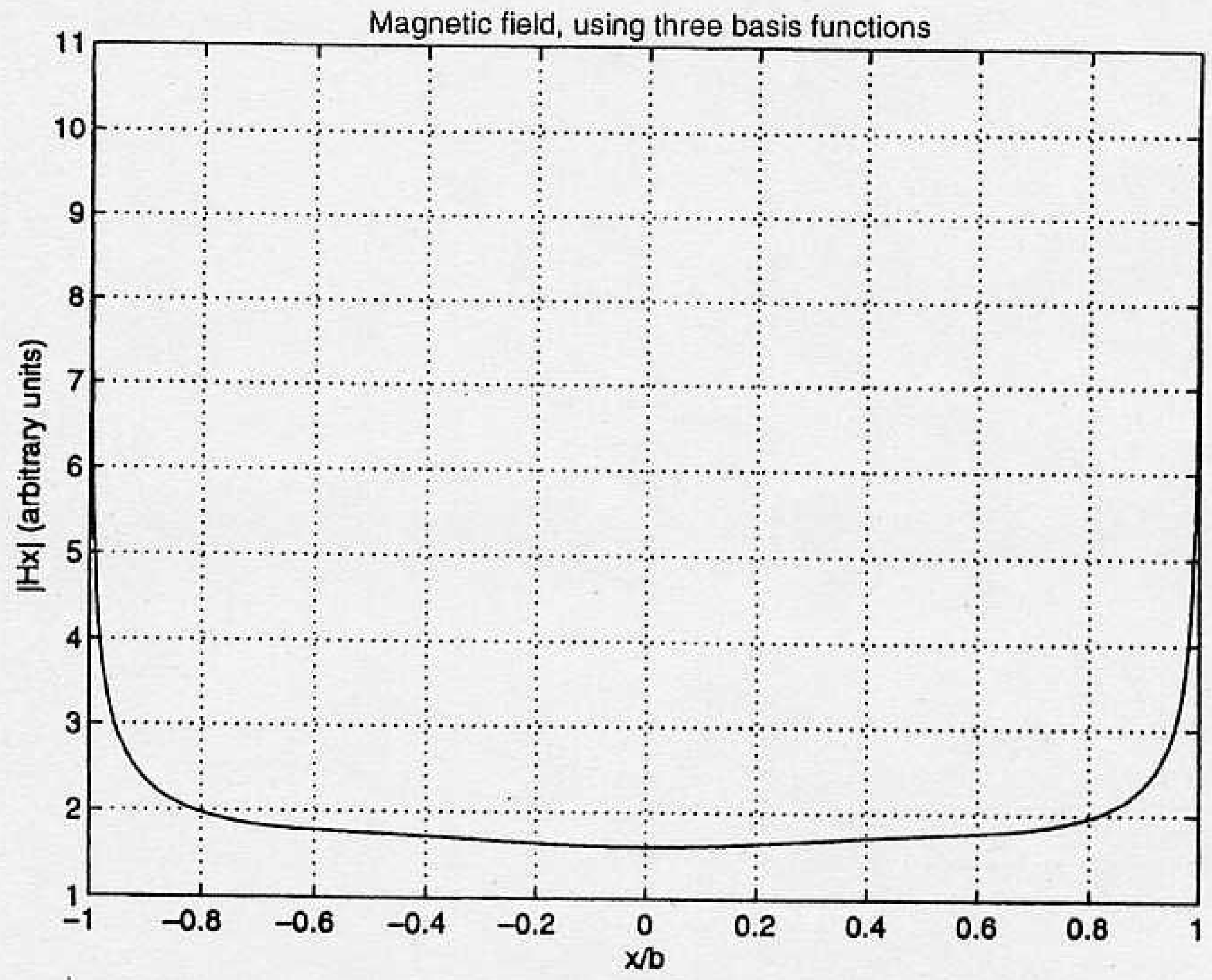

Figure 2 Magnitude of the transverse magnetic field $|H_x^1(x/a)|$ at the discontinuity as obtained from three basis functions

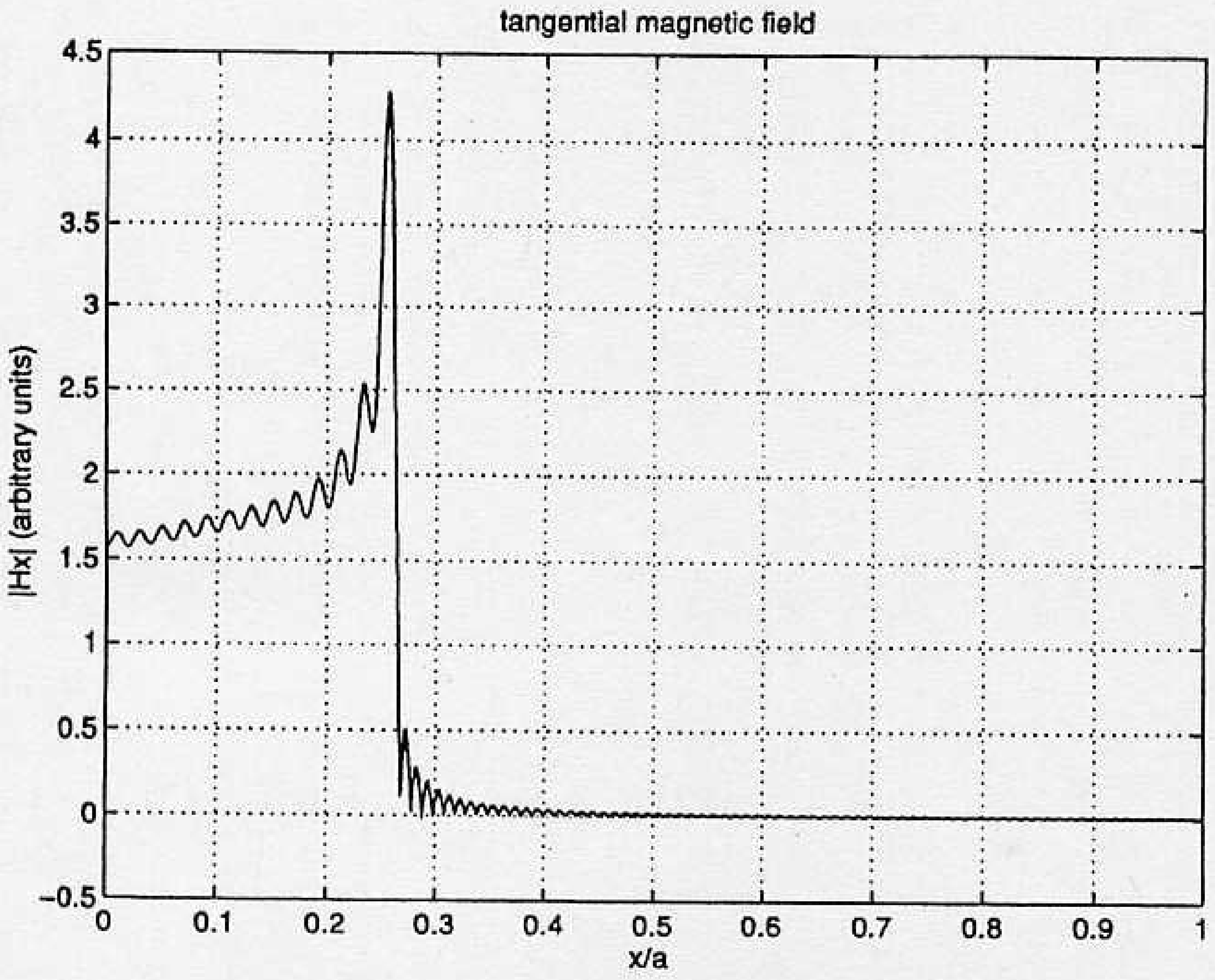

Figure 3 Magnitude of the transverse magnetic field $|H_x^1(x/a)|$ at the discontinuity as obtained from numerical summation of modal expansion [Eq. 1(a)]. Three basis functions were used and 100 modes were summed

when three basis functions are used. Note that in the MMT, the field is usually computed from its modal expansion [Eq. (2)]. For completeness, we also plotted H_x as computed from equation (2) where the modal expansion coefficients are determined from Equation (1a) (see Figure 3). It is easy to show that in the limit of an infinite number of modes, these two quantities are identical. Indeed, at the discontinuity, $H_x^1(x)$ can be written as

$$H_x^1(x) = \sum_{n=1}^{\infty} \left(\sum_{i=1}^{M} c_i \tilde{Q}_{1i}(n) \right) \Phi_{1n}(x). \tag{14}$$

If we take into account the definition of $\tilde{Q}_{1i}(n)$ as given in Eq. (8a), Eq. (14) is readily recognized as the Fourier series of the expression in the right-hand side of Eq. (13). The transition from Eq. (14) to (13) holds only when an infinite number of modes are summed. Figure 3 was obtained from summing 100 modes. Because of the discontinuity of H_x^1 at $x = b$, the numerical summation of the Fourier series exhibits the Gibbs phenomenon [6], as Figure 3 shows.

The variation of the magnitude of the reflection coefficient of the fundamental mode as the number of basis functions is increased in shown in Table 1. The results, even with one basis function, are in excellent agreement with those obtained from the MMT [3]. It is evident that the numerical solution converges with only one basis function, which demonstrates the judicious choice of the basis functions.

V. CONCLUSIONS

An accurate and efficient solution of the microstrip line discontinuity within the waveguide model was presented. An integral equation for the transverse magnetic field at the discontinuity is solved by the moment method with basis functions that include the edge conditions. One basis function is found to be sufficient to obtain accurate results for the reflection properties of the discontinuity.

TABLE 1 Magnitude of reflection coefficient $|S_{11}|$ versus number of basis functions M

M	1	2	3	4
Present Method	0.5938	0.5938	0.5938	0.5938

Ref. [3] gives a value of 0.5993

REFERENCES

1. I. Wolff, G. Kompa, and R. Mehran, "Calculation Method for Microstrip Discontinuities and T-Junctions," *Electron. Lett.*, Vol. 8, April 1972, pp. 177–179.
2. W. Menzel and I. Wolff, "A Method for Calculating the Frequency-Dependent Properties of Microstrip Discontinuities," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-25, Feb. 1977, pp. 107–112.
3. T. Itoh (Ed.), *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*, Wiley, New York, 1989.
4. F. Olyslager, "The Behavior of Electromagnetic Fields at Edges in Bi-Isotropic and Bi-Anisotropic Materials," *IEEE Trans. Antennas Propagat.*, Vol. AP-42, Oct. 1994, pp. 1392–1397.
5. I. S. Gradshteyn and I. M. Ryznik, *Tables of Integrals, Series, and Products* (5th ed.), Academic Press, New York, 1994.
6. A. V. Oppenheim, A. S. Willsky, and I. T. Young, *Signals and Systems*, Prentice-Hall, New York, 1983.

Received 6-26-96

Microwave and Optical Technology Letters, 13/5, 274–277

CCC 0895-2477/96

A MODIFIED COUPLED-MODE ANALYSIS FOR ASYMMETRIC COUPLED WAVEGUIDES

Hosung Chang and Je-Myung Jeong
Department of Radio Science and Engineering
Hanyang University
Seoul, 133-791 Korea

KEY TERMS

Coupled-mode theory, optical waveguides

ABSTRACT

We present a modified coupled-mode formulation for an asymmetric two-waveguide coupler in which the trial field is expressed as a linear combination of the fundamental modes of the individual waveguides, with an additional phase-variation term determined from the orthogonality of the normal modes. The method provides more accurate solutions than the conventional theory, and the results satisfy power conservation. © 1996 John Wiley & Sons, Inc.

I. INTRODUCTION

The coupled-mode theory has been widely applied to numerous optoelectronic components and fiber-optic devices, including optical directional couplers since implementations of the conventional coupled-mode theory for optical waveguides were developed by Marcuse [1], Yariv [2], and Taylor and Yariv [3]. Coupled-mode formulation is approximate, with assumptions that normal modes of a composite structure can be expressed as linear combinations of fundamental modes of individual waveguides and the slowly varying envelope approximation (SVEA) is valid. Conventional coupled-mode theory provides reasonably accurate propagation constants for weakly coupled symmetric waveguides when an overlap