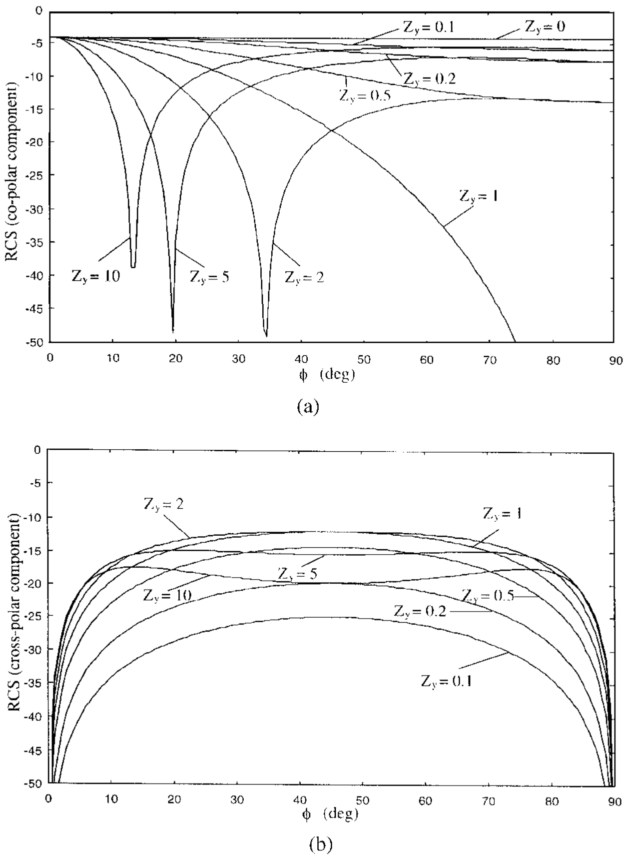

Figure 3 (a) Monostatic copolar RCS of a square anisotropic flat plate ($L = 76$ mm) illuminated at normal incidence ($Z_x = 0$ and $Z_y/\zeta = 0, 0.1, 0.2, 0.5, 1, 2, 5, 10$). (b) Monostatic cross-polar RCS of a square anisotropic flat plate ($L = 76$ mm) illuminated at normal incidence ($Z_x = 0$ and $Z_y/\zeta = 0, 0.1, 0.2, 0.5, 1, 2, 5, 10$)

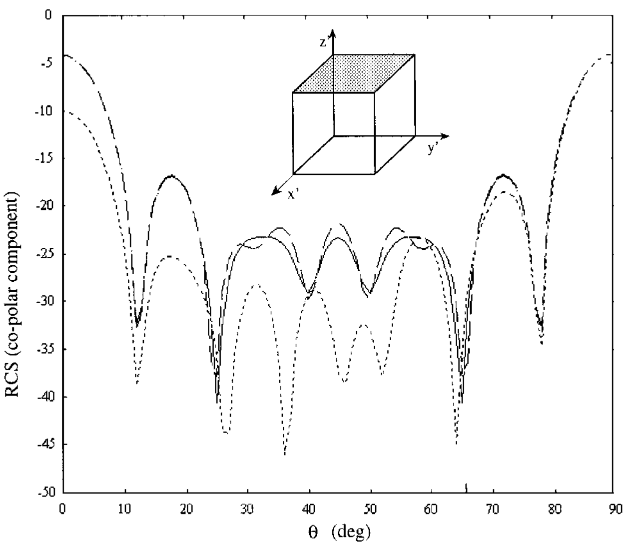

Figure 4 Monostatic copolar RCS of a cube (side $L = 76$ mm) in the θ-polarization case. Solid line: perfectly conducting cube, PO; dashed line: perfectly conducting cube, PO plus ILDCs; dotted line: perfectly conducting cube with top face anisotropic, PO

calculations by resorting to suitable ILDCs [6] for edges in anisotropic impedance surfaces. As emphasized in [6], they can be determined once either exact [7, 8] or approximate [9] solutions, in terms of Sommerfeld's integrals, are known for the corresponding canonical wedge problem, which locally approximates the actual configuration of the scatterer. This topic will be the object of future work.

REFERENCES

1. D. G. Michelson and E. V. Jull, "Depolarizing Trihedral Corner Reflectors for Radar Navigation and Remote Sensing," *IEEE Trans. Antennas Propagat.*, Vol. AP-43, No. 5, May 1995, pp. 513–518.
2. R. Cecchini, R. Coccioli, and G. Pelosi, "PERIODIC^3: A Software Package for the Analysis of Artificially Anisotropic Surfaces," *IEEE Antennas Propagat. Mag.*, Vol. 37, No. 2, April 1995, pp. 83–86.
3. W. B. Gordon, "Far-Field Approximation to the Kirchhoff–Helmholtz Representation of the Scattered Fields," *IEEE Trans. Antennas Propagat.*, Vol. AP-23, 1975, pp. 590–592.
4. W. B. Gordon, "Near-Field Calculations with Far Field Formulas," *IEEE AP-S International Symposium 1996* Baltimore, MD, July 21–26, 1996, pp. 950–953.
5. G. T. Ruck (Ed.), *Radar Cross Section Handbook*, Plenum Press, New York, 1970, p. 524.
6. G. Pelosi, S. Maci, R. Tiberio, and A. Michaeli, "Incremental Length Diffraction Coefficients for an Impedance Wedge," *IEEE Trans. Antennas Propagat.*, Vol. 40, No. 10, Oct. 1992, pp. 1201–1210.
7. G. Manara, P. Nepa, and G. Pelosi, "Electromagnetic Scattering by a Right-Angled Anisotropic Impedance Wedge," *Electron. Lett.*, Vol. 32, No. 13, 1996, pp. 1179–1180.
8. P. Nepa, G. Manara, and G. Pelosi, "A UTD Solution for the Diffraction at an Edge in a Planar Anisotropic Impedance Surface: Oblique Incidence Case," *Microwave Opt. Technol. Lett.*, Vol. 13, No. 2, pp. 55–59, Oct. 1996.
9. G. Pelosi, G. Manara, and P. Nepa, "Electromagnetic Scattering from an Anisotropic Impedance Wedge at Oblique Incidence," *IEEE Antennas and Propagation Society International Symposium*, Baltimore, MD, July 21–26, 1996, pp. 788–791.

CCC 0895-2477/97

A TECHNIQUE TO LOCATE MINIMA IN SINGULAR-VALUE DECOMPOSITION FOR EIGENVALUE PROBLEMS IN ELECTROMAGNETICS

Smain Amari,[1] Jens Bornemann,[1] and Rüdiger Vahldieck[1]
[1] Laboratory for Lightwave Electronics, Microwaves and Communications LLiMiC
Department of Electrical and Computer Engineering
University of Victoria
Victoria, British Columbia, Canada V8W 3P6

Received 20 September 1996; revised 25 November 1996

ABSTRACT: *An efficient technique to locate minima (zeros) in the smallest singular value of a singular value decomposition, as used to solve eigenvalue problems in electromagnetics, is presented. The efficiency of the technique depends only on the availability of a sufficiently accurate representation of the smallest singular value; the sharpness of the peaks is of minor relevance. The technique is applied to determine* *the cutoff wave numbers of a ridge circular waveguide to demonstrate its efficiency.* © *1997 John Wiley & Sons, Inc.* Microwave Opt Technol Lett 14: 318–321, 1997.

Key words: *circular waveguides; singular-value decomposition; eigenvalues*

I. INTRODUCTION

Linear eigenvalue problems are widespread in electromagnetics and other branches of physics, engineering, and applied mathematics. The numerical solution of such problems is often reduced to finding the roots of a determinant, or equivalently, the zeros of the smallest singular value of the corresponding matrix [1].

In integral-equation formulation of eigenvalue problems in electromagnetics, the method of moments, which transforms the integral equation into a set of linear equations in the expansion coefficients, is often the method of choice [2, 3]. Similarly, the mode-matching technique (MMT) leads to a homogeneous set of linear equations in the modal expansion coefficients [4]. Other methods, such as the method of lines (MoL), the finite-element technique (FET) and the spectral-domain approach (SDA) also lead to a homogeneous set of linear equations. It is therefore evident that most eigenvalue problems in electromagnetics involve the same issue: locate the zeros of a determinant.

A well-known problem is that the determinant often exhibits poles that are not easy to handle numerically. An alternative solution is offered by the singular value decomposition of the matrix in question [1]. In this approach, the zeros of the smallest singular value, which is pole free, are located instead. However, despite the absence of the poles in this technique, some of the minima are extremely sharp. In addition, it is in general much harder to determine a minimum of a function than one of its zeros.

To solve this problem, we present a technique that incorporates both the advantage of having a pole-free function and the ease of locating the zero of a function instead of a minimum. The gist of the technique consists of noticing that the derivative of the smallest singular value changes sign at a minimum. Starting from an accurate enough description of the smallest singular value, by computing it at a large enough number of points, a cubic spline is used to approximate its first derivative. Despite the fact that approximating a derivative is less reliable than approximating a function, the changes in sign of the derivative are less problematic. The solutions of the original eigenvalue problem are then determined from the zeros of the first derivative where a sign change from negative to positive takes place.

To illustrate the technique, the cutoff wave numbers of a ridge circular waveguide are determined and compared with available data.

II. FIRST DERIVATIVE WITH THE USE OF CUBIC SPLINES

The subject of cubic splines is discussed in ample detail in many books on numerical analysis; only essential points are summarized here [4, 5].

The idea of a cubic spline consists of replacing an original function $f(x)$ by local third-order polynomials and requiring the continuity of the function and its first two derivatives at the grid points. Let us assume that the function is given at a set of N points x_i, where it assumes values y_i. In the interval $[x_i, x_{i+1}]$, the original function is replaced by the polynomial [4]

$$W(x) = A(x)y_i + B(x)y_{i+1} + C(x)z_i + D(x)z_{i+1}. \quad (1)$$

Here

$$A(x) = \frac{x_{i+1} - x}{x_{i+1} - x_i}, \quad (2a)$$

$$B(x) = \frac{x - x_i}{x_{i+1} - x_i}, \quad (2b)$$

$$C(x) = \frac{1}{6}(A^3(x) - A(x))(x_{i+1} - x_i)^2 \quad (2c)$$

and

$$D(x) = \frac{1}{6}(B^3(x) - B(x))(x_{i+1} - x_i)^2. \quad (2d)$$

The quantities z_i are determined from requiring the first derivative to be continuous at the points x_i. A tridiagonal set of linear equations in z_is results [4]. Once these are determined, the first derivative at the grid points x_i are obtained from differentiating Eq. (1). Note that the tridiagonal set is solvable only if two additional conditions are given. If the second derivative is assumed to vanish at the end points x_1 and x_N, the cubic spline is referred to as natural [5].

In the present work, we apply the natural cubic spline to locate the minima in the smallest singular value in the solution for the cutoff wave numbers of transverse-electric modes of a ridge circular waveguide by an integral equation formulation. Only the modes with an electric wall along the plane of symmetry are considered.

III. CUTOFF WAVE NUMBERS OF A RIDGE CIRCULAR WAVEGUIDE

The problem of determining the cutoff wave numbers of a ridge circular waveguide can be formulated in variety of ways. Here, because we are mostly interested in the actual determination of the cutoff wave numbers as the zeros of the smallest singular value, the nature of the technique to calculate the electromagnetic field is of little relevance. In this particular case, the coupled-integral-equations technique (CIET) was used [6].

A typical plot of the smallest singular value as a function of $k_c a$, where a is the radius of the empty circular waveguide, is shown in Figure 1. The dips in the plot are the locations of the cutoff wave numbers. The curve in this figure was obtained from 500 data points.

Instead of locating these minima directly, we proceed to determine the values of the first derivative at the 500 data points with the use of cubic splines. Furthermore, we are only interested in the changes in the sign of the derivative; its actual values are of little importance. Figure 2 shows a plot of the sign of the first derivative thus obtained along with the smallest singular value. It is obvious that each minimum corresponds to a change in sign of the derivative from negative to positive. For example, the first sign change is not a minimum, as it occurs in the order positive to negative instead of negative to positive. Note also that the location of the minima are accurately predicted from the sign of the derivative.

Given the data in Figure 2, it is straightforward to determine the locations of the minima. For that, we analyze the sign of the derivative at two consecutive points x_i and x_{i+1}. Once a sign change is found, its location is a root if the

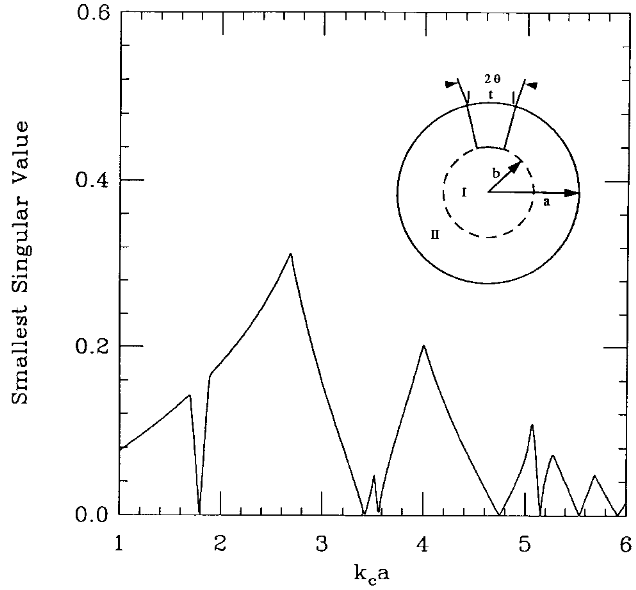

Figure 1 Smallest singular value as a function of $k_c a$ of a ridge circular waveguide (see inset). $\theta = 5°$ and $b/a = 0.8$

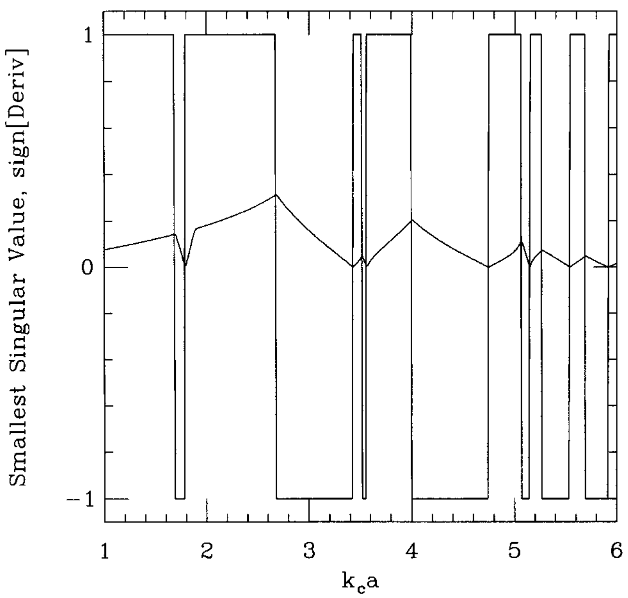

Figure 2 Smallest singular value and sign of first derivative as obtained from cubic splines. Dimensions are those of Figure 1

derivative shows the proper sign change, that is, from negative to positive. The procedure is continued until the point x_N is reached.

A more stringent test of the technique is provided by the plot of the singular value, as shown in Figure 3, which corresponds to a ridge circular waveguide with $\theta = 5°$ and $b/a = 0.99$. Figure 4 shows the sign of the derivative along with the smallest singular value. It is evident that the location of the minima is again accurately predicted by the present technique. The accuracy of the technique depends on the number of data points used in generating the plots of the minimum singular value. When large matrices are used, thereby requiring large CPU times, the present technique is useful in providing reliable starting points and reduces the CPU time.

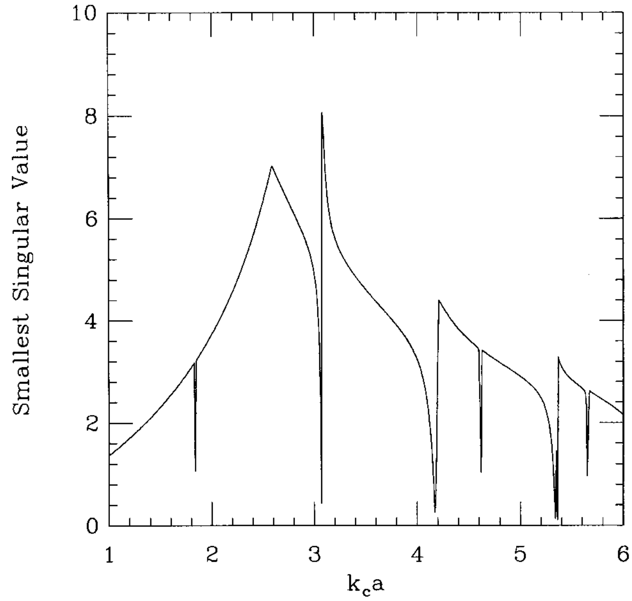

Figure 3 Cutoff wave numbers of the first two TE modes as a function of the depth of the ridge with $\theta = 5°$ and $b/a = 0.99$

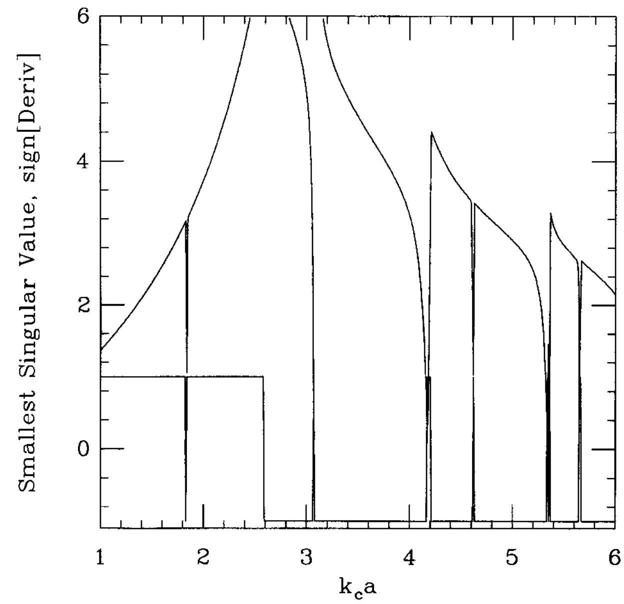

Figure 4 Smallest singular value and sign of first derivative as obtained from cubic splines. Dimensions are those of Figure 3

It takes 0.48 s to locate the seven minima in Figures 1 and 3 on an IBM RS/6000 530 machine.

IV. CONCLUSIONS

A technique to locate the minima of the smallest singular value as encountered in eigenvalue problems in electromagnetics was presented. The first derivative of the smallest singular value is computed with the use of cubic splines; the minima are determined from the changes in the sign of the first derivative from negative to positive. Numerical results for the cutoff wave numbers of TE modes of a ridge circular waveguide were used to demonstrate the efficiency of the technique.

REFERENCES

1. V. A. Labay and J. Bornemann, "Matrix Singular Value Decomposition for Pole-Free Solutions of Homogeneous Matrix Equations as Applied to Numerical Modeling Methods," *IEEE Microwave Guided Wave Lett.*, Vol. 2, No. 2, Feb. 1992, pp. 49–51.
2. R. F. Harrington, *Field Computation by the Moment Methods*, Krieger, Malabar, FL, 1987.
3. J. J. H. Wang, *Generalized Moment Methods in Electromagnetics*, Wiley, New York, 1991.
4. W. Press et al., *Numerical Recipes*, Cambridge, New York, 1986, Chap. 3.
5. W. Cheney and D. Kincaid, *Numerical Mathematics and Computing*, Brooks/Cole, Monterey, CA: 1985.
6. S. Amari, J. Bornemann, and R. Vahldieck, "Application of the Coupled Integral Equations Technique to Ridge Waveguides," *IEEE Trans. Microwave Theory Tech.*, to be published.

CCC 0895-2477/97

A NEW SOURCE FORMULATION FOR FDTD SIMULATION OF HIGH-FREQUENCY INTEGRATED CIRCUITS WITH AND WITHOUT GROUND PLANES

Xudong Wang[1] and R. Jennifer Hwu,[1]
[1] Electrical Engineering Department
University of Utah
Salt Lake City, Utah 84112

Received 10 October 1996; revised 19 November 1996

ABSTRACT: *A new voltage source formulation for FDTD is reported in this article. By stimulating the voltage source right in the plane of the strip line, this new source formulation can be used in the simulation of a wide variety of high-frequency integrated circuits, including those without ground planes. Compared with existing source formulations, the new source formulation is also relatively simple for most circuit simulations, because, since its relation to the actual source voltage is fairly straightforward. In addition to numerical calculations to show the validation, we present in this article the applications of this method in advanced high-frequency integrated circuits such as quasioptical diode array and coplanar waveguide nonlinear transmission lines. © 1997 John Wiley & Sons, Inc.* Microwave Opt Technol Lett 14: 321–324, 1997.

Key words: *finite-difference time domain; quasioptical diode array; high-frequency integrated circuits*

1. INTRODUCTION

In recent years the finite-difference–time-domain (FDTD) method has been widely used for modeling high-frequency integrated circuits [1–7]. Source formulation is an essential element in the FDTD technique. Several different source formulations are employed for the simulation of microstrip lines [2, 5–7]. Among these source formulations, the hard voltage source [2], added voltage source [5], and current source [6] use the same excitation region to excite a microstrip line. To be specific, voltage or current is imposed between the metal strip line and the ground plane in these methods. Therefore, these methods cannot be used directly in the FDTD simulation of high-frequency integrated circuits without ground planes. Very recently, a new method has been developed for which the line feed consists of obtaining the static map of the electric field in a plane perpendicular to the propagation vector [7]. In this article a new source formulation is reported. By imposing the voltage source directly in the plane of the strip line, our new source formulation can be used easily in the FDTD simulation of a large variety of high-frequency integrated circuits with and without ground planes, such as microstrip lines, coplanar waveguides, slot lines, etc.

2. NOVEL EXCITATION TECHNIQUE

Figure 1 shows the new source formulation developed by us for modeling various high-frequency integrated circuits. The stimulus is considered as an ideal voltage source in this new approach. For the case of a microstrip line, rather than exciting the line from the rectangular area underneath the metallization of the microstrip as for previous methods [2, 5, 6], the voltage source is imposed across a gap in the metal strip in our approach. This idea is consistent with the classical circuit theory in that voltages are usually imposed in series with source impedances and circuits, while currents are in parallel.

The electric field E across the gap has the following relation with the voltage of the stimulus:

$$E_S = \frac{V_S}{\Delta x}, \tag{1}$$

where E_S is the imposed electric field, V_S is the voltage source, and Δx is the length of the gap.

Because we apply a hard source to the gap, the tangent components of the scattering electric fields in the gap are forced to be zero by the hard source. The source formulation acts as though there were a perfect conductor plane in the gap that is right next to the source. In other words, the hard source imposes the effect of perfect conductor cells, so the insertion of the gap does not affect the modeling of the strip line and no discontinuities are created by the gap. It thus can be considered, as there is no gap in the strip line and signals in the circuit can propagate fluently both in the incident and reflected directions. As a result, the perfectly matched layer (PML) absorbing boundary condition (ABC) used at the source end can effectively absorb the backward traveling wave reflected from the terminal.

The PML structure in this case can be considered as the internal impedance of the voltage source. Because the PML is perfectly matched to the microstrip line, the source impedance, therefore, equals the characteristic impedance of the microstrip line. The equivalent circuit of a microstrip line with this new source formulation can therefore be constructed as shown in Figure 2, where Z_0 is the characteristic impedance of the microstrip line.

When circuits are stimulated in the same way as in Figure 2 (as most practical circuits do), the electric field is directly related to the actual voltage source by Eq. (1). This relation is relatively straightforward compared to those of [5] and [6]. Note that the current of the current source is V_s/Z_0 [5], and the voltage of the added voltage source is $V = \Delta t \cdot V_s/\varepsilon w Z_0$, where w is the width of the strip line [6]. Z_0 needs to be known before the current source and added voltage source can be related to the actual voltage of a circuit. Although the hard voltage source [2], like our method, can be easily related to the actual voltage of a circuit, there is no absorbing boundary condition that can be used effectively in the source plane. It is clear that our new source formulation is simpler and more convenient to use than previous methods. It should be pointed out that our new voltage source