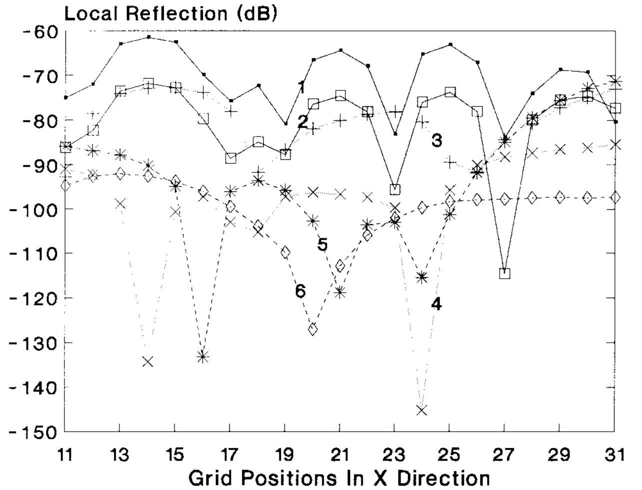

Fig. 3. Normalized local reflections for the E_z-field component of the wave $(\theta = \pi/6)$ along the line $(X, 11, 20.5)$, where Curve 1: $N = 10$, $m = 2$, Curve 2: $N = 15$, $m = 2$, Curve 3: $N = 10$, $m = 3$, Curve 4: $N = 15$, $m = 3$, Curve 5: $N = 10$, $m = 4$, and Curve 6: $N = 15$, $m = 4$.

absorber. Noting that when $N = 15$, the total computational domain is simultaneously changed to 50 × 50 × 51, and the X-axis of Fig. 3 is given according to the case $N = 10$. It can be seen from Fig. 3 that by adjusting the parameters used in the GMIPML absorber, better absorbing performance can be achieved.

IV. Conclusions

By introducing the material-independent quantities $\mathbf{D}$ and $\mathbf{B}$ into the FDTD model, a GMIPML absorber aimed for absorbing electromagnetic waves propagating in 3-D arbitrary anisotropic media consisting of both permittivity and permeability tensors is developed. In contrast with the previous PML absorbers, the conductivities σ^D and σ^B, instead of σ^E and σ^H, are used. The main reason of using σ^D and σ^B in the proposed absorber is due to the fact that these conductivities are independent of the anisotropy of the material. As a consequence, Berenger's PML can be simply and effectively extended to 3-D arbitrary anisotropic materials. Furthermore, due to the special feature (i.e., the material independence) of the proposed GMIPML absorber, it can also be used to absorb electromagnetic waves propagating in materials consisting of loss, dispersion, and nonlinearity [14] with slight modifications. Furthermore, unsplit-field formulations (i.e., without splitting the $\mathbf{D}$ and $\mathbf{B}$ fields) of the GMIPML absorber have been recently proposed [15] by the author.

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Accurate Analysis of Periodic Structures with an Additional Symmetry in the Unit Cell from Classical Matrix Eigenvalues

Smain Amari, Rüdiger Vahldieck, and Jens Bornemann

***Abstract*—Dispersion diagrams of periodic structures with an additional symmetry in the unit cell are investigated by the example of a parallel-plate waveguide loaded with irises of zero thickness. The propagation constants of the Floquet modes are determined from the *classical* eigenvalues of a non-Hermitian matrix.**

***Index Terms*—Eigenfunctions, eigenvalues, integral equations, periodic structures.**

I. Introduction

The growing interest in photonic bandgap (PBG) materials has created a demand for efficient methods of analysis of periodic structures [1]. Periodic structures have been the subject of numerous

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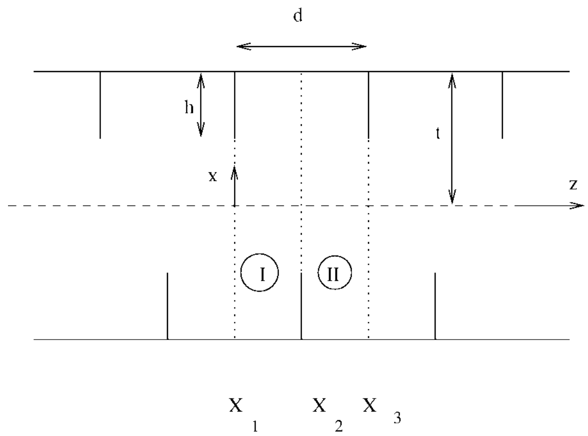

Fig. 1. Side view of a parallel-plate waveguide with glide reflection symmetry.

investigations due to their importance in slow-wave structures and corrugated antennas [2], [3]. The effect of periodicity on the propagation properties is commonly taken into account through expansions in space harmonics [2].

The present approach is based on the coupled-integral-equation technique (CIET), which was recently proposed in [4]. Although isolated discontinuities, mainly of vanishing thickness, are well treated in classical texts of waveguide theory, e.g. [5], it is the combination of multiple discontinuities that reveals the advantages of this technique over classical approaches.

In this paper, a structure with glide reflection symmetry was purposely chosen to show how additional symmetries in the unit cell of a periodic structure can be straightforwardly included in the approach. A similar structure was investigated by Hessel and Oliner using coupled-mode theory where the propagation constant is determined from a determinant equation [6]. Here, the propagation constants are determined from the *classical* eigenvalues of a square non-Hermitian matrix instead of a determinant.

II. Theory

The cross section of a periodic structure with glide reflection symmetry is shown in Fig. 1. The period is d. The structure is invariant under a translation by $d/2$ along the z-axis followed by a reflection $x \to -x$. It can be shown that the modes of this structure can be divided into two groups whose elements are eigenfunction of a glide reflection operator G with eigenvalues g_1 and g_2 such that

$$g_1 = e^{-\theta(d/2)} \quad g_2 = -e^{-\theta(d/2)} \tag{1}$$

the details of which may be found in [6]. In the following, the modes themselves will be referred to as g_1 and g_2 for simplicity. We limit the analysis to TM modes with no y-dependence.

The transverse components of the electromagnetic fields in regions I and II can be expanded in series of modes of the uniform sections. Since the structure is not invariant under a reflection $x \to -x$, both even and odd modes of the uniform parallel-plate waveguide are needed. Let $\phi_m^e(x)$ and $\phi_m^o(x)$ denote the normalized even and odd modes of the empty parallel-plate waveguide with no y-dependence. The wave admittance and propagation constants are denoted by Y_m^e, Y_m^o, k_{zm}^e, and k_{zm}^o, respectively.

Let us assume that the exact distributions of E_x at $z = 0$, $z = d/2$, and $z = d$ are given by the three unknown functions: $X_1(x)$, $X_2(x)$, and $X_3(x)$.

The Floquet condition leads to the relationship

$$X_3(x) = e^{-\theta d} X_1(x). \tag{2}$$

In the present case, the glide reflection symmetry leads to the additional relation (for mode g_1)

$$X_2(x) = e^{-\theta d/2} X_1(-x). \tag{3}$$

Therefore, we are left with only one unknown function, namely $X_1(x)$.

Following the CIET [4], we derive an integral equation for the function $X_1(x)$, which is then solved by the moment method. To this end, $X_1(x)$ is expanded in a series of the form

$$X_1(x) = \sum_{i=1}^{M} c_i B_i(x) = \sum_{i=1} c_i [B_i^e(x) + B_i^o(x)]. \tag{4}$$

Here, $B_i^e(x)$ and $B_i^o(x)$ are the even and odd parts of the ith basis function. It is not efficient to apply Galerkin's method in this case since the basis functions are used to expand the electric field at $z = 0$, whereas the integral equation is obtained from the continuity of the magnetic field at $z = d/2$. We need to project the integral equation over a function in the range of H_y at $z = d/2$. We thus use the testing functions

$$T_i(x) = B_i^e(x) - B_i^o(x). \tag{5}$$

The propagation constants θ are then determined from the following matrix eigenvalue problem:

$$[K][c] = \cosh(\theta d/2)[L][c]. \tag{6}$$

The entries of the matrices $[K]$ and $[L]$ are given by

$$[K]_{ij} = \sum_{n=1}^{\infty} Y_n^e \frac{\tilde{B}_i^e(n)\tilde{B}_j^e(n)}{\tan(k_{zn}^e d/2)} + \sum_{n=1}^{\infty} Y_n^o \frac{\tilde{B}_i^o(n)\tilde{B}_j^o(n)}{\tan(k_{zn}^o d/2)} \tag{7}$$

$$[L]ij = \sum_{n=1}^{\infty} Y_n^e \frac{\tilde{B}_i^e(n)\tilde{B}_j^e(n)}{\sin(k_{zn}^e d/2)} - \sum_{n=1}^{\infty} Y_n^o \frac{\tilde{B}_i^o(n)\tilde{B}_j^o(n)}{\sin(k_{zn}^o d/2)} \tag{8}$$

where

$$\tilde{B}_p(n) = \int_{-t}^{t-h} \phi_n^p(x) X(x)\, dx, \qquad p = e, o. \tag{9}$$

Although the matrices $[K]$ and $[L]$ are both real and symmetric, they do not necessarily commute. The product $[L]^{-1}[K]$ is *not* symmetric and, therefore, the eigenvalues are, in general, complex.

The eigenvalue matrix equation for g_2 is obtained from (6) by changing the sign of the matrix $[L]$.

A major advantage of the present formulation lies in the fact that the propagation constants are determined from the *classical* eigenvalues of a matrix. The dispersion relation of many modes, including complex and evanescent modes, can be easily computed.

III. Numerical Results

To guarantee efficiency in the numerical solution, edge-conditioned basis functions are used. The dispersion diagram for the two eigenmodes of the glide reflection operator G were determined in the interval $[-2\pi, 2\pi]$.

Fig. 2 shows the first few branches of the dispersion diagram obtained from the present technique with three basis functions. The presence of a bandgap in the diagram is typical of periodic structures.

The fact that the individual dispersion curves of the two modes g_1 and g_2 are not invariant under a translation $\beta \to \beta + (2\pi/d)$ is clearly visible in Fig. 2. However, the overall dispersion diagram is indeed periodic with a period 2π. It is also evident that the dispersion curves of the two modes are interchanged under the same translation. This property was used by Hessel and Oliner to investigate the periodicity of a periodic structure with glide reflection symmetry [6].

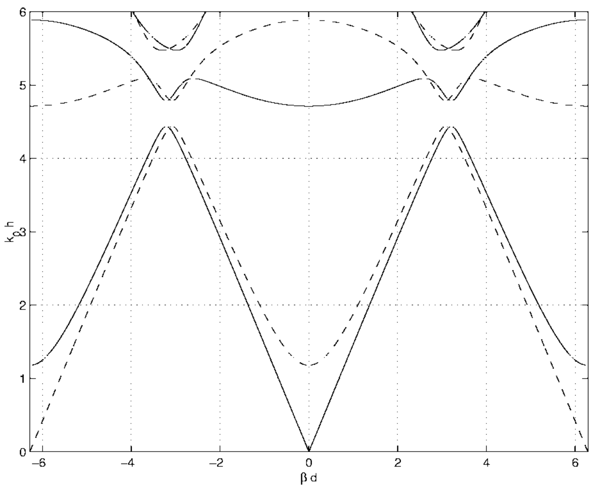

Fig. 2. $k_0 - \beta$ diagram of the first few Floquet modes. Mode g_1 (solid line) and mode g_2 (dashed line). $h = 1.5$ cm, $t = 2$ cm, and $d = 0.5$cm.

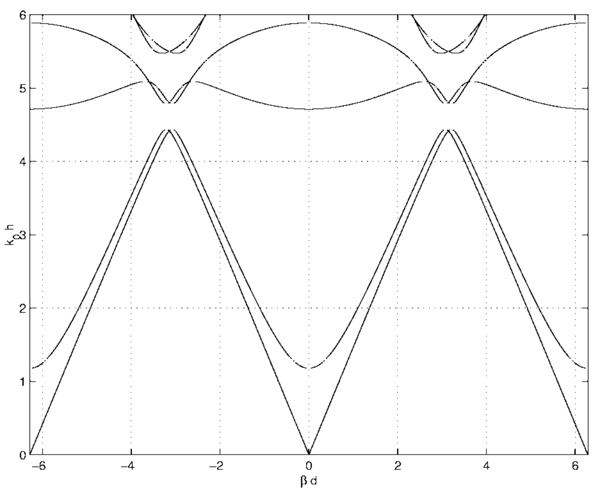

Fig. 3. Overall $k_0 - \beta$ diagram showing the periodicity to be 2π. These are the same dimensions as in Fig. 2.

The important fact that the periodicity of the overall dispersion diagram is 2π is better illustrated in Fig. 3 for the same dimensions as in Fig. 2. Note that Fig. 3 can be obtained from a direct analysis in which the glide reflection symmetry is not taken into account.

Although most investigations of periodic structures focus on the propagating modes, the present approach allows us to rapidly and accurately determine the attenuated and complex modes as well. Fig. 4 shows a plot of the real and imaginary parts of θ versus normalized frequency for the same dimensions as in Fig. 2. Higher order complex modes were also found, but are not reported here. An interesting feature of the complex modes is the presence of propagation constants of the form $\theta d = \alpha d + j2\pi$ (straight upper thick line). This is in contrast to similar modes in periodically loaded transmission lines, where propagation constants of the form $\theta d = \alpha d + j\pi$ (π instead of 2π) are also present [7, p. 367].

IV. Conclusions

Propagation properties of a periodic structure with a symmetry in the unit cell were analyzed by the example of a structure with reflection symmetry using the CIET. The propagation constants

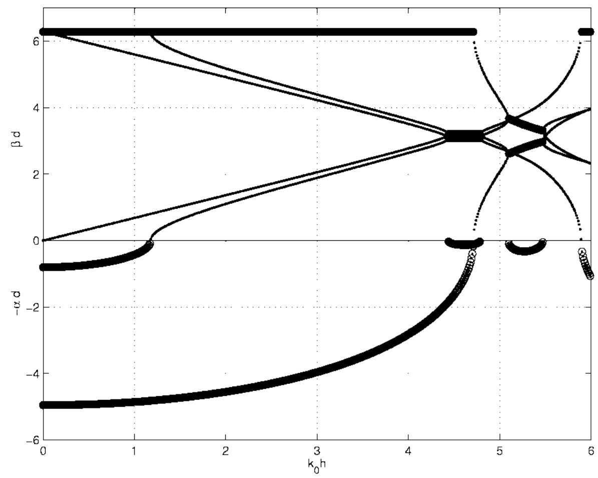

Fig. 4. Imaginary and real parts of θ as a function of frequency (k_0h). The thick lines indicate complex modes. These are the same dimensions as in Fig. 2.

are determined from the *classical* eigenvalues of a non-Hermitian generalized matrix eigenvalue problem.

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