THE MODE MATCHING TECHNIQUE

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I. Background

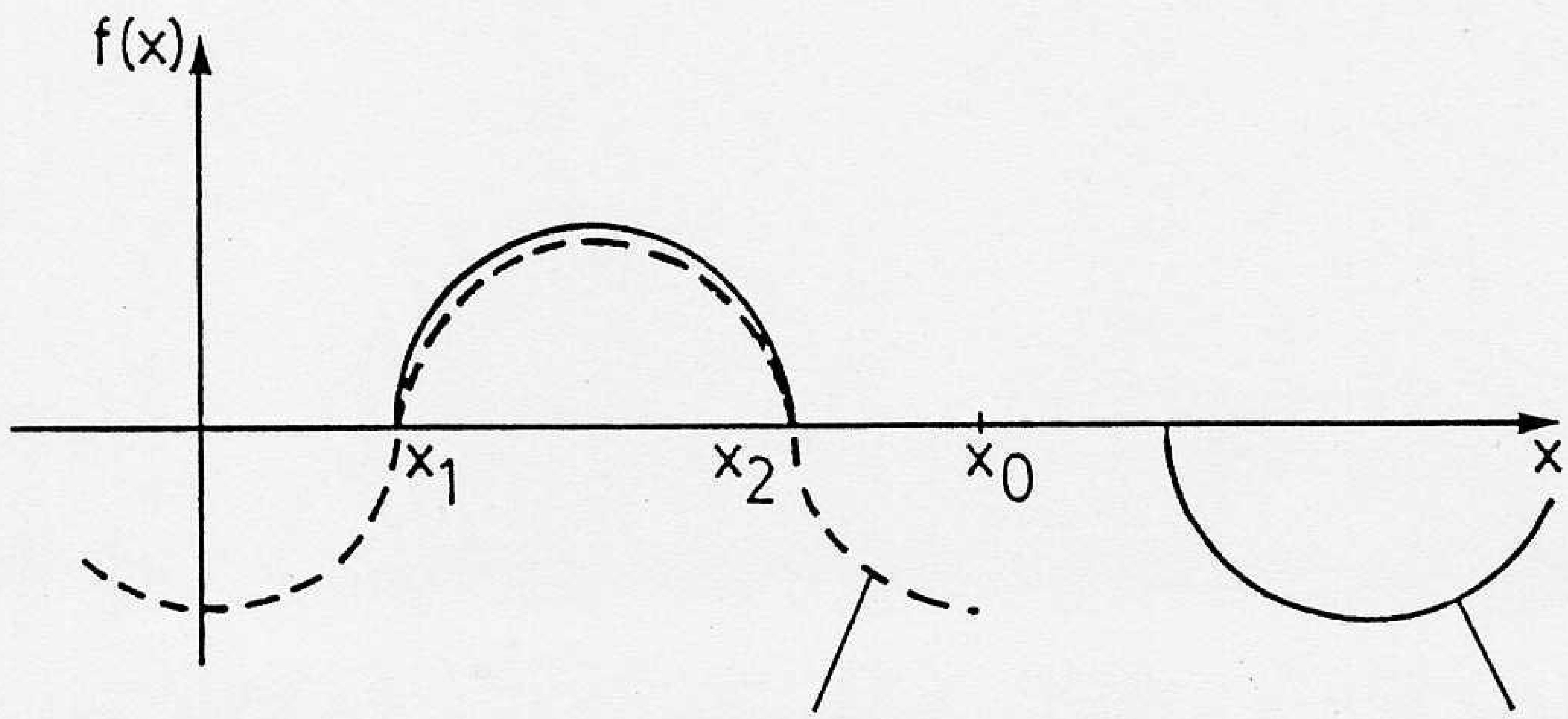
Fourier series: $f_1(x) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi}{x_0} x$

To solve for coefficients a_n, multiply by $\sin \frac{m\pi}{x_0} x$ and integrate

$$\int_0^{x_0} \sin\left(\frac{m\pi}{x_0}x\right) f_1(x)dx = \sum_{n=1}^{\infty} a_n \int_0^{x_0} \sin\left(\frac{m\pi}{x_0}x\right) \sin\left(\frac{n\pi}{x_0}x\right) dx$$

Since these sin functions constitute an orthogonal function system, the integral on the right yields $x_0/2$ if $m = n$, and vanishes otherwise. Hence

$$\frac{2}{x_0}\int_0^{x_0} \sin\left(\frac{m\pi}{x_0}x\right) f_1(x)dx = a_m.$$



$f_2(x) = f_2(x + n2[x_2 - x_1])$ $f_1(x) = f_1(x + n2x_0)$

Assume that $f_1(x)$ is an unknown function but in a different interval $[x_1,\ x_2]\epsilon[0, x_0]$, $f_1(x)$ equals $f_2(x)$ which can also be expressed by a Fourier series.

$$x_1 \leq x \leq x_2: \quad f_1(x) = f_2(x) = \sum_{k=1}^{\infty} b_k \sin\left\{\frac{k\pi}{x_2 - x_1}(x - x_1)\right\}$$

Then

$$a_m = \frac{2}{x_0}\sum_{k=1}^{\infty} b_k \int_0^{x_0} \sin\left(\frac{m\pi}{x_o}x\right) \sin\left\{\frac{k\pi}{x_2 - x_1}(x - x_1)\right\} dx$$

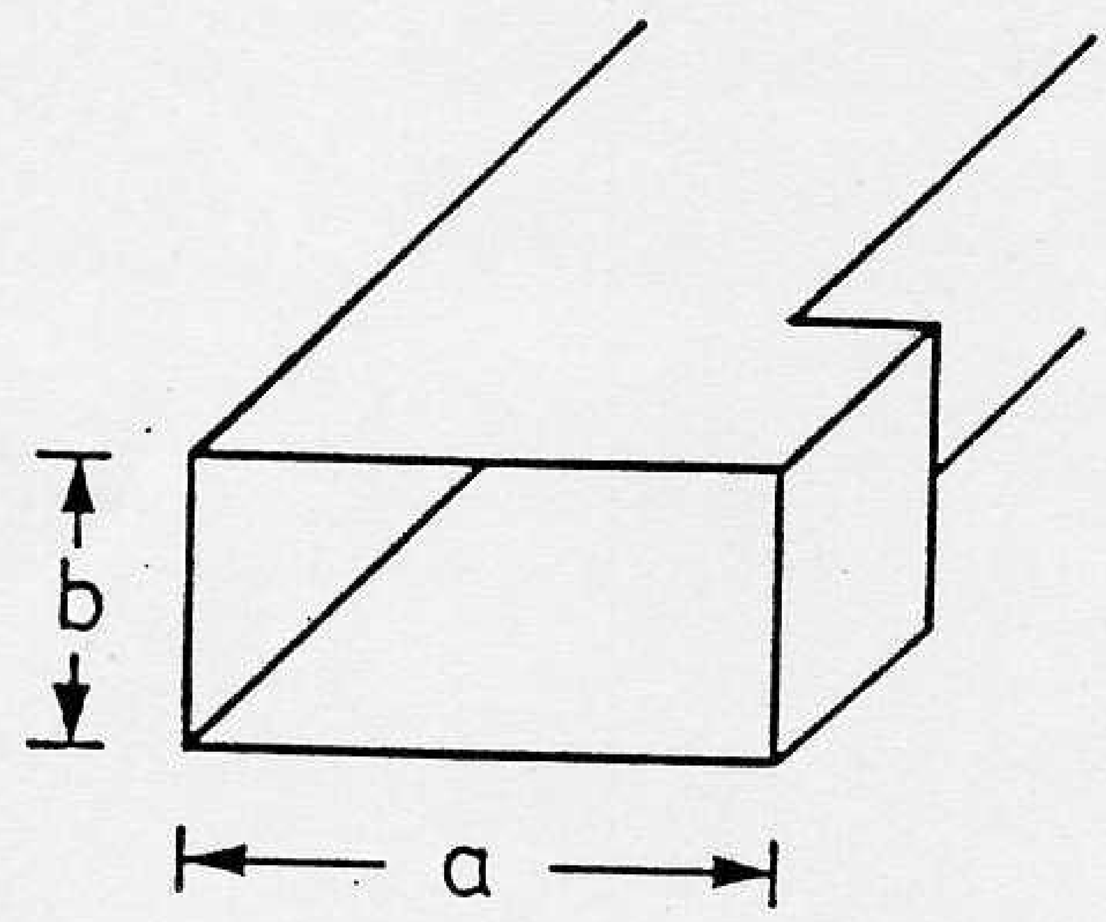
Coefficients b_k can be calculated similarly

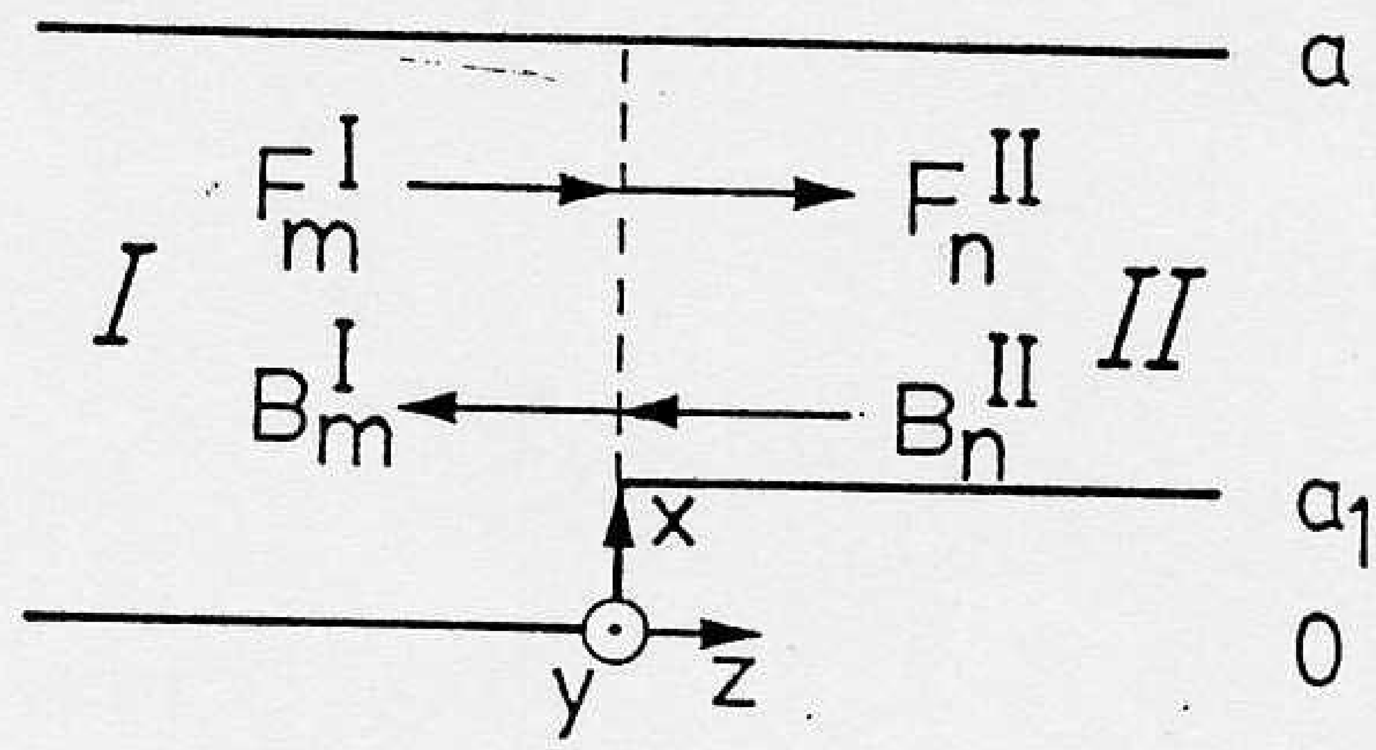
$$b_k = \frac{2}{x_2 - x_1}\sum_{m=1}^{\infty} a_m \int_{x_1}^{x_2} \sin\left\{\frac{k\pi}{x_2 - x_1}(x - x_1)\right\} \sin\left(\frac{m\pi}{x_0}x\right) dx$$

These two equations basicly correspond to the matching conditions for the electric and magnetic field components at a discontinuity.

II. H-Plane Discontinuity

1. Modal Scattering Matrix





$$E_y^I = \sum_{m=1}^{M} T_m^I \sin\left(\frac{m\pi}{a}x\right)\left(F_m^I e^{-jk_{zm}^I z} + B_m^I e^{+jk_{zm}^I z}\right)$$

$$H_x^I = -\sum_{m=1}^{M} T_m^I Y_m^I \sin\left(\frac{m\pi}{a}x\right)\left(F_m^I e^{-jk_{zm}^I z} - B_m^I e^{+jk_{zm}^I z}\right)$$

$$k_{zm}^I = \begin{cases} +\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{m\pi}{a}\right)^2} & , \quad \text{propagating mode} \\ -j\sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu_0\epsilon_0} & , \quad \text{evanescent mode} \end{cases}$$

$$Y_m^I = \frac{k_{zm}^I}{\omega\mu_0}$$

$$E_y^{II} = \sum_{n=1}^{N} T_n^{II} \sin\left\{\frac{n\pi}{a-a_1}(x-a_1)\right\}\left(F_n^{II} e^{-jk_{zn}^{II} z} + B_n^{II} e^{+jk_{zn}^{II} z}\right)$$

$$H_x^{II} = -\sum_{n=1}^{N} T_n^{II} Y_n^{II} \sin\left\{\frac{n\pi}{a-a_1}(x-a_1)\right\}\left(F_n^{II} e^{-jk_{zn}^{II} z} - B_n^{II} e^{+jk_{zn}^{II} z}\right)$$

$$(k_{zn}^{II})^2 = \omega^2\mu_0\epsilon_0 - \left(\frac{n\pi}{a-a_1}\right)^2 \quad , \quad Y_n^{II} = \frac{k_{zn}^{II}}{\omega\mu_0}$$

Matching the transverse field components at the discontinuity ($z = 0$)

$$\left\{ \begin{array}{ll} E_y^I = 0 & , \quad 0 \le x \le a_1 \\ E_y^I = E_y^{II} & , \quad a_1 \le x \le a \end{array} \right\}$$

$$H_x^I = H_x^{II} \quad , \quad a_1 < x \le a$$

$$E_Y \; : \; \sum_{m=1}^{M} T_m^I \sin\left(\frac{m\pi}{a}x\right)(F_m^I + B_m^I) = \sum_{n=1}^{N} T_n^{II} \sin\left\{\frac{n\pi}{a-a_1}(x-a_1)\right\}(F_n^{II} + B_n^{II})$$

$$H_x \; : \; \sum_{m=1}^{M} T_m^I Y_m^I \sin\left(\frac{m\pi}{a}x\right)(F_m^I - B_m^I) =$$

$$= \sum_{n=1}^{N} T_n^{II} Y_n^{II} \sin\left\{\frac{n\pi}{a-a_1}(x-a_1)\right\}(F_n^{II} - B_n^{II})$$

E_y : multiply by $\sin\left(\frac{m'\pi}{a}x\right)$ and integrate from 0 to a

H_x : multiply by $\sin\left\{\frac{n'\pi}{a-a_1}(x-a_1)\right\}$ and integrate from a_1 to a

$$E_y \; : \; \frac{a}{2} T_m^I (F_m^I + F_m^{II}) =$$

$$= \sum_{n=1}^{N} T_n^I \left\{\int_0^{a_1} 0dx + \int_{a_1}^{a} \sin\left(\frac{m\pi}{a}x\right)\sin\left\{\frac{n\pi}{a-a_1}(x-a_1)dx\right\}(F_n^{II} + B_n^{II})\right.$$

$$H_x \; : \; \sum_{m=1}^{M} T_m^I Y_m^I \int_{a_1}^{a} \sin\left\{\frac{n\pi}{a-a_1}(x-a_1)\right\}\sin\left(\frac{m\pi}{a}x\right)dx(F_m^I - B_m^I) =$$

$$= \frac{a-a_1}{2} T_n^{II} Y_n^{II} (F_n^{II} - B_n^{II})$$

Normalization

Scattering parameters are defined as the ratio of normalized quantities. For a given wave amplitude of $F_m^I = 1\sqrt{W}(B_m^I = 0)$, the complex power P_m^I carried by that wave is normalized so that

$$P_m^I = \frac{1}{2}\int_{A^I} (\vec{E}_m^I \times \vec{H}_m^{I*}) d\vec{A} = \frac{1}{2}\int_{y_l}^{y_u}\int_{x_l}^{x_u} (E_{xm}^I H_{ym}^{I*} - E_{ym}^I H_{xm}^{I*}) dx dy$$

$$= \left\{ \begin{array}{ll} 1W & \text{propagating mode} \\ jW & \text{evanescent TE mode} \\ -jW & \text{evanescent TM mode} \end{array} \right\}$$

In this example $E_x = H_y = 0$.

Therefore it follows that $\quad T_m^I = 2\sqrt{\frac{\omega\mu_0}{abk_{zm}^I}}$

and similarly $\quad T_n^{II} = 2\sqrt{\frac{\omega\mu_0}{(a-a_1)bk_{zn}^{II}}}.$

$$E_y \ : \ F^I_m + B^I_m = \sum_{n=1}^{N} (L_E)_{mn}(F^{II}_n + B^{II}_n)$$

$$H_x \ : \ \sum_{m=1}^{M} (L_H)_{nm}(F^I_m - B^I_m) = F^{II}_n - B^{II}_n$$

$$(L_E)_{mn} = 2\sqrt{\frac{k^I_{zm}}{a(a-a_1)k^{II}_{zn}}} \int_{a_1}^{a} \sin\left(\frac{m\pi}{a}x\right) \sin\left\{\frac{n\pi}{a-a_1}(x-a_1)\right\} dx$$

$$= (L_H)_{nm}$$

In the notation of vectors and matrices the two equations read

$$\underline{F}^I + \underline{B}^I = \underline{\underline{L}}_E(\underline{F}^{II} + \underline{B}^{II})$$
$$\underline{\underline{L}}_H(\underline{F}^I - \underline{B}^I) = \underline{F}^{II} - \underline{B}^{II}.$$

The S-matrix expresses the scattered waves as a function of incident waves.

$$\begin{bmatrix} \underline{B}^I \\ \underline{F}^I \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_{11} & \underline{\underline{S}}_{12} \\ \underline{\underline{S}}_{21} & \underline{\underline{S}}_{22} \end{bmatrix} \begin{bmatrix} \underline{F}^I \\ \underline{B}^I \end{bmatrix}$$

$$\underline{\underline{S}}_{11} = [\underline{\underline{L}}_E\underline{\underline{L}}_H + \underline{\underline{I}}]^{-1}[\underline{\underline{L}}_E\underline{\underline{L}}_H + \underline{\underline{I}}]$$

$$\underline{\underline{S}}_{12} = 2[\underline{\underline{L}}_E\underline{\underline{L}}_H + \underline{\underline{I}}]^{-1}\underline{\underline{L}}_E$$

$$\underline{\underline{S}}_{21} = \underline{\underline{L}}_H[\underline{\underline{I}} - \underline{\underline{S}}_{11}]$$

$$\underline{\underline{S}}_{22} = \underline{\underline{I}} - \underline{\underline{L}}_H\underline{\underline{S}}_{12} \qquad \underline{\underline{I}} = \text{unit matrix}$$

Modal Scattering Matrix (for $M = N = 2$)

$$[\underline{\underline{S}}] = \begin{bmatrix} S_{11}(1,\ 1) & S_{11}(1,\ 2) & S_{12}(1,\ 1) & S_{12}(1,\ 2) \\ S_{11}(2,\ 1) & S_{11}(2,\ 2) & S_{12}(2,\ 1) & S_{12}(2,\ 2) \\ & & & \\ S_{21}(1,\ 1) & S_{21}(1,\ 2) & S_{22}(1,\ 1) & S_{22}(1,\ 2) \\ S_{21}(2,\ 1) & S_{21}(2,\ 2) & S_{22}(2,\ 1) & S_{22}(2,\ 2) \end{bmatrix}$$

e.g. $S_{21}(2,\ 1)$ is the transmission coefficient of the second mode (TE_{20}) in region II related to the first mode (TE_{10}) incident at port I.

The modal scattering matrix of a lossless reciprocal multiport is symmetric and orthogonal.

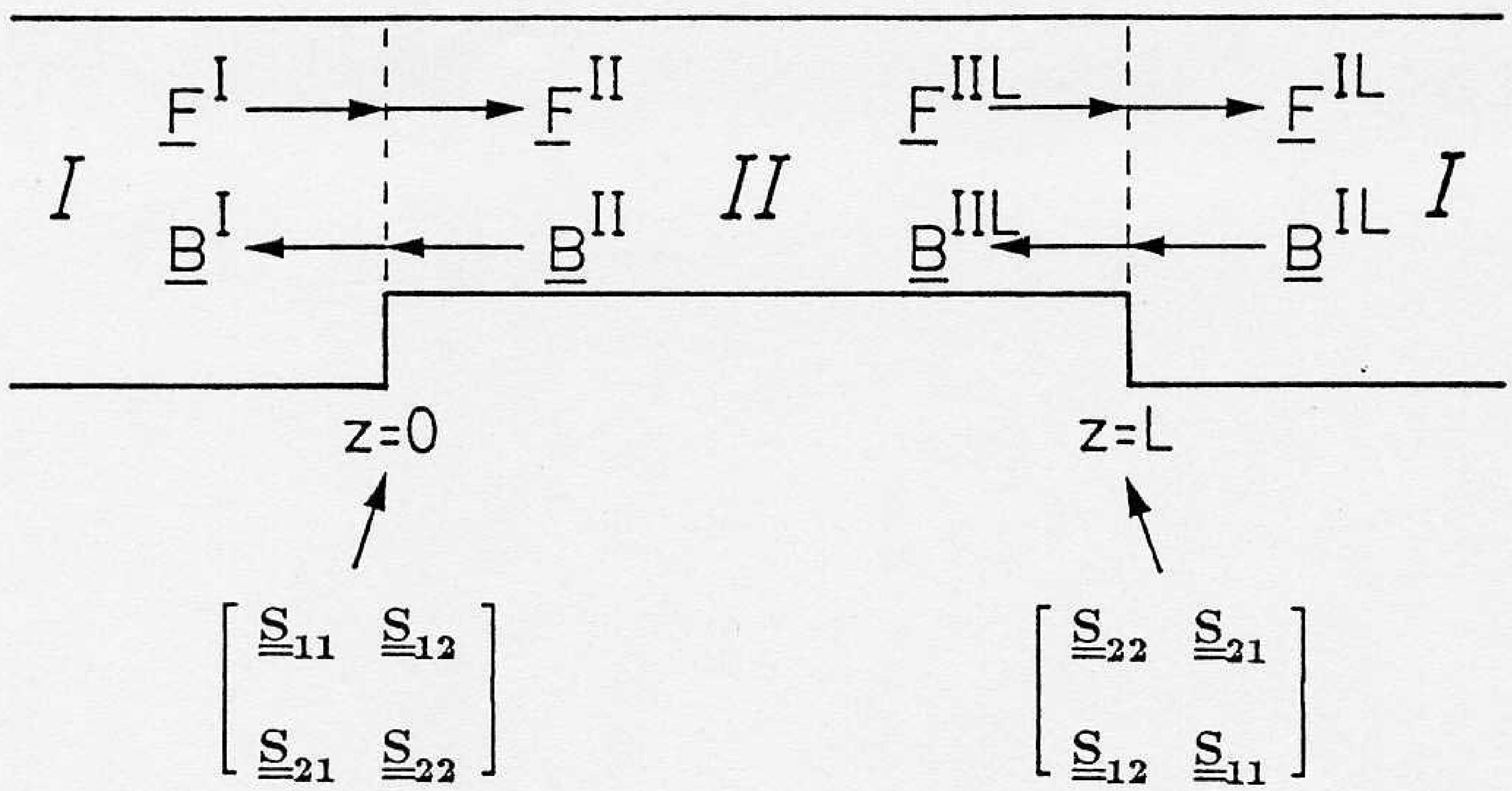
$$\underline{\underline{S}} = \underline{\underline{S}}^T = \underline{\underline{S}}^{-1}$$

2. Discontinuity of Finite Length

S-matrix of waveguide section II of length L (without discontinuities)

$$(\underline{\underline{S}}) = \begin{bmatrix} \underline{\underline{O}} & \underline{\underline{D}} \\ \underline{\underline{D}} & \underline{\underline{O}} \end{bmatrix} \quad \text{where } \underline{\underline{D}} = \text{ Diag } \{e^{-jk_{zn}^{II} L}\}$$

e.g. $\underline{F}^{IIL} = \underline{\underline{D}}\,\underline{F}^{II}$, $\underline{B}^{II} = \underline{\underline{D}}\,\underline{B}^{IIL}$

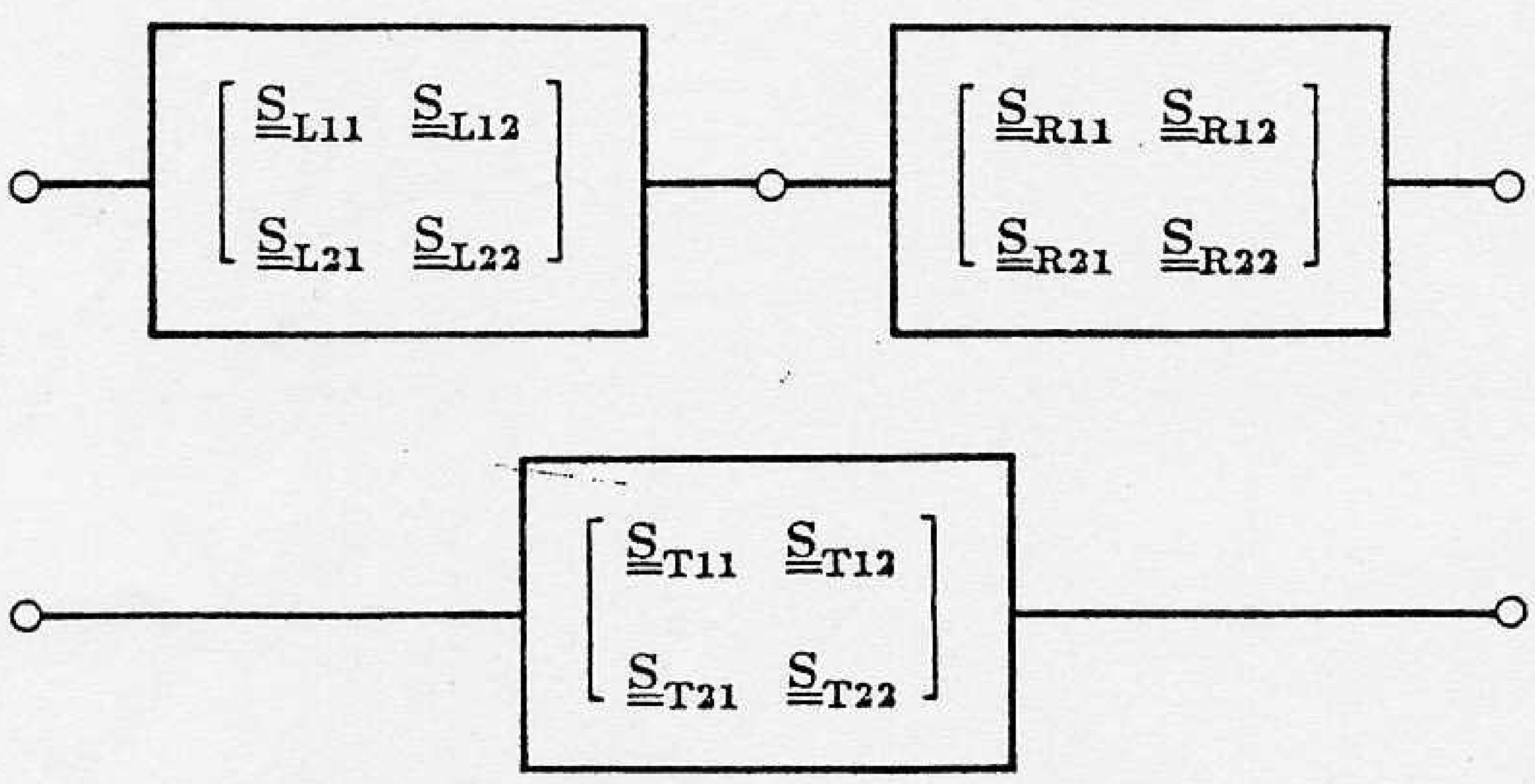


Overall S-matrix $\underline{\underline{S}}_0$ including discontinuities

$$\underline{\underline{S}}_{011} = \underline{\underline{S}}_{022} = \underline{\underline{S}}_{11} + \underline{\underline{S}}_{12}\,\underline{\underline{D}}\,[\underline{\underline{I}} - \underline{\underline{S}}_{22}\,\underline{\underline{D}}\,\underline{\underline{S}}_{22}\,\underline{\underline{D}}]^{-1}\underline{\underline{S}}_{22}\,\underline{\underline{D}}\,\underline{\underline{S}}_{21}$$

$$\underline{\underline{S}}_{021} = \underline{\underline{S}}_{012} = \underline{\underline{S}}_{12}\,\underline{\underline{D}}\,[\underline{\underline{I}} - \underline{\underline{S}}_{22}\,\underline{\underline{D}}\,\underline{\underline{S}}_{22}\,\underline{\underline{D}}]^{-1}\underline{\underline{S}}_{21}$$

3. Cascading Scattering Matrices



$$\underline{\underline{S}}_{T11} = \underline{\underline{S}}_{L11} + \underline{\underline{S}}_{L12}\, \underline{\underline{S}}_{R11}\, \underline{\underline{W}}\, \underline{\underline{S}}_{L21}$$

$$\underline{\underline{S}}_{T12} = \underline{\underline{S}}_{L12}(\underline{\underline{I}} + \underline{\underline{S}}_{R11}\, \underline{\underline{W}}\, \underline{\underline{S}}_{L22})\underline{\underline{S}}_{R12}$$

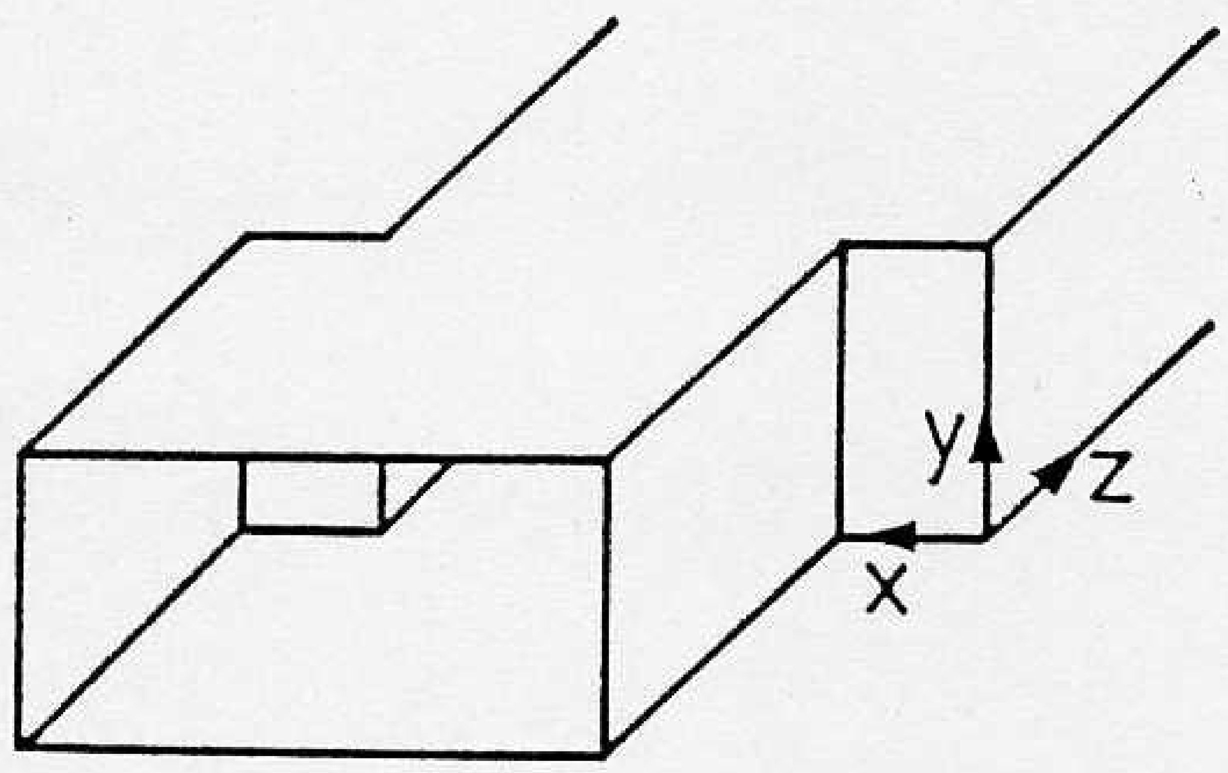
$$\underline{\underline{S}}_{T21} = \underline{\underline{S}}_{R21}\, \underline{\underline{W}}\, \underline{\underline{S}}_{L21}$$

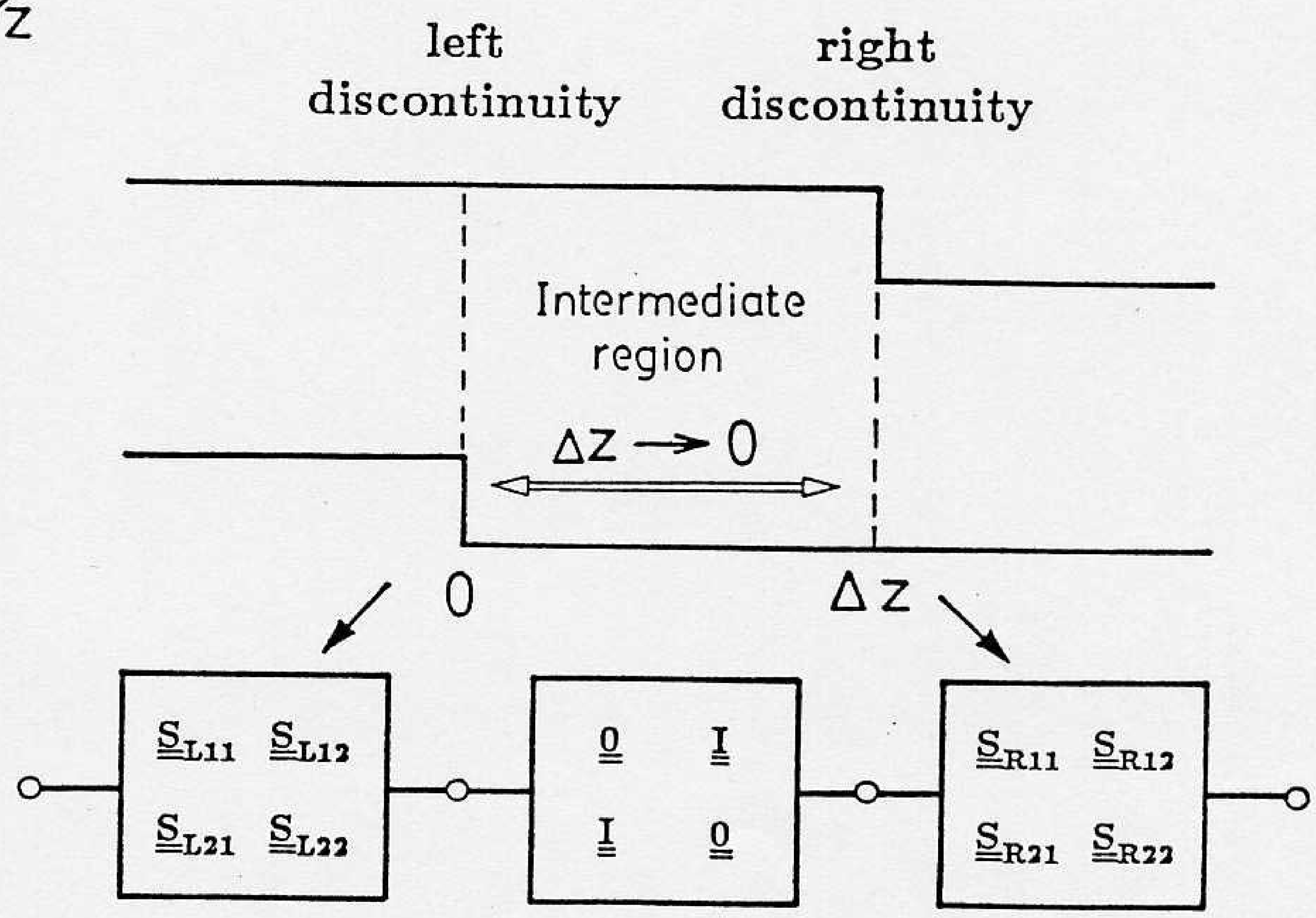
$$\underline{\underline{S}}_{T22} = \underline{\underline{S}}_{R22} + \underline{\underline{S}}_{R21} \underline{\underline{W}}\, \underline{\underline{S}}_{L22}\, \underline{\underline{S}}_{R12}$$

$$\underline{\underline{W}} = [\underline{\underline{I}} - \underline{\underline{S}}_{L22}\underline{\underline{S}}_{R11}]^{-1}$$

4. Intermediate Region

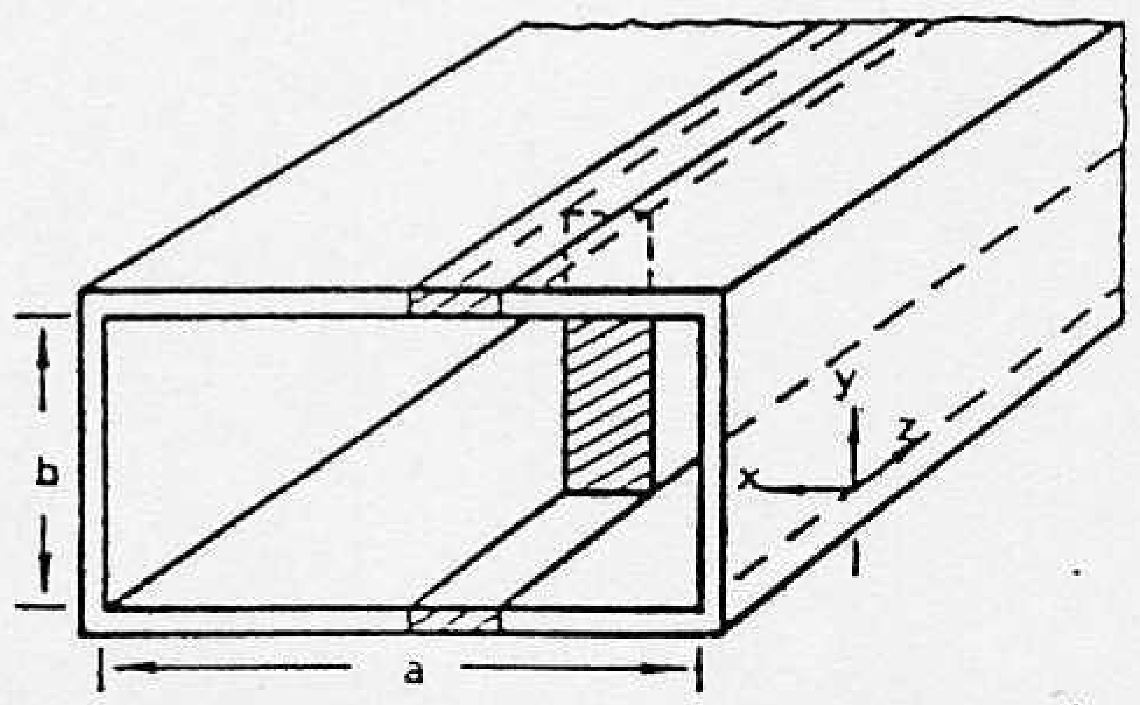
An intermediate region must be introduced if one cross-section is not a subset of the cross-section to which it is connected.

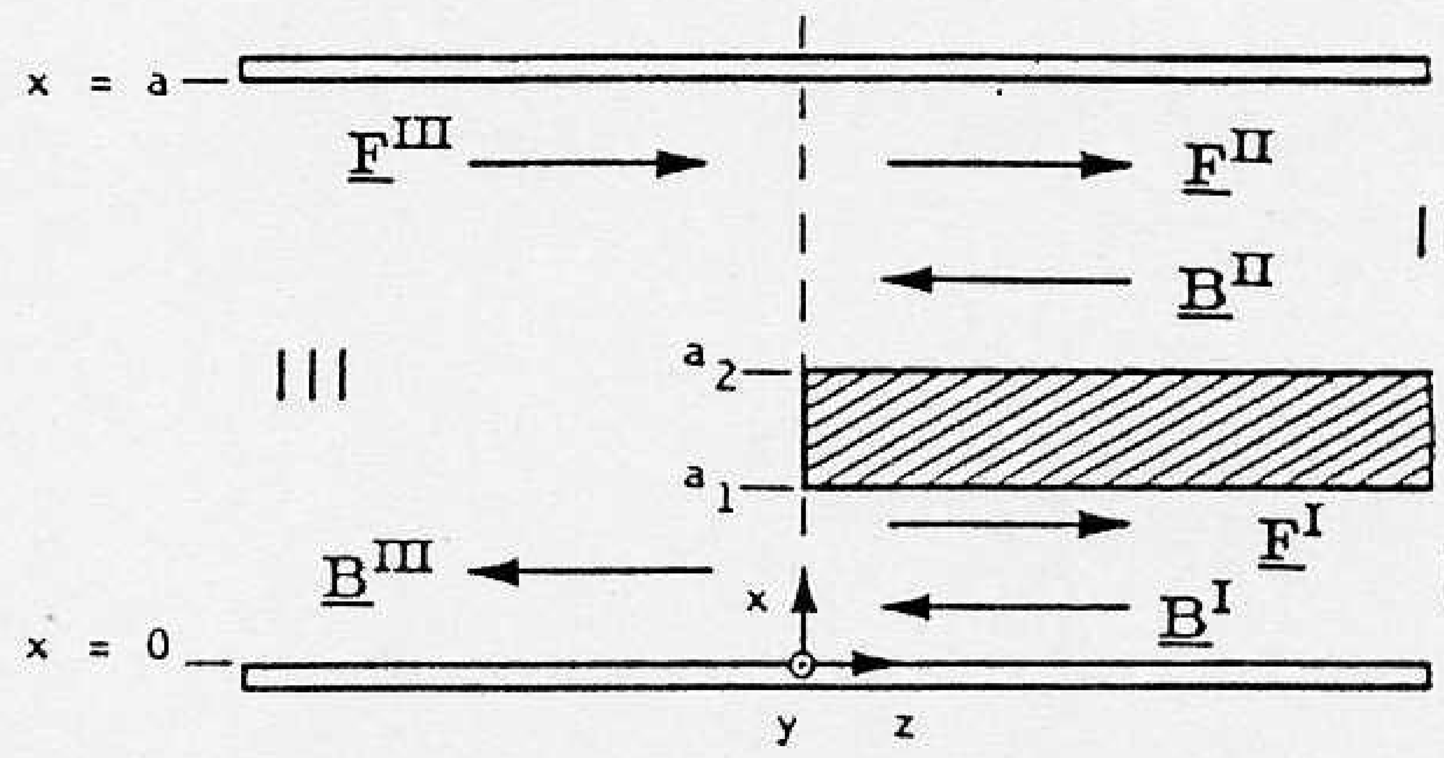




The original discontinuity is obtained by letting the length of the intermediate region go to zero.

III. Waveguide Bifurcation





$$E_y^{III} = \sum_{m=1}^{M} T_m^{III} \sin\left(\frac{m\pi}{a}x\right)\left(F_m^{III} e^{-jk_{zm}^{III}z} + B_m^{III} e^{+jk_{zm}^{III}z}\right)$$

$$E_y^{II} = \sum_{n=1}^{N} T_n^{II} \sin\left\{\frac{n\pi}{a-a_2}(x-a_2)\right\}\left(F_n^{II} e^{-jk_{zn}^{II}z} + B_n^{II} e^{+jk_{zm}^{II}z}\right)$$

$$E_y^{I} = \sum_{i=1}^{I} T_i^{I} \sin\left(\frac{i\pi}{a_1}x\right)\left(F_i^{I} e^{-jk_{zi}^{I}z} + B_i^{I} e^{+jk_{zi}^{I}z}\right)$$

Matching the E_y and H_x field components and $z = 0$ yields

$$\underline{F}^{III} + \underline{B}^{III} = \underline{\underline{L}}_E^{I}(\underline{F}^{I} + \underline{B}^{I}) + \underline{\underline{L}}_E^{II}(\underline{F}^{II} + \underline{B}^{II})$$

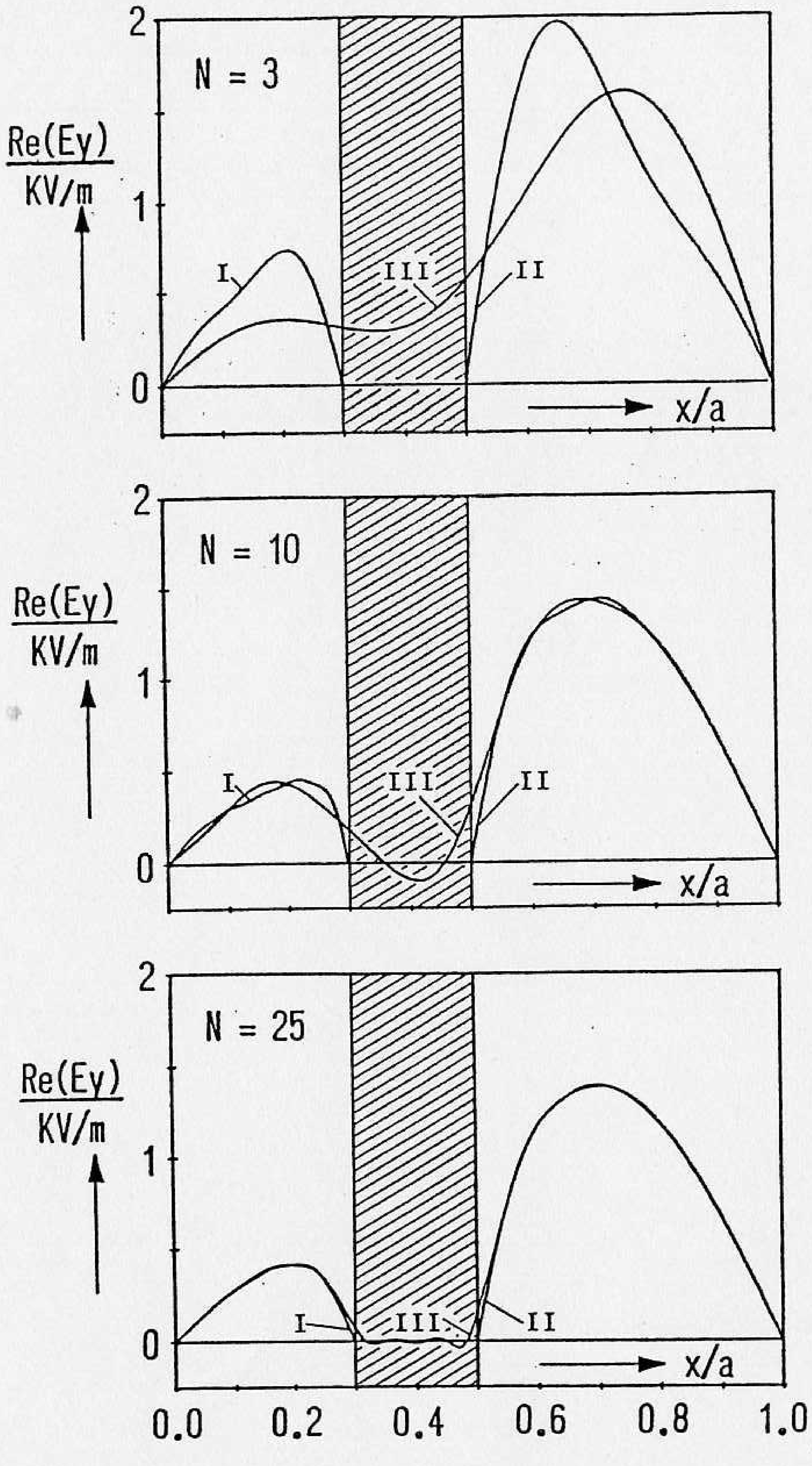
$$\underline{\underline{L}}_H^{I}(\underline{F}^{III} - \underline{B}^{III}) = \underline{F}^{I} - \underline{B}^{I}$$

$$\underline{\underline{L}}_H^{II}(\underline{F}^{III} - \underline{B}^{III}) = \underline{F}^{II} - \underline{B}^{II}$$

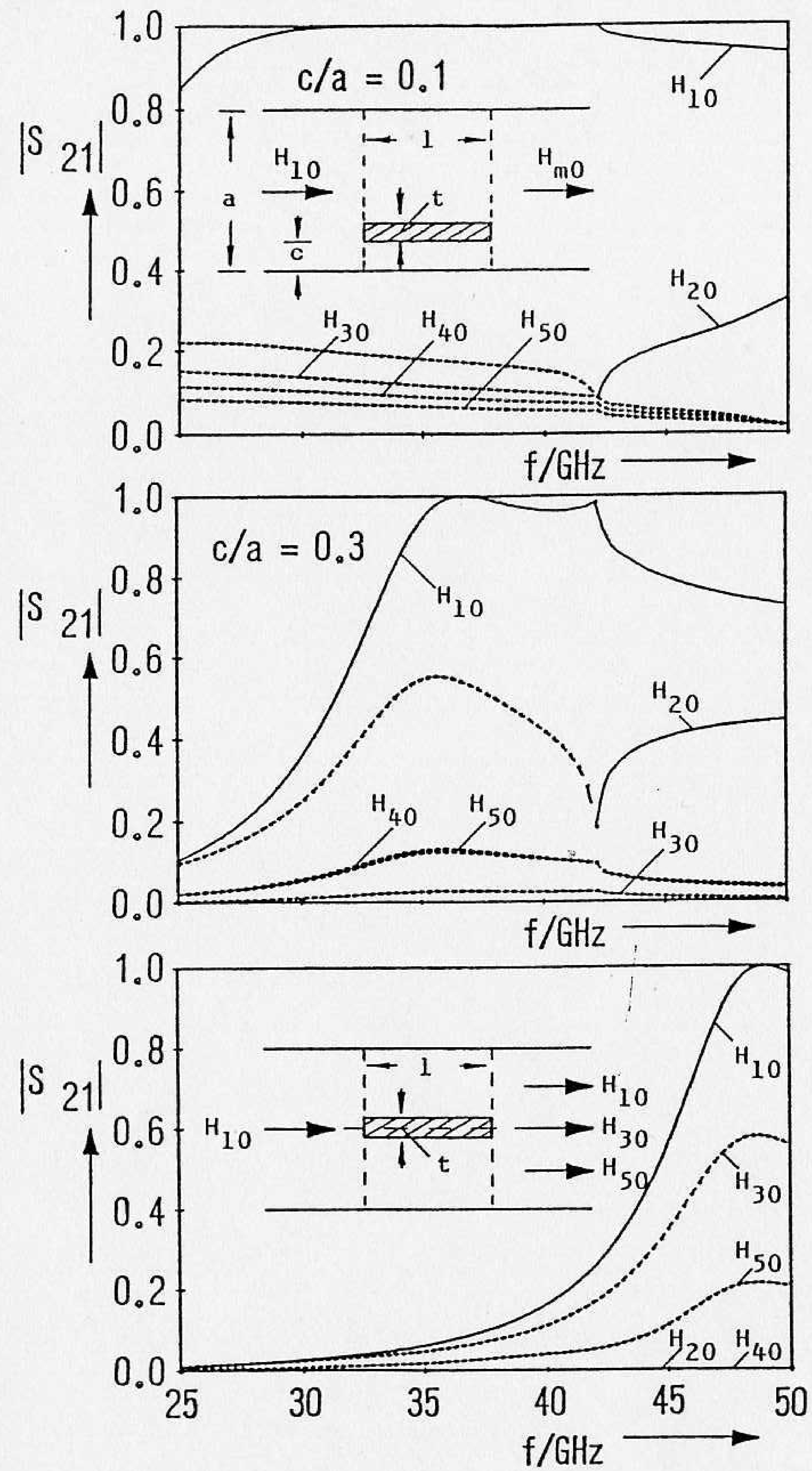
where

$$(\underline{\underline{L}}_E^{I})_{mi} = 2\sqrt{\frac{k_{zm}^{III}}{a a_1 k_{zi}^{I}}} \int_0^{a_1} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{i\pi}{a_1}x\right) dx = (\underline{\underline{L}}_H^{I})_{im}$$

$$(\underline{\underline{L}}_E^{II})_{mn} = 2\sqrt{\frac{k_{zm}^{III}}{a(a-a_2)k_{zn}^{II}}} \int_{a_2}^{a} \sin\left(\frac{m\pi}{a}x\right) \sin\left\{\frac{n\pi}{a-a_2}(x-a_2)\right\} dx = (\underline{\underline{L}}_H^{II})_{nm}$$



Real part of electric field component at waveguide bifurcation (M = N = I)



Transmission coefficients $TE_{10} \rightarrow TE_{mo}$ (— propagating, - - - evanescent mode)

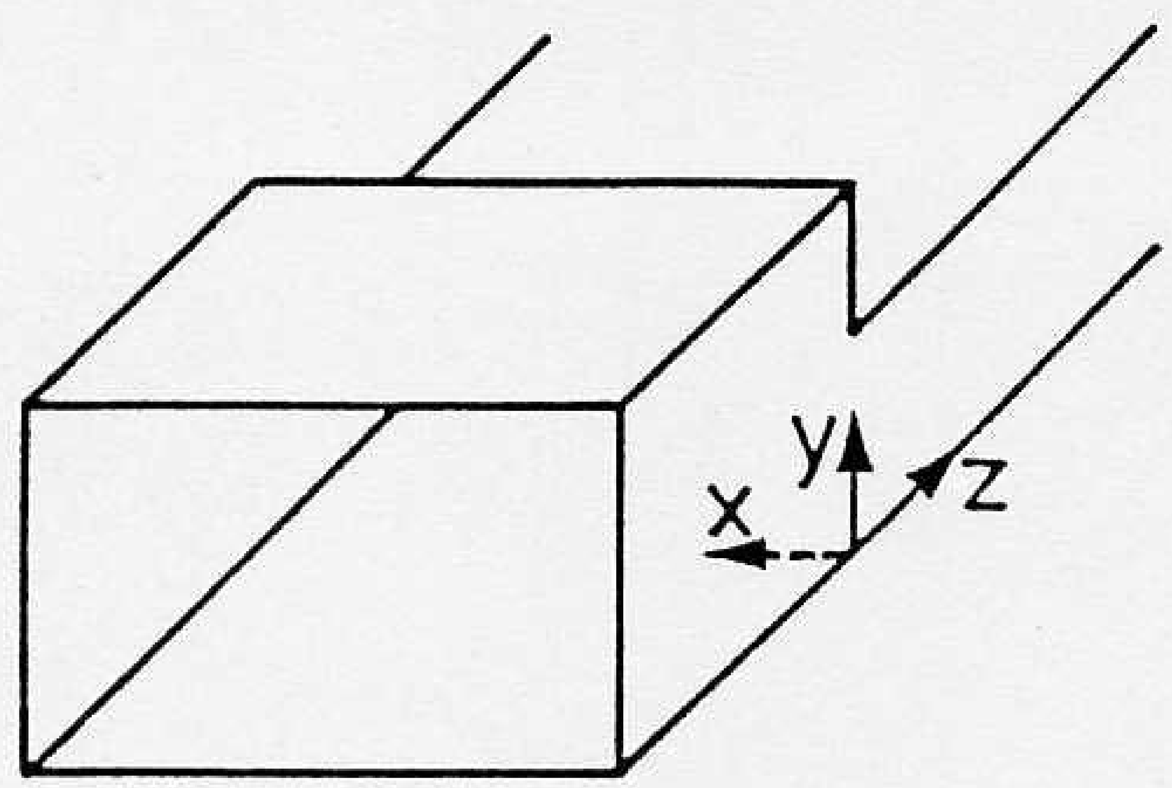
IV. E-Plane Discontinuity

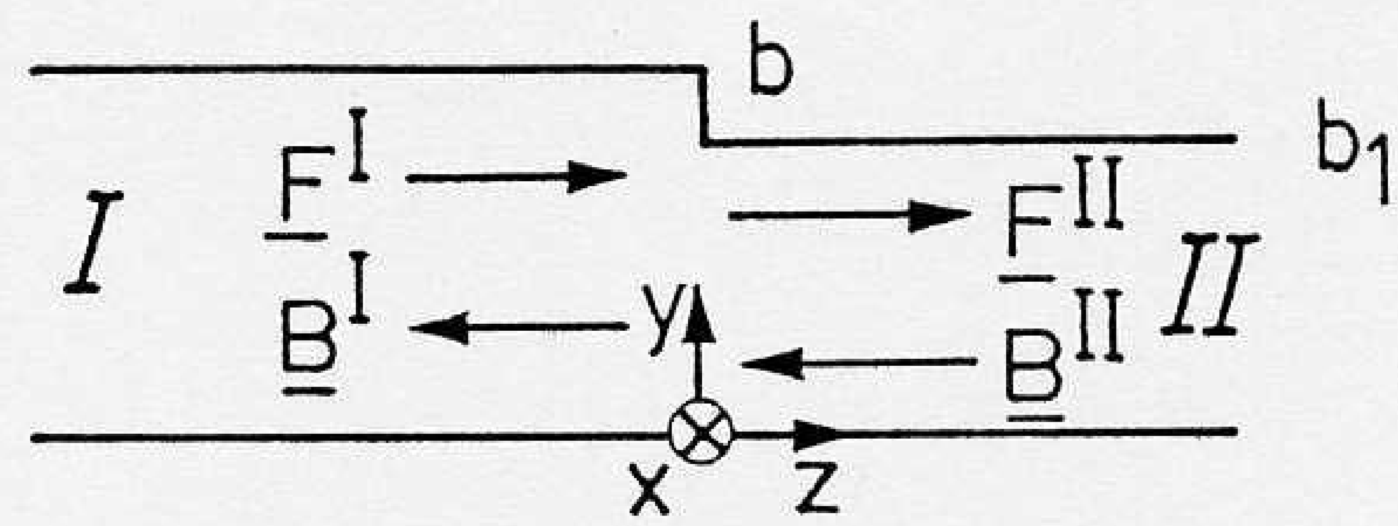
Field Components

Incident TE_{10} mode has	E_y	H_x	H_z
Discontinuity adds	E_z	H_y	

Five field components ($E_x \equiv 0$) : TE^x_{mn} modes

Since there is no change in x-direction: TE^x_{1n} modes





Vector potentials

$$\vec{E} = \nabla \times \vec{A}_h$$

$$\vec{H} = \frac{j}{\omega\mu} \nabla \times \nabla \times \vec{A}_h$$

Let $\vec{A}_h = A_{hx}\vec{e}_x$, then

$$E_x = 0 \qquad H_x = \frac{j}{\omega\mu_0}[k_0^2 A_{hx} + \frac{\partial^2}{\partial_{x^2}} A_{hx}]$$

$$E_y = \frac{\partial A_{hx}}{\partial_z} \qquad H_y = \frac{j}{\omega\mu_0}\frac{\partial^2}{\partial_x \partial_y} A_{hx}$$

$$E_z = -\frac{\partial A_{hx}}{\partial_y} \qquad H_z = \frac{j}{\omega\mu_0}\frac{\partial^2}{\partial_x \partial_z} A_{hx}$$

$$A^I_{hx} = \sum_{n=0}^{N} T^I_n \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{n\pi}{y}\right)}{\sqrt{1+\delta_{on}}} (F^I_n e^{-jk^I_{zn}z} - B^I_n e^{+jk^I_{zn}z})$$

$$A^{II}_{hx} = \sum_{i=0}^{I} T^{II}_n \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{i\pi}{y}\right)}{\sqrt{1+\delta_{oi}}} (F^{II}_i e^{-jk^{II}_{zi}z} - B^{II}_i e^{+jk^{II}_{zi}z})$$

$$k^I_{zn} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \qquad T^I_n = 2\sqrt{\frac{\omega\mu_0}{abk^I_{zn}[k_0^2 - \left(\frac{\pi}{a}\right)^2]}}$$

$$k^{II}_{zi} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{i\pi}{b_1}\right)^2} \qquad T^{II}_i = 2\sqrt{\frac{\omega\mu_0}{ab_1k^{II}_{zi}[k_0^2 - \left(\frac{\pi}{a}\right)^2]}}$$

E_y and H_z are matched at $z = 0$. In this special case of constant waveguide width, the condition for the third component to be matched (H_y) is identical to that for H_z.

$$\underline{F}^I + \underline{B}^I = \underline{\underline{L}}_E(\underline{F}^{II} + \underline{B}^{II})$$

$$\underline{\underline{L}}_H(\underline{F}^I - \underline{B}^I) = \underline{F}^{II} - \underline{B}^{II}$$

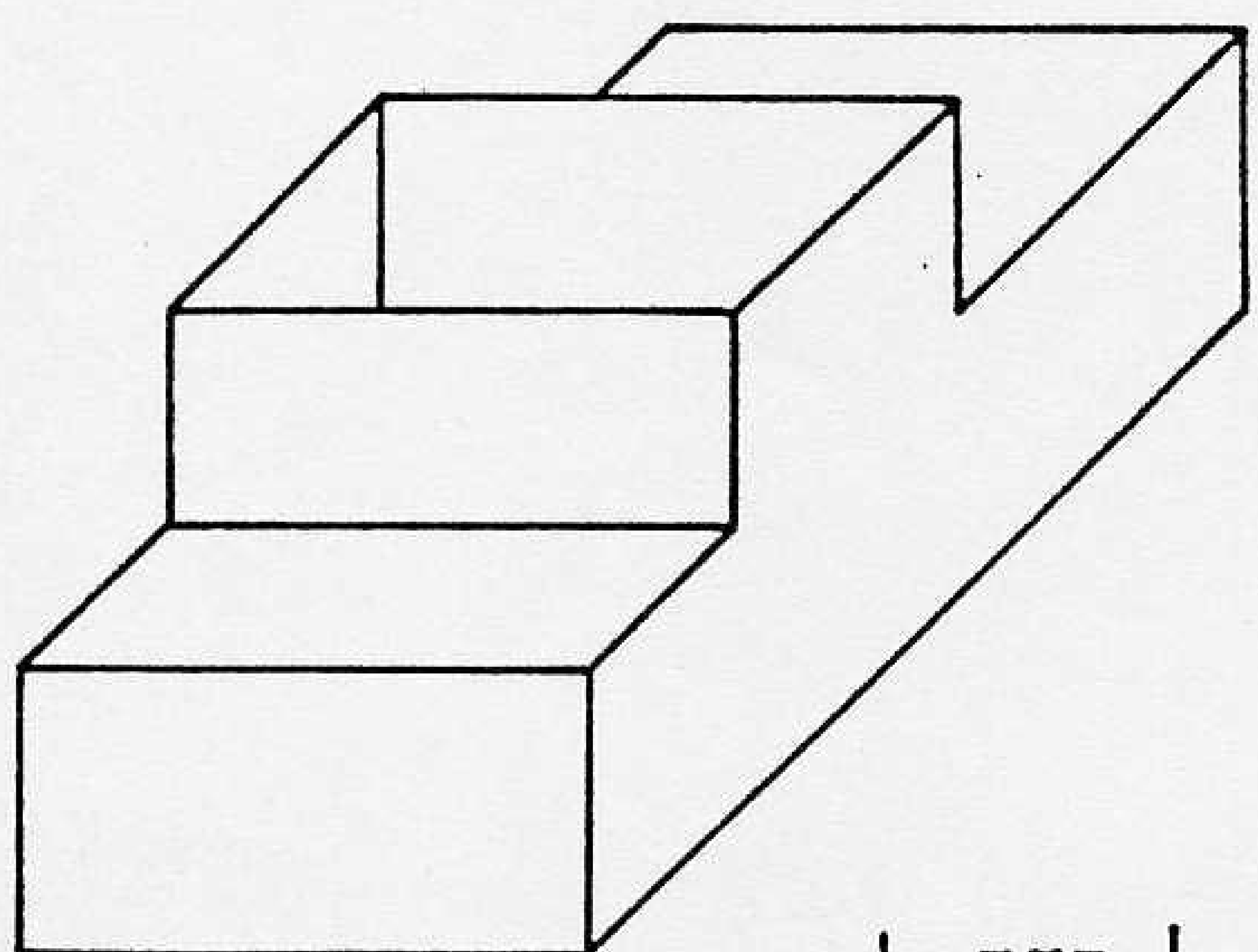
$$(\underline{\underline{L}}_E)_{ni} = 2\sqrt{\frac{k^{II}_{zi}}{bb_1 k^I_{zn}}} \int_0^{b_1} \frac{\cos\left(\frac{n\pi}{b}y\right)}{\sqrt{1+\delta_{on}}} \; \frac{\cos\left(\frac{i\pi}{b_1}y\right)}{\sqrt{1+\delta_{oi}}} dy = (\underline{\underline{L}}_H)_{in}$$

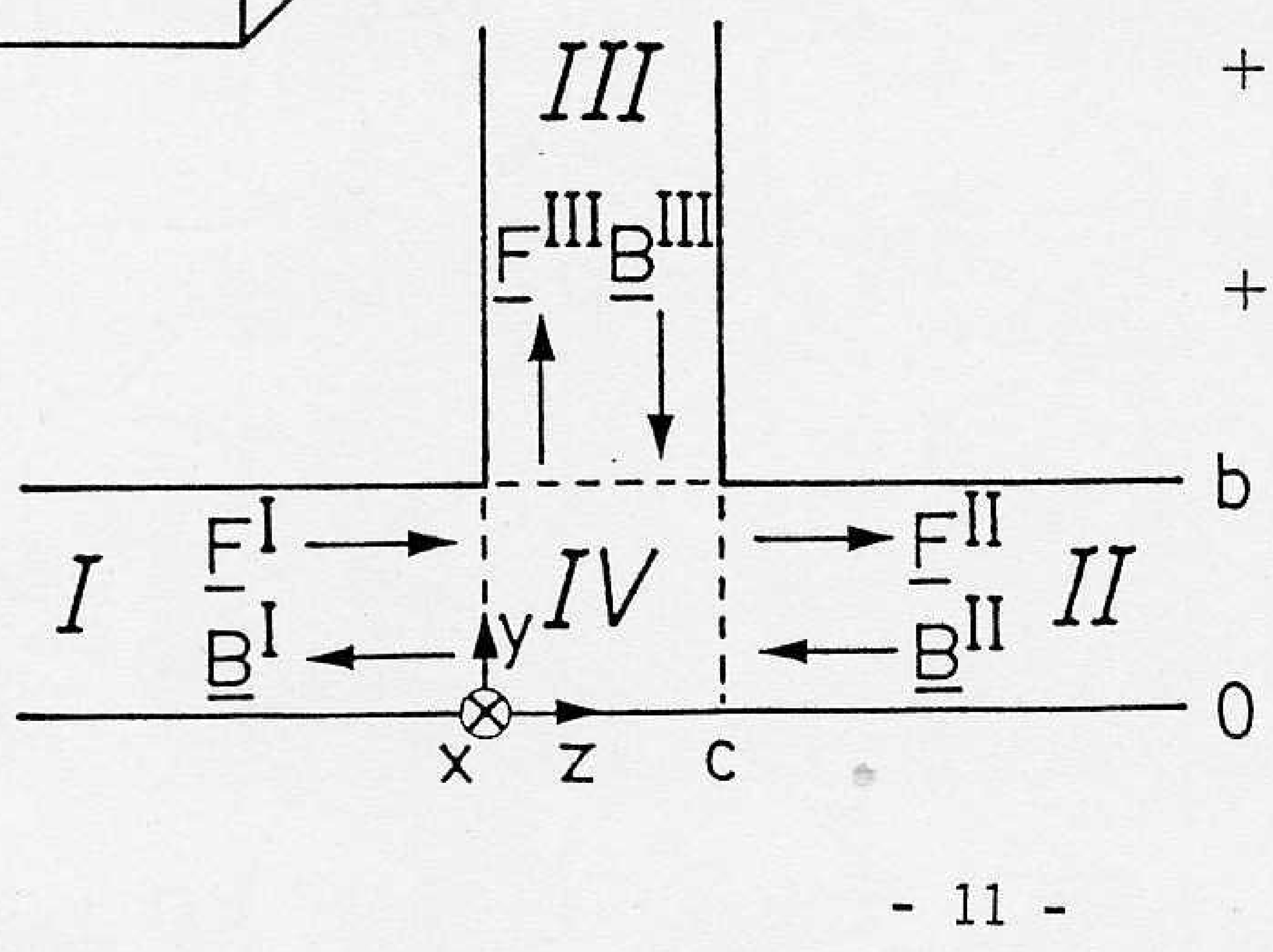
V. T-Junctions (Resonator Method)

$$A^I_{hz} = \sum_{n=0}^{N} T^I_n \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{n\pi}{b}y\right)}{\sqrt{1+\delta_{on}}} (F^I_n e^{-jk^I_{zn}z} - B^I_n e^{+jk^I_{zn}z})$$

$$A^{II}_{hz} = \sum_{i=0}^{I} T^{II}_i \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{i\pi}{b}y\right)}{\sqrt{1+\delta_{oi}}} (F^{II}_i e^{-jk^{II}_{zi}(z-c)} - B^{II}_i e^{+jk^{II}_{zi}(z-c)})$$

$$A^{III}_{hz} = \sum_{k=0}^{K} T^{III}_k \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{k\pi}{c}z\right)}{\sqrt{1+\delta_{ok}}} (F^{III}_k e^{-jk^{III}_{yk}(y-b)} - B^{III}_k e^{+jk^{III}_{yk}(y-b)})$$





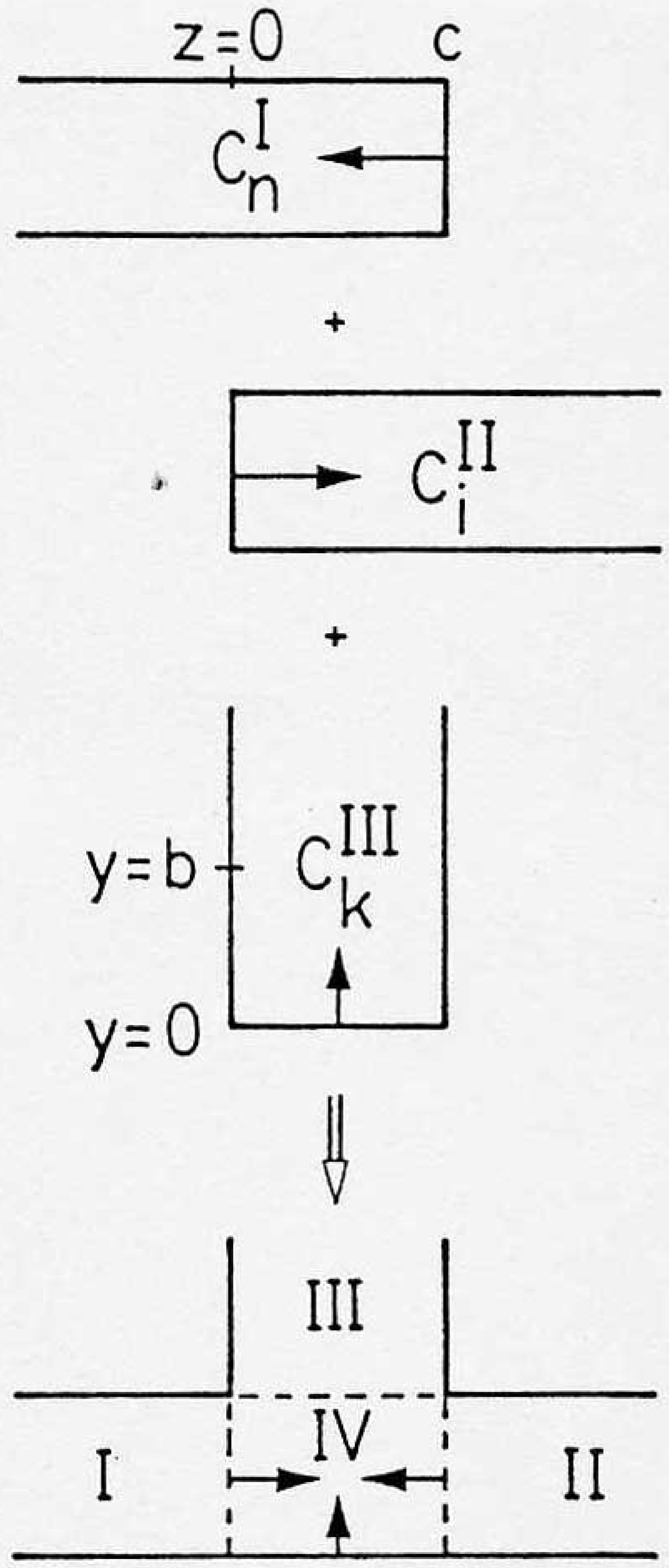
Resonator Region

$$A^{IV}_{hz} = A^{IV}_{hz1} + A^{IV}_{hz2} + A^{III}_{hz3}$$

$$= \sum_n C^I_n \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{n\pi}{b}y\right)}{\sqrt{1+\delta_{on}}} \cos\{k^I_{zn}(z-c)\}$$

$$+\sum_i C^{II}_i \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{i\pi}{b}y\right)}{\sqrt{1+\delta_{oi}}} \cos(k^{II}_{zi} z)$$

$$+\sum_k C^{III}_k \sin\left(\frac{\pi}{a}x\right) \frac{\cos\left(\frac{k\pi}{c}z\right)}{\sqrt{1+\delta_{ok}}} \cos(k^{III}_{yk} y)$$



How to find coefficients C_n^I, C_i^{II}, C_k^{III}?

Match

1. $E_y^I = E_y^{IV}$ at $z = 0$
   $\Rightarrow \quad C_n^I = f\{T_n^I,\ F_n^I,\ B_n^I\}$

2. $E_y^{II} = E_y^{IV}$ at $z = c$
   $\Rightarrow \quad C_i^{II} = f\{T_i^{II},\ F_i^{II},\ B_i^{II}\}$

3. $E_y^{III} = E_y^{IV}$ at $y = b$
   $\Rightarrow \quad C_k^{III} = f\{T_k^{III},\ F_k^{III},\ B_k^{III}\}$

To find the three equations from which to derive the scattering matrix, match the H_x field components at the three interfaces.

1. $\underline{H_x^I = H_x^{IV} \text{ at } z = o}$

$$\underline{F}^I - \underline{B}^I = \underline{\underline{D}}^I(\underline{F}^I + \underline{B}^I) + \underline{\underline{L}}^{I,\,II}(\underline{F}^{II} + \underline{B}^{II}) + \underline{\underline{L}}^{I,\,III}(\underline{F}^{III} + \underline{B}^{III})$$

2. $\underline{H_x^{II} = H_x^{IV} \text{ at } z = c}$

$$\underline{F}^{II} - \underline{B}^{II} = \underline{\underline{L}}^{II,\,I}(\underline{F}^I + \underline{B}^I) + \underline{\underline{D}}^{II}(\underline{F}^{II} + \underline{B}^{II}) + \underline{\underline{L}}^{II,\,III}(\underline{F}^{III} + \underline{B}^{III})$$

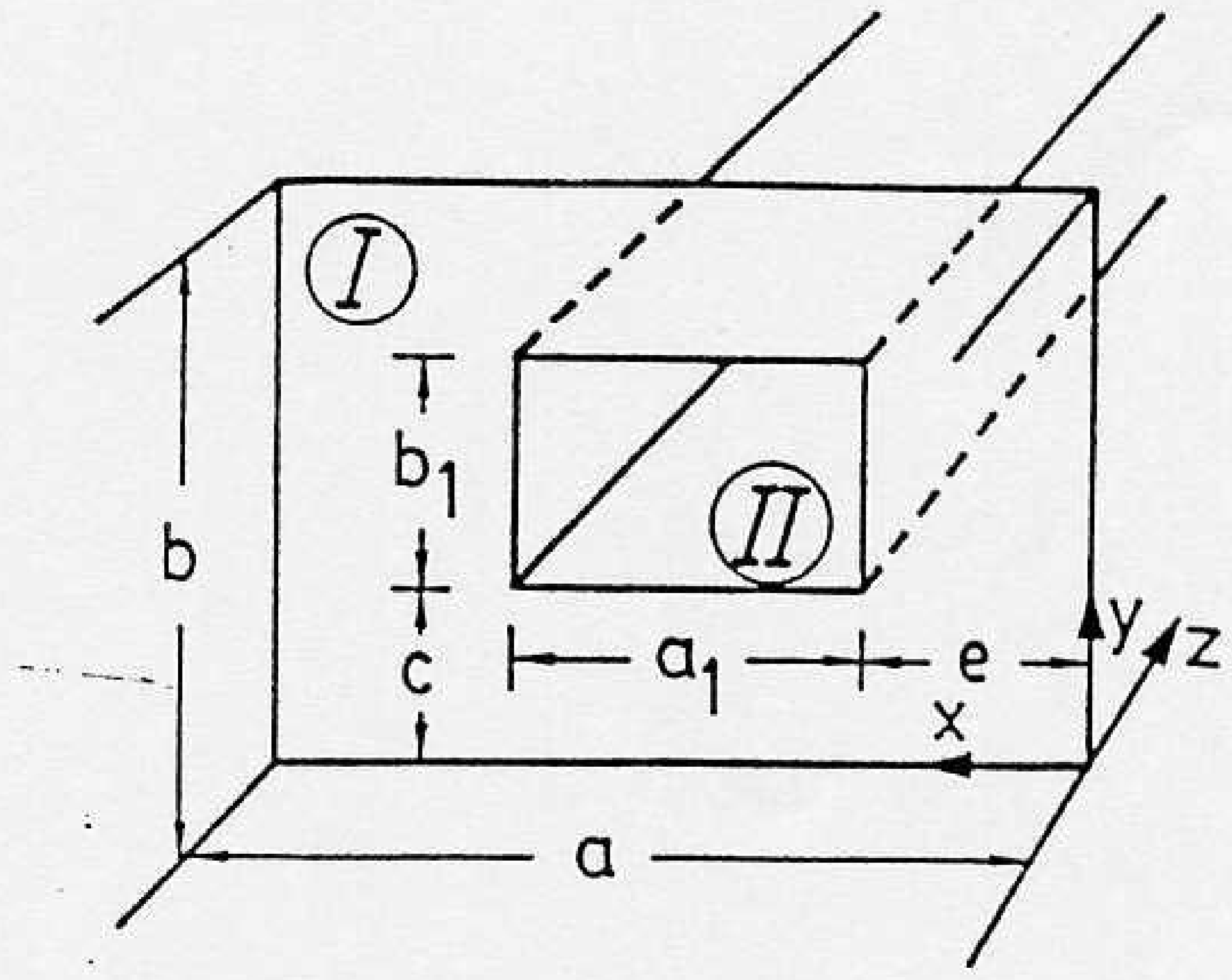
3. $\underline{H_x^{III} = H_x^{IV} \text{ at } y = b}$

$$\underline{F}^{III} - \underline{B}^{III} = \underline{\underline{L}}^{III,\,I}(\underline{F}^I + \underline{B}^I) + \underline{\underline{L}}^{III,\,II}(\underline{F}^{II} + \underline{B}^{II}) + \underline{\underline{D}}^{III}(\underline{F}^{III} + \underline{B}^{III})$$

T-junction scattering matrix

$$\begin{bmatrix} \underline{B}^I \\ \underline{F}^{II} \\ \underline{F}^{III} \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_{11} & \underline{\underline{S}}_{12} & \underline{\underline{S}}_{13} \\ \underline{\underline{S}}_{21} & \underline{\underline{S}}_{22} & \underline{\underline{S}}_{23} \\ \underline{\underline{S}}_{31} & \underline{\underline{S}}_{32} & \underline{\underline{S}}_{33} \end{bmatrix} \begin{bmatrix} \underline{F}^I \\ \underline{B}^{II} \\ \underline{B}^{III} \end{bmatrix}$$

VI. Double Plane Steps



Two Ways of Solution

1. Generalized TE_{mn} - TM_{mn} Mode Analysis (6 field components)

TE modes ($E_z = 0$) TM modes ($H_z = 0$)

$$\vec{E}_{TE} = \nabla \times (A_{hz}\vec{e}_z) \qquad \vec{E}_{TM} = \frac{1}{j\omega\epsilon} \nabla \times \nabla \times (A_{ez}\vec{e}_z)$$

$$\vec{H}_{TE} = \frac{-1}{j\omega\mu} \nabla \times \nabla \times (A_{hz}\vec{e}_z) \qquad \vec{H}_{TM} = \nabla \times (A_{ez}\vec{e}_z)$$

$$\vec{E} = \vec{E}_{TM} + \vec{E}_{TE} = \frac{1}{j\omega\epsilon} \nabla \times \nabla \times (A_{ez}\vec{e}_z) + \nabla \times (A_{hz}\vec{e}_z)$$

$$\vec{H} = \vec{H}_{TE} + \vec{H}_{TM} = \frac{-1}{j\omega\mu} \nabla \times \nabla \times (A_{hz}\vec{e}_z) + \nabla \times (A_{ez}\vec{e}_z)$$

By matching the four transverse field components E_x, E_y, H_x, H_y at the discontinuity, four matrix equations with four unknowns are obtained.

Matrix size: $(N_{TE} + N_{TM}) \times (N_{TE} + N_{TM})$

2. TE^x_{mn} Mode Analysis (5 field components)

$$\vec{E} = \nabla \times (A_{hx}\vec{e}_x)$$
$$\vec{H} = \frac{-1}{j\omega\mu} \nabla \times \nabla (A_{hx}\vec{e}_x)$$

By matching the three transverse field components E_y, H_x, $H_y (E_x \equiv 0)$ at the discontinuity, three matrix equations with two unknowns are obtained

a) Conventional Method
Match E_y and H_x only and ignore H_y. This procedure provides excellent results as long as resonant effects do not occur in the discontinuity plane.

b) <u>Modified Method</u>

Match E_y and H_x or H_y alternatively if resonant effects occur.

In both cases, the matrix size is only $N_{TE} \times N_{TE}$.

$$E_y \ : \ \underline{F}^I + \underline{B}^I = \underline{\underline{L}}_E(\underline{F}^{II} + \underline{B}^{II})$$

$$H_x \ : \quad \underline{\underline{L}}_{Hx}(\underline{F}^I - \underline{B}^I) = \underline{F}^{II} - \underline{B}^{II} \qquad (\underline{\underline{L}}_{Hx} = \underline{\underline{L}}_E^T)$$
$$H_y \ : \quad \underline{\underline{L}}_{Hy}(\underline{F}^I - \underline{B}^I) = \underline{F}^{II} - \underline{B}^{II}$$

The equations for H_x and H_y are treated as one equation with a new matrix $\underline{\underline{L}}_H$ where

$(\underline{\underline{L}}_H)_{qp} = (\underline{\underline{L}}_{Hx})_{qp}$, if mode q or mode p is a TE^z_{mo} type

$(\underline{\underline{L}}_H)_{qp} = (\underline{\underline{L}}_{Hy})_{qp}$, if neither mode q nor mode p is a TE^z_{mo} type

Hint: If both mode indices m, n are involved, assign a new index p for a combination m, n by rearranging the modes with respect to increasing cutoff frequencies.

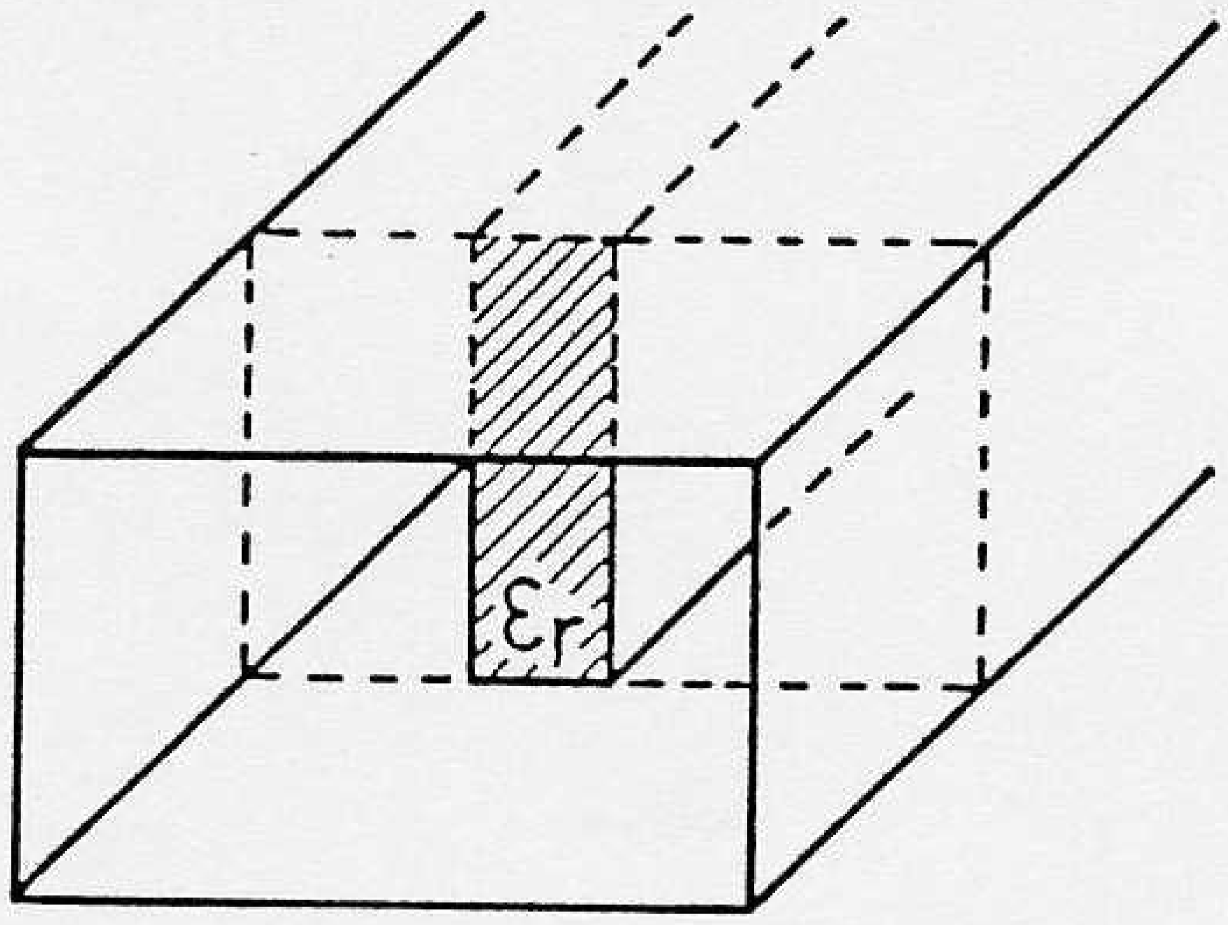
$$A^I_{hx} = \sum_{m=1}^{M} \sum_{n=0}^{N} T^I_p \sin\left(\frac{m\pi}{a}x\right) \frac{\cos\left(\frac{n\pi}{b}y\right)}{\sqrt{1+\delta_{on}}} \cdot (F^I_p e^{-jk^I_{zp}z} - B^I_p e^{+jk^I_{zp}z})$$

$$A^{II}_{hx} = \sum_{i=1}^{I} \sum_{k=0}^{K} T^{II}_q \sin\left\{\frac{i\pi}{a_1}(x-e)\right\} \frac{\cos\left\{\frac{k\pi}{b_1}(y-c)\right\}}{\sqrt{1+\delta_{ok}}} \cdot (F^{II}_q e^{-jk^{II}_{zq}z} - B^{II}_q e^{+jk^{II}_{zq}z})$$

VII. Steps to Cross-Sections with Unknown Eigenfunctions

1. Dielectric-Slab-Loaded Waveguide

Since the dielectric-slab-loaded waveguide combines two different propagation media (air and dielectric), the eigenfunctions of this structure are hybrid in general (6 field components). However, some of these eigenmodes are of the TE_{mo} type (E_y, H_x, H_z), which alone would be excited by an incident TE_{10} wave. (In this case, neither the structure nor the field configuration of the incident mode show changes with respect to the y-direction.)



Matching the tangential field components (E_y, H_z) at the air-dielectric interfaces leads to a transcendental equation for the propagation constants of the dielectric-slab-loaded waveguide. Once the propagation constants are obtained, the power normalization factors can be calculated, and TE_{mo}-type mode matching can be applied.

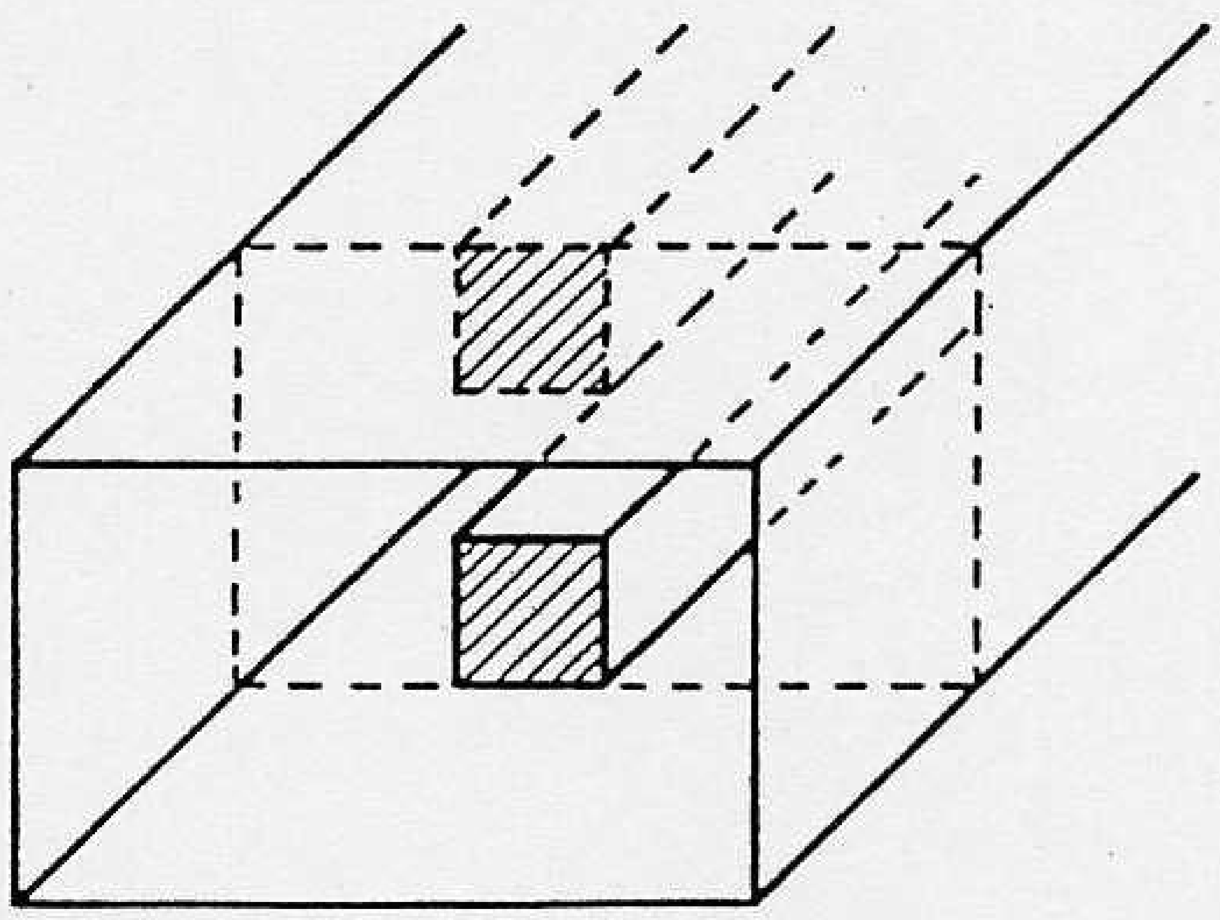
2. Ridge Waveguide

The eigenmodes of the ridge waveguide have to be calculated first, e.g., by using the transverse resonance method.

As in the case of the rectangular waveguide the mode spectrum of the ridge waveguide consists of TE and TM waves. In order to match the electromagnetic fields at the discontinuity, however, six field components are required.

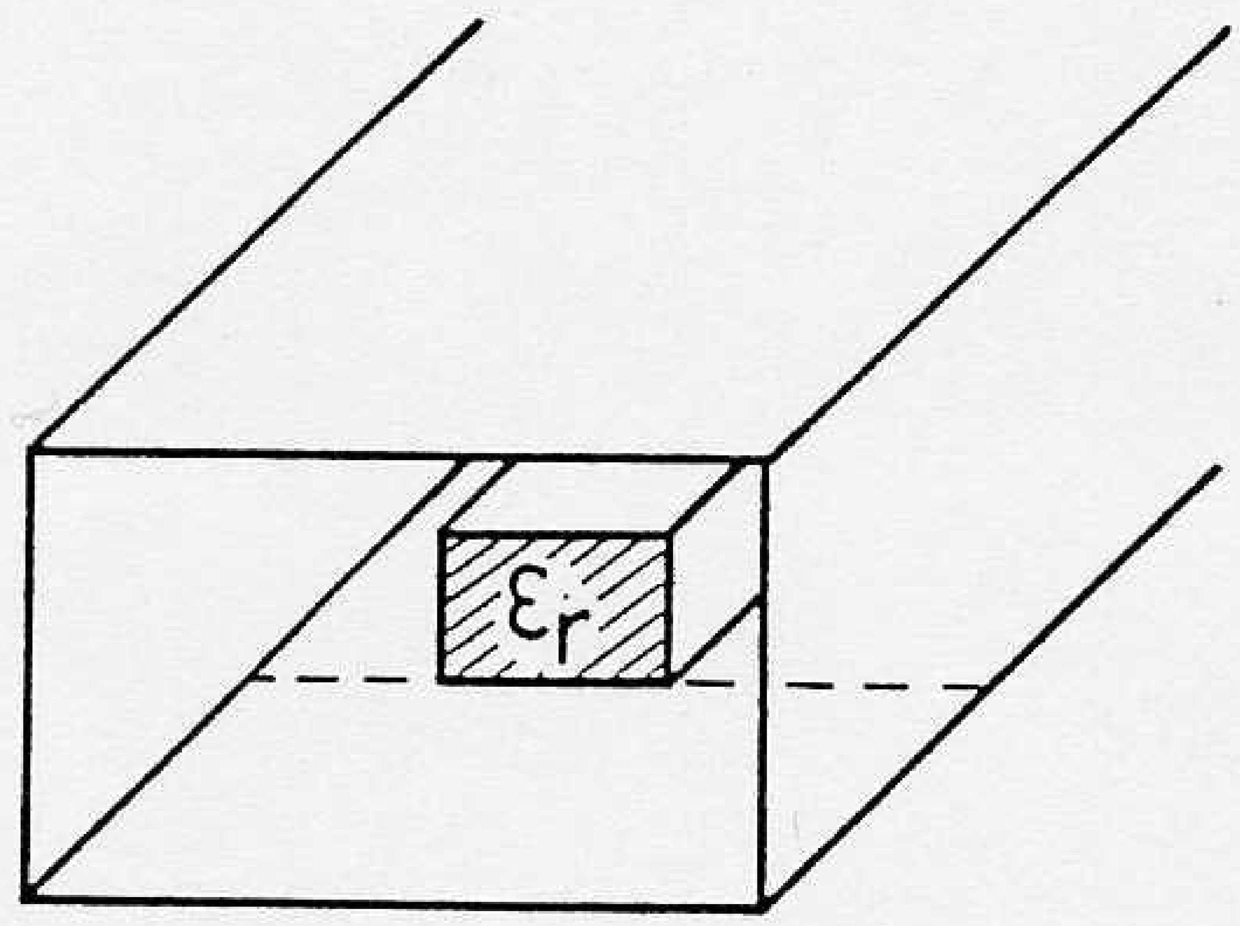
$$\vec{E} = \frac{1}{j\omega\epsilon} \nabla \times \nabla \times (A_{ez}\vec{e}_z) + \nabla \times (A_{hz}\vec{e}_z)$$

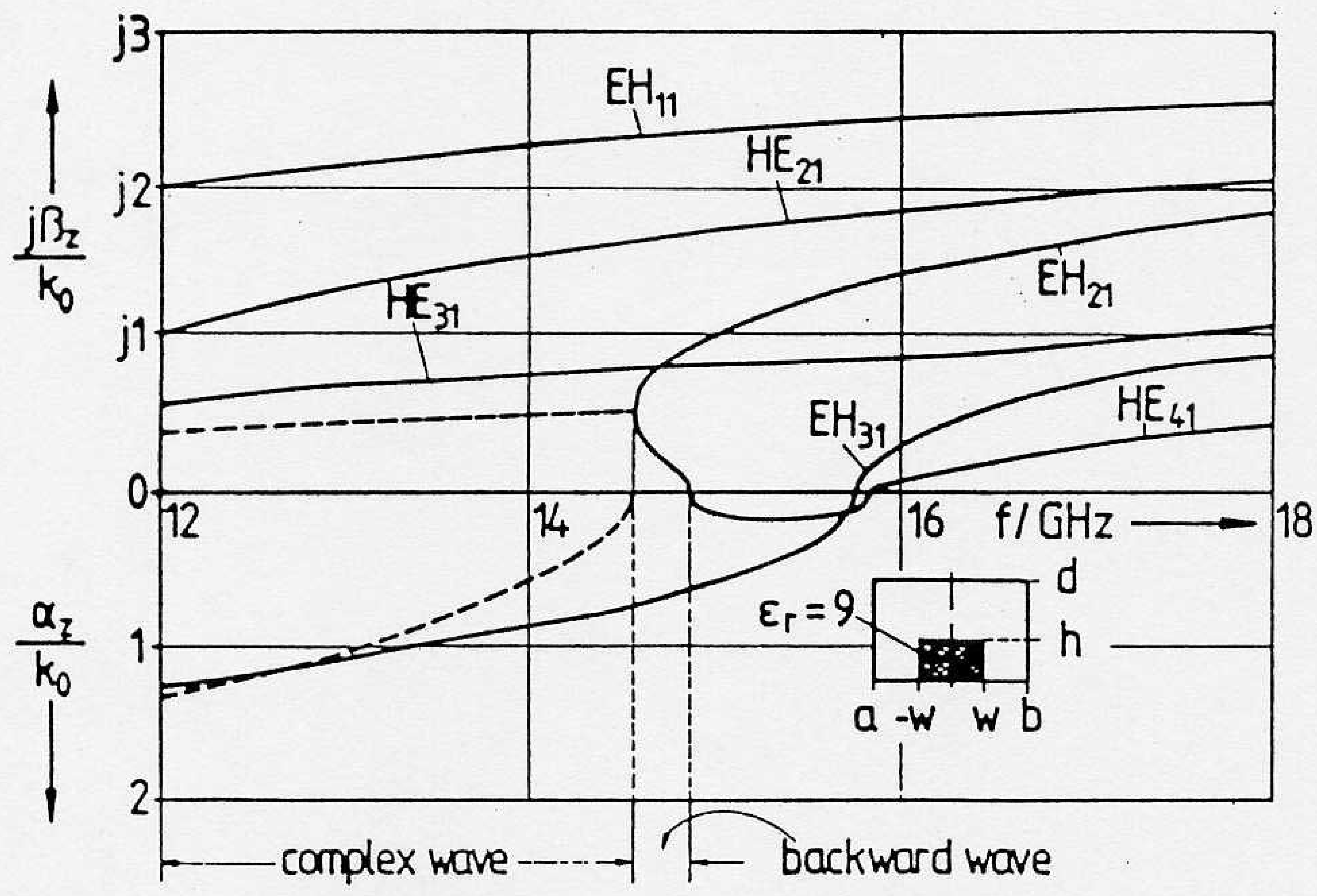
$$\vec{H} = \frac{-1}{j\omega\mu} \nabla \times \nabla \times (A_{hz}\vec{e}_z) + \nabla \times (A_{ez}\vec{e}_z)$$



3. Shielded Dielectric Image Guide

The shielded dielectric image guide shows discontinuities in both cross-section directions. It also consists of two different propagation media (air and dielectric). As a result, the mode spectrum is hybrid (eigenmodes with 6 field components) including complex waves and backward waves.





Dispersion diagram of shielded dielectric image guide (Strube and Arndt, IEEE Trans. MTT-33, May 1985).