

circumference. The dots are from the well-known solution in a series of cylindrical harmonics. Agreement is seen to be very good, even including the small "bump" at 180° . Convergence depends on use of a sufficiently small lattice spacing compared with wavelength. A rough experimental maximum for h is $\lambda/8\pi$.

III. DISCUSSION OF THE METHOD

SLIM is a technique for solution of unbounded electrostatic (and conjecturally, electrodynamic) field problems that requires no inversion of matrices. This makes the method simple to program and use. Its simplicity should be especially helpful in otherwise complex problems involving oddly shaped boundaries and three-dimensional fields.

The method's characteristic self-consistency also tends to create confidence in the accuracy of the results (as compared, for instance, with ordinary absorbing boundary conditions, which give results whose accuracy is unknown). If one finds a set of charges and fields which are self-consistent (in the sense that the fields imply the charges and vice versa) and which satisfies the boundary conditions, it seems quite likely that this solution will be correct. On the basis of intuition and experience, we conjecture that in all cases in which SLIM converges, the results are correct to within accuracy of computation.

An important question is whether SLIM actually offers any saving of computational resources as compared with other methods that require matrix inversion. MOM is undesirable for large problems because it results in full matrices, inversion of which requires on the order of N^3 operations (where N is the number of unknown charges or currents). FD or FE methods with various absorbing boundary conditions (or the MEI boundary conditions) result in sparse matrices requiring on the order of only N^2 computational operations. In comparison, SLIM eliminates the matrix-inversion steps and reduces the storage of unknowns to minimal size. However, one must still evaluate on the order of N^2 Green's functions: once for each field point for each source point. Depending on the complexity of the Green's function, this step may take longer than the matrix-inversion step, which makes the elimination of that step less important. On the other hand, the evaluation of the Green's functions only has to be done once for each structure. Once the Green's functions have been found and stored, further computational steps can take place quickly. Thus, the second and subsequent iterations should require only on the order of N operations. Moreover, in scattering problems, the Green's functions are characteristic of the scatterer, not of the illumination. Thus, if one is finding radar cross sections with many angles of illumination, all the angles after the first one should require computational steps on the order of only the first power of N .

As in any technique in which simplicity and lightness are pushed to the limit, a question of reliability must arise. It is quite possible that SLIM is less robust than the adaptive absorbing boundary condition techniques (with matrix inversion) described by Jin and Liu [9]. This question requires further study.

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Using Selective Asymptotics to Accelerate Dispersion Analysis of Microstrip Lines

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Abstract—A selective asymptotic technique (SAT) to accelerate the elements of the impedance matrix in the conventional spectral-domain approach (SDA) is presented. Instead of using the full asymptotic expression of the Green's functions, only those parts which cannot be evaluated in closed form are approximated by their asymptotic expressions. The resulting expressions are more accurate and systematic, as no additional parameter is introduced. The technique is applied to determine the effective dielectric constant of an open microstrip line to demonstrate its efficiency.

Index Terms—Dispersive media, microstrip lines, numerical integration, spectral-domain approach.

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I. INTRODUCTION

The accurate determination of the dispersion properties of microstrip lines is essential to modern integrated microwave circuits. Such an analysis is most frequently carried out using the spectral-domain approach (SDA) within the zero-thickness approximation due to its speed and ease of implementation [1]. Other methods of analysis used by many researchers include those of [2]–[5].

The accuracy of the results obtained from the SDA heavily depends on the accuracy by which the elements of its impedance matrix are evaluated, especially for open structures where these are given by improper (infinite upper limit) integrals. In practice, these integrals are computed using a finite upper limit, which is increased until convergence is achieved, or a mapping, which reduces the infinite range of integration to a finite one.

Recently, an asymptotic technique was used by Park and Balanis [6], [7], to extract the “tail” of these integrals in closed form, thereby substantially reducing the central processing unit (CPU) time. However, by using the full asymptotic expression of the Green’s function, it was necessary to deal with asymptotic integrands which are nonintegrably diverging at the origin ($\alpha = 0$), especially for diagonal elements of the impedance matrix. The asymptotic contributions of the off-diagonal terms were determined in closed form with no additional parameter.

In this paper, we introduce a selective asymptotic technique (SAT) to further improve on the work reported in [6] and [7]. Asymptotic contributions of both diagonal and off-diagonal elements of the impedance matrix are extracted in closed form by avoiding the introduction of divergences which are not present in the original Green’s function. By evaluating the Fourier transforms of the basis functions only once at the beginning of the calculation, and storing them in the computer’s memory, we also show that the CPU time is dramatically reduced. The main numerical effort is reduced to evaluating the determinant of the impedance matrix instead of computing its elements. Asymptotic forms of Green’s function, which do not exhibit these singularities, were also used in [4] within the quasi-TEM approximation.

To validate the approach, the effective dielectric constant of an open microstrip line is determined and compared to data from the literature. Excellent agreement is documented, along with a considerable reduction in CPU time.

II. THE SAT

The basic idea behind the SAT is to extract from a given quantity as little asymptotic forms as required in order to evaluate the integrals in closed form without introducing new numerical pathologies. The application of the technique depends on the problem at hand and is best illustrated by an example.

Consider the following improper integral:

$$I = \int_0^{\infty} \frac{\tanh(x)}{1+x^2} dx \quad (1)$$

whose exact value is 1.048 257 5. In directly extracting the asymptotic contribution to this integral, the limits $\tanh(x) \rightarrow 1$, $x \rightarrow \infty$ and $1+x^2 \rightarrow x^2$, $x \rightarrow \infty$ are used, leading to

$$I_{\text{asympt}} = \int_0^{\infty} \frac{dx}{x^2}. \quad (2)$$

This integral obviously diverges, and the process fails. To remedy the situation, we introduce a parameter in the problem and rewrite the original integral in the form

$$I \approx \int_0^{x_u} \frac{\tanh(x)}{1+x^2} dx + \int_{x_u}^{\infty} \frac{dx}{x^2} \quad (3)$$

TABLE I
RESULTS OF (4) AND (6) AS A FUNCTION OF x_u

x_u	Eq. 4	Equ. 6
0.1	10.004967	1.476095
0.2	5.019482	1.392883
0.3	3.375800	1.321806
0.5	2.107368	1.214517
0.8	1.476472	1.122527
1.0	1.305428	1.090826
1.2	1.211139	1.072544
1.4	1.156163	1.062127
1.6	1.122612	1.056211
2.0	1.087275	1.050922
2.5	1.068457	1.048963
3.0	1.060035	1.048452
4.0	1.053295	1.048274

or

$$I \approx \int_0^{x_u} \frac{\tanh(x)}{1+x^2} dx + \frac{1}{x_u}. \quad (4)$$

This is the approach used in [6] and [7], and we note that the introduction of the quantity x_u makes it parameter dependent.

In the SAT, we extract from the integrand as little asymptotic forms as possible and, yet, evaluate the resulting contributions in closed form. In this specific example, we take the asymptotic form of the numerator *only* and leave the denominator unchanged. Subtracting and adding the asymptotic term, we rewrite the original integral in the form

$$I = \int_0^{\infty} \frac{\tanh(x)-1}{1+x^2} dx + \int_0^{\infty} \frac{dx}{1+x^2}. \quad (5)$$

The second term can be evaluated in closed form and is equal to $\pi/2$. Therefore,

$$I = \int_0^{\infty} \frac{\tanh(x)-1}{1+x^2} dx + \frac{\pi}{2}. \quad (6)$$

The remaining integrand decreases much faster than the original one and can be computed with much less numerical effort. Not only is (6) exact, it is also parameter free since an additional parameter such as x_u in (4) is not required here.

Table I shows typical results as a function of x_u . Note that for the sole purpose of comparing (4) and (6), the parameter x_u has been artificially introduced in (6). Numerical integration was carried out using eight Gaussian points. These results show that the SAT gives smaller errors for all values of x_u . More importantly, however, it is demonstrated that the accuracy of the approximation increases with the value of x_u . Whereas the method of [6] and [7] in (4) cannot efficiently handle extremely large values of x_u , as this will result in the original improper integral of (1), our method in (6) permits $x_u \rightarrow \infty$. Moreover, in this case, the integral in (6) should not be evaluated directly, but by introducing the mapping $x = \tan[(\pi/2)y]$ with $y \in [0, 1]$. Therefore, no upper limit is needed, and the SAT is clearly parameter free.

III. APPLICATION OF SAT TO SPECTRAL-DOMAIN GREEN’S FUNCTION

We focus attention on the structure shown in Fig. 1, which consists of an open and infinitely long microstrip line of width W on top of a lossless substrate of thickness H and dielectric constant ϵ_1 . We also assume that the metallic strip and the ground plane are perfectly conducting and neglect the thickness of the strip.

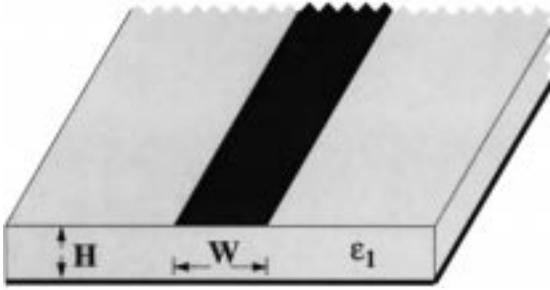


Fig. 1. Cross section of an open microstrip line.

In applying the SDA to this structure, we assume that the dominant mode is propagating along the z -axis with a propagation constant β at a frequency ω . Assuming a surface-current density on the conducting strip, a relationship between the tangential electric field at the interface between the two dielectrics ($y = H$) and the surface-current density is derived from the continuity conditions of the components of the electromagnetic field. In the spectral domain, this relationship involves the Greens' dyadics of the structure [1].

The Green's dyadics of the open microstrip line are easily shown to be of the following forms:

$$\begin{aligned} \tilde{G}_{zz}(\alpha, \beta) &= \frac{1}{\alpha^2 + \beta^2} \left[\frac{\beta^2 \gamma_1 \gamma_2}{\epsilon_1 \gamma_2 \coth(\gamma_1 h) + \gamma_1} - \frac{k_0^2 \alpha^2}{\gamma_1 \coth(\gamma_1 h) + \gamma_2} \right] \end{aligned} \quad (7)$$

$$\begin{aligned} \tilde{G}_{xx}(\alpha, \beta) &= \frac{1}{\alpha^2 + \beta^2} \left[\frac{\alpha^2 \gamma_1 \gamma_2}{\epsilon_1 \gamma_2 \coth(\gamma_1 h) + \gamma_1} - \frac{k_0^2 \beta^2}{\gamma_1 \coth(\gamma_1 h) + \gamma_2} \right] \end{aligned} \quad (8)$$

and

$$\begin{aligned} \tilde{G}_{xz}(\alpha, \beta) &= \frac{\alpha \beta}{\alpha^2 + \beta^2} \left[\frac{\gamma_1 \gamma_2}{\epsilon_1 \gamma_2 \coth(\gamma_1 h) + \gamma_1} + \frac{k_0^2}{\gamma_1 \coth(\gamma_1 h) + \gamma_2} \right]. \end{aligned} \quad (9)$$

Here, $\gamma_i^2 = \alpha^2 + \beta^2 - k_i^2$ and $k_0^2 = \omega^2 \epsilon_0 \mu_0$.

To determine the propagation constant, we expand the surface-current density in series of basis functions and apply Galerkin's method to obtain a set of linear equations in the expansion coefficients

$$[Z_{mm}][c] + [Z_{mn}][d] = 0 \quad (10)$$

$$[Z_{nm}][c] + [Z_{nn}][d] = 0. \quad (11)$$

The entries of the impedance matrix $[Z]$ are given in terms of integrals of the form

$$Z_{mm} = \int_0^\infty \tilde{J}_{zm}(\alpha) \tilde{G}_{zz}(\alpha, \beta) \tilde{J}_{zm}(\alpha) d\alpha \quad (12)$$

$$Z_{nm} = \int_0^\infty \tilde{J}_{xn}(\alpha) \tilde{G}_{xz}(\alpha, \beta) \tilde{J}_{zm}(\alpha) d\alpha \quad (13)$$

$$Z_{nn} = \int_0^\infty \tilde{J}_{xn}(\alpha) \tilde{G}_{xx}(\alpha, \beta) \tilde{J}_{xn}(\alpha) d\alpha. \quad (14)$$

The evaluation of these integrals constitutes a major part in the numerical solution. To overcome this numerical problem, we extract the tail of these integrals using the asymptotic forms of the integrands whose integrals are evaluated in closed form. By adding and subtracting the asymptotic contributions to the original integrals, the effective range of numerical integration is substantially reduced. However, it is important that the extraction of the asymptotic terms does not introduce new numerical pathologies.

To apply the concept of SAT, we first note that when α is large, both γ_1 and γ_2 are approximately equal to α , whereas $\tanh(\gamma_1 h)$ approaches unity

$$\gamma_i = \sqrt{\alpha^2 + \beta^2 - k_i^2} \approx |\alpha|, \quad \alpha^2 \gg (\beta^2 - k_i^2) \quad (15)$$

$$\coth(\gamma_1 h) \approx 1. \quad (16)$$

However, we do not use the asymptotic form of the denominator, as this would lead to a singular integrand at the origin for the zeroth-order Bessel function, as discussed in [6] and [7]. Instead, we purposely leave the denominator in its exact form for the time being:

$$\tilde{G}_{zz}^\infty(\alpha, \beta) = \frac{\alpha}{\alpha^2 + \beta^2} \left[\frac{\beta^2}{1 + \epsilon_1} - \frac{k_0^2}{2} \right] \quad (17)$$

$$\tilde{G}_{xx}^\infty(\alpha, \beta) = \frac{\alpha^3}{\alpha^2 + \beta^2} \frac{\beta^2}{1 + \epsilon_1} \quad (18)$$

$$\tilde{G}_{xz}^\infty(\alpha, \beta) = \frac{\alpha^2}{\alpha^2 + \beta^2} \frac{\beta}{1 + \epsilon_1}. \quad (19)$$

We, therefore, rewrite the original integral Z_{mm} , e.g., in the form

$$\begin{aligned} Z_{mm} &= \int_0^\infty \tilde{J}_{zm}(\alpha) [\tilde{G}_{zz}(\alpha, \beta) - \tilde{G}_{zz}^\infty(\alpha, \beta)] \tilde{J}_{zm}(\alpha) d\alpha \\ &\quad + \int_0^\infty \tilde{J}_{zm}(\alpha) \tilde{G}_{zz}^\infty(\alpha, \beta) \tilde{J}_{zm}(\alpha) d\alpha. \end{aligned} \quad (20)$$

Note that the expression of Z_{mm} given in (20) is still exact, as we only added and subtracted the same quantity.

For the technique to be efficient, the last integral in the above must be known in closed form. When weighted Chebyshev polynomials are used to approximate the surface-current density with the proper edge condition, typical integrals involve Bessel functions leading to expressions of the form [7]

$$I_{mn} = \int_0^\infty \tilde{G}_{zz}^\infty(\alpha, \beta) J_m(\alpha W/2) J_n(\alpha W/2) d\alpha. \quad (21)$$

Depending on the values of m and n , we use different expressions for $\tilde{G}_{zz}^\infty(\alpha, \beta)$. When $m + n \neq 0$, it is possible to drop the term β^2 from the denominator and write I_{mn} as

$$I_{mn} = \int_0^\infty \frac{J_m(\alpha W/2) J_n(\alpha W/2)}{\alpha} d\alpha. \quad (22)$$

This integral can be evaluated in closed form and is given by [8, p. 404]

$$I_{mn} = \frac{2}{\pi} \frac{\sin\left[\frac{(m-n)\pi}{2}\right]}{m^2 - n^2}, \quad m + n \neq 0. \quad (23)$$

Furthermore, in the case of the open microstrip line, only Bessel functions of even orders are used. The integrals I_{mn} vanish, except when $m = n$ [6].

The case where $m = n = 0$ must be treated separately, as (23) is not valid for these values of m and n . Instead of using the asymptotic form $\tilde{G}_{zz}^\infty(\alpha, \beta)$ in (22), which introduces a singularity at the origin, we leave the nonzero term β^2 in the denominator to obtain

$$I_{00} = \int_0^\infty \frac{\alpha J_0(\alpha W/2) J_0(\alpha W/2)}{\alpha^2 + \beta^2} d\alpha. \quad (24)$$

It is worth emphasizing that by keeping the exact form of the denominator, we avoid the introduction of a singularity at $\alpha = 0$ for $m = n = 0$ [6]. A similar expression is given in the appendix of [5] where an "arbitrary" real number is suggested to deal with the divergence; the value of such a parameter is uniquely specified here. The integral in (24) can be evaluated in closed form [9, p. 702] as follows:

$$I_{00} = I_0(\beta W/2) K_0(\beta W/2). \quad (25)$$

TABLE II
EFFECTIVE DIELECTRIC CONSTANT FROM THIS PAPER AND
[6] USING FIVE BASIS FUNCTIONS FOR J_z AND
FOUR FOR J_x ($\epsilon_r = 8$, $W/H = 1$, $\mu_r = 1$)

H/λ_0	This work	[6]
0.005	5.4678	5.4752
0.05	6.1275	6.1316
0.1	6.7580	6.7572
0.3	7.6614	7.6551
0.7	7.9139	7.9151
1.0	7.9529	7.9556

Here, I_0 and K_0 are modified Bessel functions of the first and second kind of order zero. This completes the evaluation of the asymptotic terms. Note that we are not particularly concerned with the Green's function G itself, but rather with the evaluation of the integrals, which naturally involve G , as they affect the solution.

IV. NUMERICAL RESULTS

SAT is used to determine the effective dielectric constant of the microstrip line of Fig. 1.

The effective dielectric constant of a microstrip line of aspect ratio $W/H = 1$ was determined for different values of the ratio H/λ_0 . Table II summarizes the results obtained from five basis functions for J_z and four basis functions for J_x . The agreement between the results from this paper and those presented in [6] is excellent.

The numerical evaluation of the integrals with the proper asymptotic term subtracted and integrated in closed form was carried out using Gauss quadratures. All wavenumbers α , β are scaled in unit of the wavenumber in free space k_0 . The upper limit of integration over the thus scaled variable α was determined as the maximum of $2\epsilon_r$ and $8\pi\lambda_0/H$. These two numbers are chosen to guarantee that all terms in the Green's dyadics have reached their asymptotic expressions, thereby leaving vanishing integrands for values of α larger than this upper limit.

The root of the determinant was located using the bisection method where the root is first bracketed starting from a value equal to the dielectric constant.

To reduce CPU times, the Fourier transforms of the basis functions are evaluated only once at the beginning of the program and stored in the computer's memory. Indeed, these are independent of β and keep the same values at each iteration in the search for the root of the determinant. Overall, 96 Gaussian points were used over the interval of integration, thereby requiring a memory space of the same size for each basis function. Within this implementation, the CPU time required to determine the effective dielectric constant is less than 100 ms per frequency point on an ULTRAPARC machine.

V. CONCLUSIONS

An SAT was introduced and applied to accelerate the analysis of dispersion properties of microstrip lines by the spectral-domain approach. Asymptotic forms of integrands are selectively extracted and evaluated in closed form without introducing additional numerical pathologies. Numerical results obtained from this approach agree well with those in the literature. A substantial reduction in CPU time is achieved by computing the Fourier transforms of the basis functions only once, in addition to subtracting the asymptotic parts of the integrands.

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On the Use of Linear-Prediction Techniques to Improve the Computational Efficiency of the FDTD Method for the Analysis of Resonant Structures

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Abstract—Linear-prediction (LP) techniques are used to accurately and efficiently compute the frequencies and damping factors of microwave resonant structures from their transient response, which was previously obtained by using the finite-difference time-domain (FDTD) method. The LP equations are formulated in terms of a total least squares (TLS) problem and are solved by using the singular-value decomposition (SVD) algorithm. This approach confers robustness to the LP method, improves the spectral resolution, and provides a simple criterion for selecting the order of the LP model. We illustrate these characteristics of the LP method by applying it to two types of problems: the determination of the propagation constants of waveguides loaded with lossy dielectrics, and the calculation of the resonant frequencies of cylindrical cavities loaded with dielectric ring resonators.

Index Terms—FDTD, Maxwell solver, numerical methods.

I. INTRODUCTION

The finite-difference time-domain (FDTD) method is a powerful numerical technique, which is currently used for the analysis of a

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