An Effective Mesh-Generation Strategy for Image Representation Using Data-Dependent Triangulation

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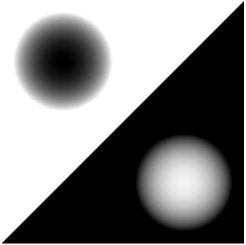
Outline

Motivation

- growing interest in (triangle) mesh representations of images
- mesh representations effective at capturing geometric structure in images (e.g., image edges)
- mesh representations proven beneficial in many applications, including:
  - pattern recognition
  - computer vision
  - feature detection
  - image coding
  - tomographic reconstruction
- two popular classes of mesh representations are those based on:
  - Delaunay triangulations (DTs)
  - data-dependent triangulations (DDTs)
- representations based on DDTs offer much greater flexibility but more difficult to generate

- mesh model completely characterized by:
  1. set P of **sample points** $P = \{p_i\}_{i=0}^{|P|-1} \subset \Lambda$
  2. corresponding function values $Z = \{z_i\}_{i=0}^{|P|-1}$, where $z_i = \phi(p_i)$
  3. set F of (triangle) faces formed by triangulation of P (i.e., connectivity of triangulation of P)



Image

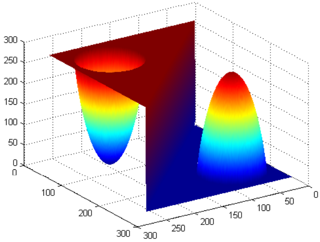
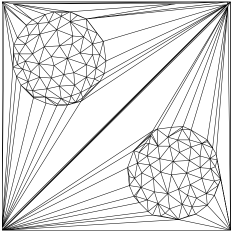
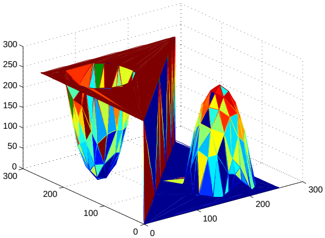


Image Modelled as Surface

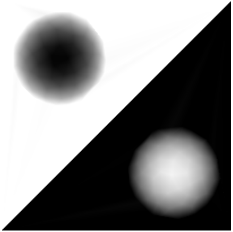
Mesh Approximation of Image (Sampling Density 0.25%)



Triangulation of Image Domain



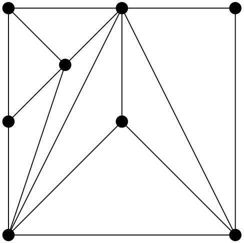
Resulting Triangle Mesh



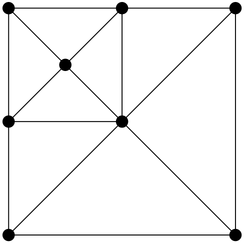
Reconstructed Image

Triangulation

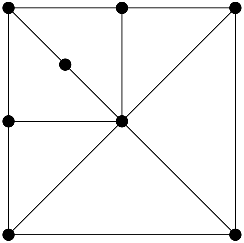
- A **triangulation** of a set V of vertices is a set T of triangles such that:
  - the union of the vertices of all triangles in T is V;
  - the interiors of any two triangles in T are disjoint; and
  - the union of the triangles in T is the convex hull of V.



Triangulation



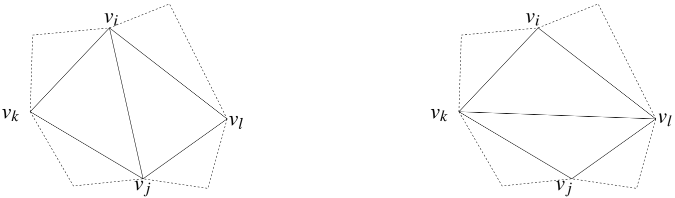
Triangulation



Invalid Triangulation

Edge Flips

- An edge e in a triangulation is said to be **flippable** if it has two incident faces and the union of these two faces is a ***strictly convex*** quadrilateral q.
- If an edge e is flippable, a valid triangulation is obtained if e is deleted from the triangulation and replaced by the other diagonal of quadrilateral q. This transformation is known as an **edge flip**.
- Edge flip example (edge $v_i v_j$ flipped to yield edge $v_k v_l$):



- Every triangulation of a set of points is reachable from every other triangulation (of the same set of points) by edge flips.

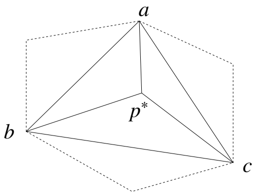
Local Optimization Procedure (LOP)

- Lawson's local optimization procedure (LOP) is algorithm for obtaining triangulation with optimal connectivity, which utilizes edge flips
- define cost function for triangulation
- want to produce triangulation of lowest cost
- edge said to be optimal if:
  1. not flippable; or
  2. flippable and flipping edge would not strictly decrease cost of triangulation
- each edge that is not optimal is flipped
- process continues until all edges optimal
- in effect, triangulation cost function induces criteria for preferred edge
- consider freedom in how to choose criteria for preferred edge
- in practice, cycles can occur (e.g., due to roundoff error); skip edge if visited more than 5 times

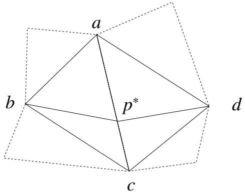
Proposed Mesh-Generation Algorithm

1. **Initial triangulation.** Initially, choose a triangulation of the extreme convex-hull points of the image domain (e.g., four corners of a rectangular image domain).
2. If the target number of sample points has been reached, go to step 7.
3. **Point selection.** Select a new point p^* to add to the triangulation, using an optimality criterion to be described shortly.
4. **Point insertion.** Insert p^* into the triangulation.
5. **Main connectivity adjustment.** Apply the LOP to adjust the connectivity of the triangulation.
6. Go to step 2.
7. **Final connectivity adjustment.** Update the connectivity of the triangulation using a simple postprocessing scheme (i.e., no propagating edge flip).

Step 4: Point Insertion



new vertex p^* is inserted inside a triangle abc



new vertex p^* is inserted on an edge ac

Step 3: Point Selection

- point selection performed in two steps:
  1. select the face f^* in the triangulation into which new point is to be inserted
     - choose f^* as face with greatest squared error
  2. choose point p^* in face f^* for insertion
     1. select set T of test points to consider as candidates for insertion, where T is 8 points in f^* with greatest absolute error
     2. choose p^* as point in T whose insertion would result in greatest decrease in squared error over face f^*

Step 5: Main Connectivity Adjustment

- shape quality of face f defined as

$$\mathrm{quality}(f) = \mathrm{area}(f) / \mathrm{diam}(f),$$

  where $\mathrm{diam}(f)$ is length of longest side of (smallest) bounding box for f and $\mathrm{area}(f)$ is area of f
- in LOP, when choosing between edge e and its flipped version e', choose e if

$$\mathrm{edgeCost}(e) \leq \mathrm{edgeCost}(e'),$$

  where

$$\mathrm{edgeCost}(e) = [\mathrm{faceErr}(f_1) + \mathrm{faceErr}(f_2)] / [\mathrm{quality}(f_1)\,\mathrm{quality}(f_2)],$$

  f_1 and f_2 are two faces incident on e, and $\mathrm{faceErr}(f)$ is squared error summed over points in face f

Step 7: Final Connectivity Adjustment

- final adjustment made to connectivity of triangulation in order to reduce squared error
- LOP not used
- for each flippable edge e in the triangulation, if flipping e results in strictly lower approximation error, e flipped
- edge flips cannot propagate

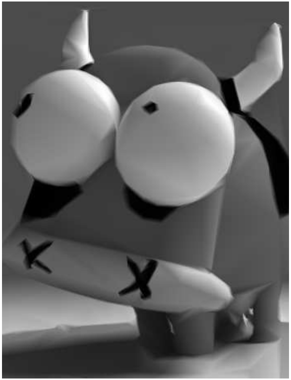
PSNR Mesh-Quality Comparison

Image	Sampling Density (%)	PSNR (dB)		
		Proposed	GPR	GH
bull	0.125	**35.97**	33.12	33.12
	0.250	**40.14**	38.23	38.28
	0.500	**42.48**	41.87	40.73
	1.000	**44.19**	43.99	42.48
ct	0.250	**33.66**	32.15	32.22
	0.500	**38.14**	37.22	37.68
	1.000	**42.66**	41.35	42.01
	2.000	**47.39**	45.33	46.63
lena	0.500	**27.43**	26.66	25.37
	1.000	**30.11**	29.12	28.51
	2.000	**32.55**	31.82	31.26
	3.000	**33.96**	33.37	32.78

- proposed method consistently yields higher quality meshes than GH and GPR schemes, often by more than 1 dB

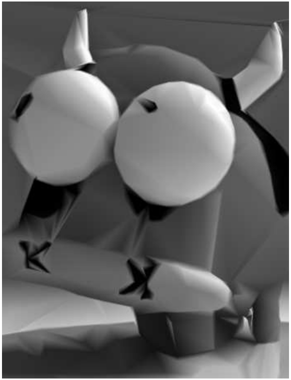
Subjective-Quality Comparison:

bull Image, Sampling Density 0.125%



Proposed (35.97 dB)

GPR (33.12 dB)

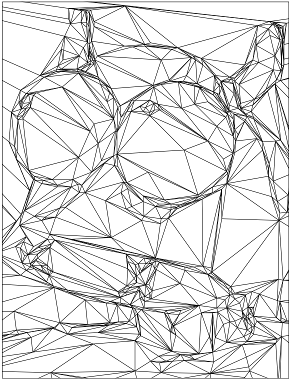


GH (33.12 dB)

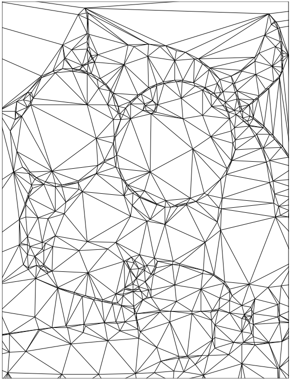
- proposed method produces less visibly distorted image reconstruction relative to GPR and GH schemes, with image edges being better preserved

Subjective-Quality Comparison (Continued):

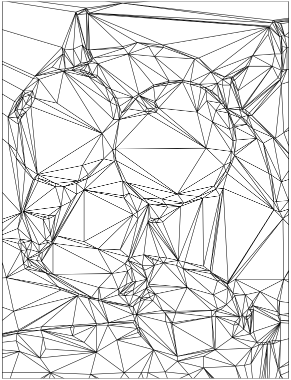
`bull` Image, Sampling Density 0.125%



Proposed



GPR



GH

- triangulation of image domain for each method shown above
- in case of proposed method, triangulation edges tend to better align with image edges

Time-Complexity Comparison

Execution times for the various methods for the `lena` image

Samp. Density (%)	Time (s)		
	Proposed	GPR	GH
0.50	3.28	43.03	1.75
1.00	3.84	43.03	2.15
3.00	4.65	42.40	2.72
3.00	5.27	42.12	3.15

- proposed method requires very substantially less time than GPR scheme (sometimes by more than order of magnitude)
- proposed method requires about 1.6 to 1.8 times more time than GH scheme, but relatively small price to pay for much higher quality meshes

Memory-Complexity Comparison

Peak mesh size for the various methods for
image width W, image height H, and sampling density D

Method	Peak Mesh Size	Relative Peak Mesh Size	
		D= 0.125%	D= 3%
Proposed	DWH	0.00125	0.03
GH	DWH	0.00125	0.03
GPR	WH	1	1

- memory usage is dominated by data structure employed to represent mesh
- proposed method requires essentially same amount of memory as GH scheme
- proposed method requires 33 to 800 times less memory than GPR scheme

Conclusions

- proposed new content-adaptive mesh-generation method for image representation
- compared performance of proposed method to two other schemes (GH and GPR)
- relative to state-of-the-art GPR scheme, proposed method shown to yield better (or comparable) quality meshes in terms of squared error and subjective quality at only very small fraction of computational and memory costs
- relative to GH scheme, proposed method shown to yield much better quality meshes in terms of squared error and subjective quality with same memory cost and only modest increase in computational cost
- beneficial to many applications that use triangle mesh representations of images