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REFERENCES

1. P. W. Hannan, D. S. Lerner, and G. H. Knittel, "Impedance Matching a Phase-Array Antenna Over Wide Scan Angles by Connecting Circuits," *IEEE Trans. Antennas Propagat.*, Vol. AP-13, Jan. 1965, pp. 28–34.
2. D. Pozar, "Analysis of an Infinite Array of Rectangular Microstrip patches with Idealized Probe Feeds," *IEEE Trans. Antennas Propagat.*, Vol. AP-32, Oct. 1984, pp. 1101–1107.
3. J. Herd, "Full Wave Analysis of Proximity Coupled Rectangular Microstrip Antenna Arrays," *Electromagn.*, Vol. 11, Jan. 1991, pp. 21–46.
4. Hewlett Packard Company, Microwave and RF Design Systems, HP 85150D, Santa Rosa, CA, May 1994.

SIMPLE AND EFFICIENT NUMERICAL EVALUATION OF THE ONE-DIMENSIONAL GENERALIZED EXPONENTIAL INTEGRAL FOR ULTRA-THIN WIRE ANTENNAS

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ABSTRACT: *A new, simple, and efficient technique for the accurate calculation of the one-dimensional generalized exponential integral is presented. The method is very robust, suitable for even ultra-thin wire antennas, which are of great interest for communications with submersible vehicles, for example.* © 1998 John Wiley & Sons, Inc. Microwave Opt Technol Lett 19: 255–257, 1998.

Key words: *generalized exponential integral; wire antennas; method of moments*

I. INTRODUCTION

The generalized exponential integral is one of the most elementary integrals in applied electromagnetics. Accurate calculation of the integral is essential in, e.g., the examination of radiating properties of various structures by the method of moments.

The one-dimensional form of the generalized exponential integral is well known from the calculation of mutual and self-impedances in wire antennas. The problem of calculating the self-impedance of a wire element of length $2z_1$ and radius a translates numerically to the evaluation of the integral

$$I_1 = \int_{-z_1}^{z_1} \frac{e^{-jk\sqrt{z^2+a^2}}}{\sqrt{z^2+a^2}}\, dz. \tag{1}$$

Note that while I_1 does not exist in the case of an infinitesimally thin wire ($a = 0$), the two-dimensional variety of the integral does exist— and can be calculated exactly [1]—even for a equaled to zero, which physically corresponds to integrating the electric field over the surface of a planar radiator.

Exact solutions to the integral, especially for the wires that are electrically thin, have long eluded antenna engineers and researchers. Although a multitude of techniques for the evaluation of I_1 can be found in the literature, most of them make certain convenient but restrictive assumptions which limit the application range of the techniques. Recently, several procedures that are general and free of the customary restrictions have appeared. Of particular importance is the technique of [2], which is shown to be stable for an a/λ as small as 10^{-19}.

This paper introduces a new and exact, yet simple, method for the evaluation of the one-dimensional generalized exponential integral, suitable for even ultra-thin ($a/\lambda \approx 10^{-30}$) wire antennas. A decomposition of the integral's kernel is used to avoid the usual numerical problems encountered when the integral is computed directly. The high efficiency of the method will be demonstrated, and comparison with benchmark results as well as those of [2] will be presented.

II. INTEGRAL CALCULATION

We split the integral I_1 into real and imaginary parts. The imaginary part presents no numerical problems, and therefore can be evaluated directly as is. The real part is more difficult to tackle because of the singularity that emerges when $a = 0$ and $z = 0$.

In order to also accurately calculate the real part of I_1, the kernel of the integral is decomposed into two parts:

$$\mathrm{Re}[I_1] = 2\int_0^{z_1} \frac{\cos\left(k\sqrt{z^2+a^2}\right) - \cos(ka)}{\sqrt{z^2+a^2}}\, dz + 2\cos(ka) \cdot \int_0^{z_1} \frac{dz}{\sqrt{z^2+a^2}}. \tag{2}$$

The first integrand in (2) is a much better behaving function than that of the real part expressed as a single component. (Actually, the first integrand is not singular anymore, as a limit, even when $a \to 0$.) The singularity has been isolated and is contained in the second integral, which we evaluate analytically:

$$\int_0^{z_1} \frac{dz}{\sqrt{z^2+a^2}} = \mathrm{ar\,sinh}\left(\frac{z_1}{a}\right). \tag{3}$$

Employing the identity

$$\mathrm{ar\,sinh}(x) \equiv \ln\left(x + \sqrt{x^2+1}\right), \tag{4}$$

the overall expression for I_1 as a result is

$$I_1 = 2\left[\int_0^{z_1} \frac{\cos\left(k\sqrt{z^2+a^2}\right) - \cos(ka)}{\sqrt{z^2+a^2}}\, dz + \cos(ka) \cdot \ln\left(\frac{z_1}{a} + \sqrt{\left(\frac{z_1}{a}\right)^2 + 1}\right)\right] - j2\int_0^{z_1} \frac{\sin\left(k\sqrt{z^2+a^2}\right)}{\sqrt{z^2+a^2}}\, dz. \tag{5}$$

The above equation corresponds to the integration passing through the origin, point $z = 0$ [cf. (1)]. When integration from an arbitrary z_1 to an arbitrary z_2 is required, I_1 becomes

$$I_2 = \int_{z_1}^{z_2} \frac{e^{-jk\sqrt{z^2+a^2}}}{\sqrt{z^2+a^2}}\, dz. \tag{6}$$

It is transparent that our integration technique is also directly applicable to this integral; a straightforward manipulation yields

$$I_2 = \int_{z_1}^{z_2} \frac{\cos\left(k\sqrt{z^2+a^2}\right) - \cos(ka)}{\sqrt{z^2+a^2}}\, dz + \cos(ka) \cdot \ln \frac{\frac{z_2}{a} + \sqrt{\left(\frac{z_2}{a}\right)^2 + 1}}{\frac{z_1}{a} + \sqrt{\left(\frac{z_1}{a}\right)^2 + 1}} - j\int_{z_1}^{z_2} \frac{\sin\left(k\sqrt{z^2+a^2}\right)}{\sqrt{z^2+a^2}}\, dz. \tag{7}$$

If the Gauss–Legendre quadrature is employed to perform the integrations in (5) and (7), respectively, only a few integration points are needed in order to obtain accurate results, as will be demonstrated in the following section.

III. NUMERICAL RESULTS

In the first application, the above-described integration technique was implemented in a method-of-moments code computing the input admittance of a center-fed straight-wire antenna (length-to-diameter ratio of 74.2, impedance-matrix order of 33) as a function of frequency. Agreement within the plotting accuracy with the results presented by Harrington in [3] was observed.

Comparison computations with the half-wave, thin-wire results of [2] using the Harrington formulation were performed. The data, as in [2], ranging from $a/\lambda = 10^{-19}$ to 10^{-4} and calculated for 63 wire segments, were reproduced within the plotting accuracy of the above. A few of the values are presented compared one-on-one in Table 1.

To test the numerical stability of our technique, we have lowered the number of wire segments as well as further reduced the thickness of the analyzed wire. The computed data are shown in Table 2. When the order of the impedance matrix (i.e., the number of wire elements that the antenna is segmented into) is gradually reduced from 63 to 33, the real part of the calculated input impedance changes by less than 0.6% and the imaginary part by 3.6%.

No more than three integration points in the integrals of (5) and (7), respectively, were needed to obtain all of the presented results. Higher counts of integration points resulted in numbers that agreed in seven significant values with the data produced by integrating in merely three points.

IV. CONCLUSIONS

A novel technique is introduced to accurately calculate the one-dimensional generalized exponential integral. The method is completely general and free of any restrictions on its application. A decomposition of the integral's kernel into two parts, one of which is evaluated analytically and the other numerically, is employed. Since the remaining integrand obtained this way is a much smoother function than that when the integral is expressed by a single term, only very few integration points are needed to perform the integration exactly, rendering the procedure highly efficient. In addition, the technique is suitable for the analysis of ultra-thin wire antennas (with the radius-to-wavelength ratio as small as 10^{-30}, possibly even smaller). Due to the simplicity of the technique, minimal effort is needed for the implementation.

TABLE 1 Performance Comparison of Our Technique and That of [2] for a Center-Fed Half-Wave Dipole and Various Values of Wire Radius *a* (Harrington's Formula)

	Z_{in} [Ω]	
$\log(a/\lambda)$	Procedure of [2]	Our Technique
−4	79.83 + j43.35	79.857 + j43.391
−9	75.23 + j42.07	75.217 + j42.104
−14	74.37 + j41.80	74.344 + j41.815
−19	73.98 + j41.66	73.974 + j41.668

TABLE 2 Convergence Performance, in Terms of Input Impedance of a Center-Fed Half-Wave Dipole for Various Values of Wire Radius *a*, of Our Technique

No. of Wire	Z_{in} [Ω] for			
Segments	$\log(a/\lambda) = -4$	$\log(a/\lambda) = -10$	$\log(a/\lambda) = -20$	$\log(a/\lambda) = -30$
63	79.857	74.959	73.924	73.618
	+j43.391	+j42.023	+j41.645	+j41.477
53	79.758	74.880	73.899	73.596
	+j43.020	+j41.378	+j41.283	+j41.040
43	79.621	74.926	73.865	73.565
	+j42.534	+j41.721	+j40.948	+j40.693
33	79.406	74.799	73.803	73.508
	+j41.825	+j40.900	+j40.549	+j40.345

Excellent agreement with benchmark results and stability of the technique are demonstrated.

REFERENCES

1. M. Gimersky, S. Amari, and J. Bornemann, “Numerical Evaluation of the Two-Dimensional Generalized Exponential Integral,” *IEEE Trans. Antennas Propagat.*, Vol. 44, Oct. 1996, pp. 1422–1425.
2. D. H. Werner, P. L. Werner, J. A. Huffman, A. J. Ferraro, and J. K. Breakall, “An Exact Solution of the Generalized Exponential Integral and Its Application to Moment Method Formulations,” *IEEE Trans. Antennas Propagat.*, Vol. 41, Dec. 1993, pp. 1716–1719.
3. R. F. Harrington, *Field Computation by Moment Methods*, IEEE Press, Piscataway, NJ, 1993, p. 72.

CPW-FED SLOT ANTENNA WITH CPW TUNING STUB LOADING

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ABSTRACT: *Adjusting the slot width in a slot antenna can determine the operating frequency bandwidth. In order to obtain a wider radiation beamwidth, a narrower slot width is needed. This paper uses a CPW tuning stub in the CPW-fed slot antenna to achieve this goal.* © 1998 John Wiley & Sons, Inc. Microwave Opt Technol Lett 19: 257–258, 1998.

Key words: *coplanar waveguide-fed slot antenna; tuning stub length*

1. INTRODUCTION

A uniplanar CPW-fed active slot antenna has been reported [1] and has many applications. Some characteristics of a slow antenna on multilayer dielectric substrates [2, 3] have also been discussed, which have a wider bandwidth than a patch antenna. In this paper, we report more characteristics of a slot antenna with a single layer substrate. The widest bandwidth can be adjusted by changing the slot width. For obtaining a wider 3 dB radiation beamwidth, a narrower slot width is chosen, and tuning stub loading after the antenna must be used. The tuning stub changes the reactance of the slot antenna, which has been used in a microstrip-slot-fed patch antenna [4] for tuning the imaginary part of the input impedance. This paper discusses the difference in radiation pattern beamwidth, resonant frequency, and bandwidth using different loading stub lengths. Experimental results show that the tuning stub length is an important factor affecting the resonant frequency and bandwidth.

2. EXPERIMENTAL RESULTS

Figure 1 shows the geometry of a CPW-fed slot antenna with tuning length stub loading. The two slots are of the same size, and are printed on a substrate of thickness (h) 1.6 mm and relative permittivity (ϵ_r) 4.2. The slot length (L) determines the resonant length, while the slot has a width (G) which may be adjusted to achieve a wider bandwidth. The optimal value of G is found as 7 mm. Resonant frequency occurs at about 2.33 GHz, and the bandwidth is 19.3% with VSWR < 2. Figure 2 shows the return loss of various slot widths with tuning lengths resulting in different resonant frequencies. As the slot width gets narrower, the tuning length needs to be longer, the resonant frequency decreases, and the antenna bandwidth gets narrower. At $G = 5$ mm, $t = 10$ mm, a resonant frequency of 2.1 GHz appears with 12.8% bandwidth.

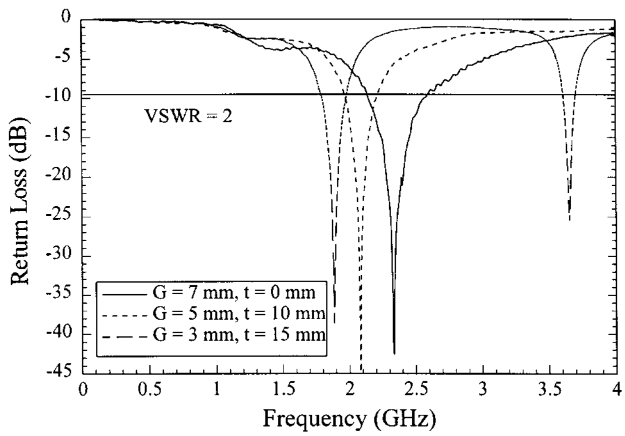

Figure 2 Return loss versus frequency for various slot widths and tuning lengths: $h = 1.6$ mm, $\epsilon_r = 4.2$, $s = 0.5$ mm, $w = 3.8$ mm, and $L = 41.5$ mm

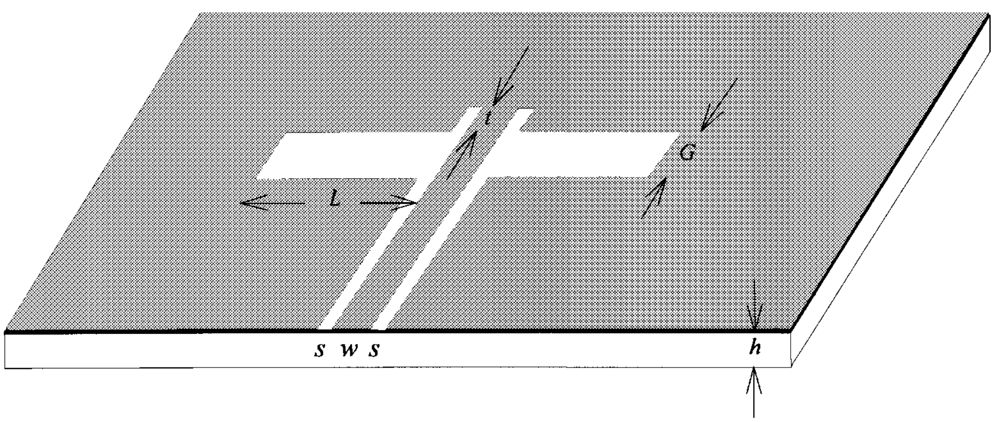

Figure 1 Geometry of slot antenna with coplanar waveguide tuning stub loading