An improved method for generating triangle-mesh models of images

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August 2014

Outline

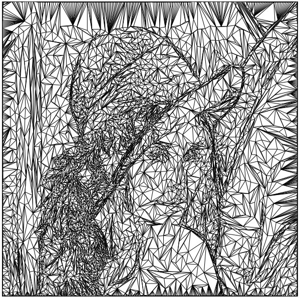
Objective and Motivation

- growing interest in image representation using nonuniform sampling
- since most images non-stationary, uniform sampling highly suboptimal
- nonuniform sampling has proven useful in many applications, including:
  - feature detection, pattern recognition, computer vision
  - image/video coding, tomographic reconstruction, restoration, interpolation
- triangle meshes facilitate nonuniform sampling and capture geometric structure
- two popular classes of triangle meshes are those based on: Delaunay triangulations and data-dependent triangulations (DDTs)
- since DDTs allow for arbitrary triangulation connectivity, offer more flexibility
- one highly effective mesh-generation technique is error diffusion (ED) method, based on Delaunay triangulations
- objective: develop improved version of ED method that utilizes DDTs instead of Delaunay triangulations in order to better exploit triangulation connectivity

Mesh Model of Image

- for image function ϕ sampled at points in Λ, mesh model consists of:
  1. set $P = \{p_i\}$ of sample points, where $P \subset \Lambda$
  2. corresponding function values $Z = \{z_i = \phi(p_i)\}$
  3. triangulation T of P
- approximating function: continuous piecewise linear function $\hat{\phi}$ that interpolates ϕ at each point in P
- sampling density of model defined as $|P| / |\Lambda|$

Original Image



Triangulation

Approximation

Error Metric

- for given number N of sample points (i.e., $|P| = N$), seek to minimize difference between $\hat{\phi}$ and ϕ as measured by mean squared error (MSE)

$$\varepsilon = |\Lambda|^{-1} \sum_{p \in \Lambda} \left(\hat{\phi}(p) - \phi(p) \right)^2$$

- MSE expressed in terms of peak-signal-to-noise ratio (PSNR) given by

$$\mathrm{PSNR} = 20 \log_{10} [(2^{\rho} - 1)/\sqrt{\varepsilon}],$$

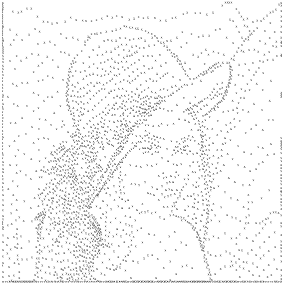
  where ρ is number of bits per sample in image ϕ
- higher PSNR corresponds to lower MSE

Error Diffusion (ED) Method

- method for generating mesh models of images proposed by Yang, Wernick, and Brankov
- method consists of two steps:
  1. select sample points using Floyd-Steinberg error diffusion with points distributed such that density approximately proportional to maximum-magnitude second-order directional derivative (MMSODD)
  2. triangulate sample points using Delaunay triangulation
- example illustrating sample-point selection (i.e., step 1):

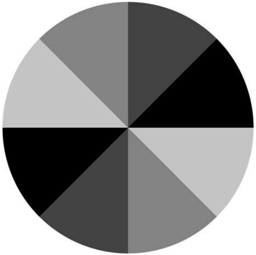

Image


Density Function (MMSODD)

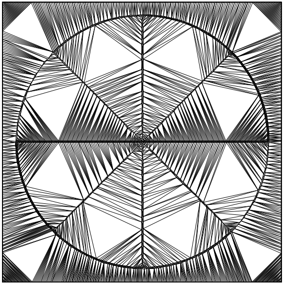

Sample Points

ED Method (Continued)

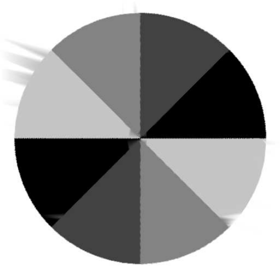
- example illustrating triangulation (i.e., step 2) and reconstruction:



Original Image



Triangulation

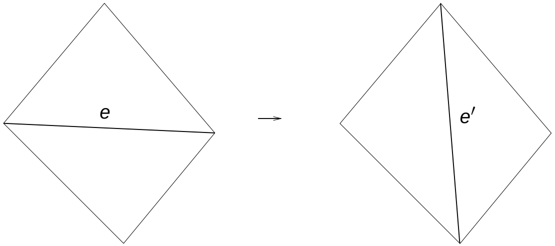


Approximation (31.65 dB)

- simple and computationally efficient
- use of Delaunay connectivity often results in many triangulation edges crosscutting image edges, leading to degraded approximation quality

Edge Flips

- edge e in triangulation said to be flippable if e has two incident faces and union of these faces is strictly convex quadrilateral q
- for flippable edge e, edge flip is operation that replaces edge e in triangulation by other diagonal of q
- example of edge flip:



- for same set P of points, every triangulation of P is reachable from every other triangulation of P via finite sequence of edge flips

Lawson Local Optimization Procedure (LOP)

- technique for selecting connectivity of triangulation so as to be optimal with respect to some prescribed criterion
- frequently used for choosing connectivity of DDTs
- repeatedly applies edge flips until all flippable edges are optimal with respect to some criterion
- squared error (SE) criterion: flippable edge e optimal if applying edge flip to e would not lead to strict decrease in squared error ε
- for most criteria, optimal solution not unique (i.e., only local optimum guaranteed)
- some local optima much better than others

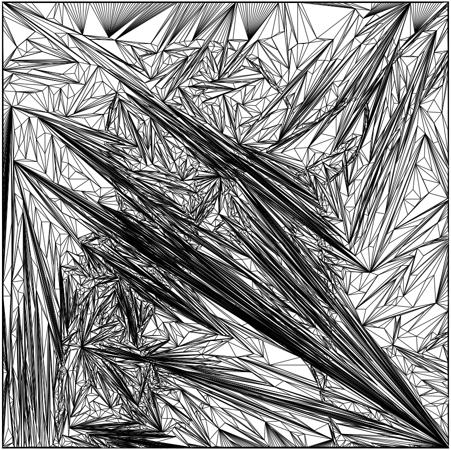
Computational Framework for Proposed Method

1. sample-point selection:
   select set P of sample points for mesh model of desired size using same sample-point selection strategy employed by ED method
2. initial mesh construction:
   for each point $p \in P$, starting with extreme convex hull points followed by remaining points in randomized order:
   1. insert p in triangulation T by deleting any faces containing p and retriangulating resulting hole
   2. adjust connectivity of T by applying LOP with triangulation optimality criterion chosen as c, where c is free parameter of framework
3. final connectivity adjustment:
   adjust connectivity of T by applying LOP with triangulation optimality criterion chosen as SE (i.e., squared error)

Rationale for Computational Framework

- choice of free parameter c is critical, as different choices of c will typically lead to vastly differing meshes
- since goal is to minimize MSE, "obvious" approach is simply to optimize for squared error (i.e., choosing c as SE)
- obvious solution yields extremely poor quality meshes
- when c is chosen as SE, LOP usually converges to extremely bad local optimum, corresponding to triangulation with many poorly-chosen sliver triangles, leading to severely degraded approximation quality
- use of SE criterion extremely sensitive to starting point for LOP
- proposed framework allows c to be chosen differently from SE
- added final connectivity-adjustment step employing SE criterion in order to further reduce squared error for final mesh

Poor Performance of Obvious Approach



Triangulation

Approximation (20.72 dB)

- use of obvious approach leads to triangulations with many very poorly chosen sliver triangles, leading to extremely poor quality approximation

Choice of Optimality Criterion c

- framework requires choice of optimality criterion c to be used by LOP after insertion of each new point in triangulation
- ten possibilities for c considered:

Criterion	Described/Proposed By
squared error (SE)	Rippa
preferred-directions Delaunay	Dyken and Floater
angle between normals (ABN)	Dyn, Levin, and Rippa
jump in normal derivatives (JND)	Dyn, Levin, and Rippa
deviations from linear polynomials (DLP)	Dyn, Levin, and Rippa
distances from planes (DP)	Dyn, Levin, and Rippa
edge-length-weighted ABN (ELABN)	Alboul, Kloosterman, Traas, and Van Damme
Garland-Heckbert hybrid (GHH)	Garland and Heckbert
shape-quality-weighted SE (SQSE)	Li and Adams
JND-weighted SE (JNDSE)	Li and Adams

Choice of Optimality Criterion c: Specific Results

Image	Samp. Density (%)	PSNR (dB)									
		SE	Delaunay	ABN	JND	DLP	DP	ELABN	GHH	SQSE	JNDSE
animal	0.5	28.45	37.32	30.44	37.09	33.34	28.92	36.40	**37.73**	37.71	37.70
	1.0	28.95	40.58	35.74	40.60	38.02	30.24	40.45	40.36	**40.67**	40.61
	2.0	34.60	42.93	36.66	42.76	40.27	29.23	42.74	42.86	43.26	**43.26**
	3.0	32.99	44.24	39.35	43.85	41.70	34.65	43.22	44.23	44.46	**44.49**
	4.0	36.52	45.23	39.91	44.79	41.58	35.89	44.32	45.27	45.46	**45.50**
cr	0.5	31.19	34.40	30.38	34.45	32.42	30.23	34.22	34.30	34.81	**34.84**
	1.0	32.41	36.33	33.01	36.35	34.42	31.16	36.37	36.48	37.13	**37.16**
	2.0	33.33	38.68	34.34	38.36	36.33	32.52	38.24	38.75	38.95	**39.01**
	3.0	34.12	39.57	34.95	39.32	36.96	33.78	39.17	39.62	39.76	**39.82**
	4.0	35.63	40.10	36.29	39.89	37.56	33.54	39.70	40.19	40.31	**40.36**
lena	0.5	17.61	21.17	19.22	20.55	19.61	18.07	20.51	21.20	21.75	**21.82**
	1.0	21.50	25.21	20.69	24.91	21.86	19.91	24.58	25.30	25.89	**25.92**
	2.0	20.72	29.48	24.36	29.09	26.25	21.04	27.67	29.26	29.91	**29.99**
	3.0	23.43	31.26	24.62	30.99	27.22	22.34	30.15	31.21	31.58	**31.62**
	4.0	23.67	32.39	26.30	32.17	29.13	24.06	31.45	32.47	32.78	**32.84**

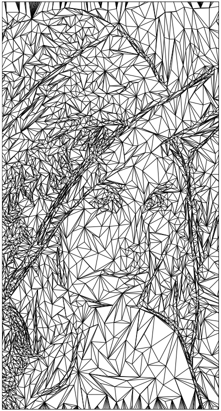
- JNDSE criterion performs best in 13/15 of above test cases

Choice of Optimality Criterion c: Overall Results

Samp. Density (%)	Mean Rank									
	SE	Delaunay	ABN	JND	DLP	DP	ELABN	GHH	SQSE	JNDSE
0.5	8.23	3.93	7.78	6.13	6.98	9.40	5.38	3.28	2.08	**1.85**
1.0	8.95	4.05	8.03	5.60	7.10	9.75	5.00	3.35	1.90	**1.28**
2.0	8.98	3.80	8.00	5.08	7.10	9.90	5.85	3.30	1.85	**1.15**
3.0	9.18	3.90	8.00	4.95	7.03	9.80	6.00	3.15	1.95	**1.05**
4.0	9.10	3.88	8.03	5.03	7.03	9.85	5.98	3.13	1.95	**1.05**
All	8.89	3.91	7.97	5.36	7.05	9.74	5.64	3.24	1.95	**1.28**

- 200 test cases: 40 images (mix of photographic, medical, and computer generated), 5 sampling densities per image
- in each test case, results ranked (with 1 being best), yielding mean rankings shown in table above
- in terms of average ranking, JNDSE criterion clear winner, especially for higher sampling densities
- JNDSE criterion performs best in 164/200 (82%) and second best in 26/200 (13%)
- proposed method: choose c as JNDSE

Poor Performance of SE: Example (lena image, sampling density 2%)

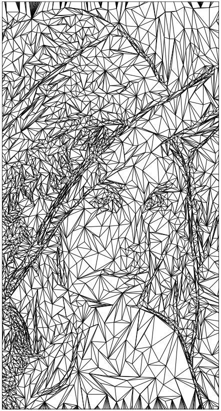
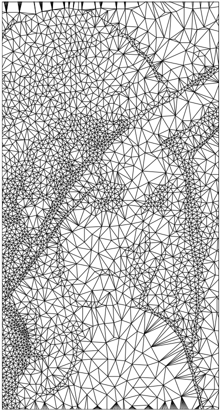
SE		JNDSE	
			
Approximation (20.72 dB)	Triangulation	Approximation (29.99 dB)	Triangulation

Proposed Method vs. ED Method: PSNR

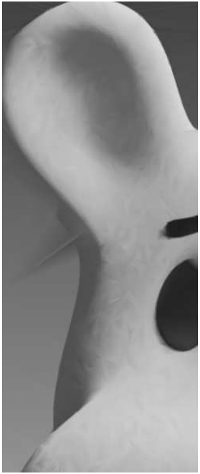
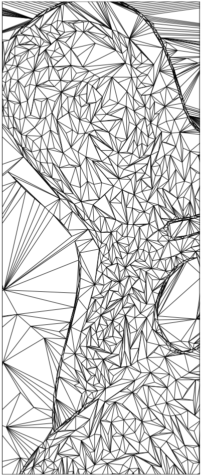
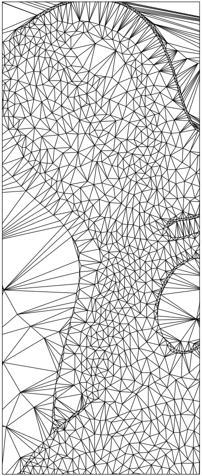
Image	Samp. Density (%)	PSNR (dB)	
		Proposed	ED
animal	0.50	**37.70**	33.86
	1.00	**40.61**	37.66
	2.00	**43.26**	40.46
	3.00	**44.49**	41.91
	4.00	**45.50**	42.23
cr	0.50	**34.84**	31.96
	1.00	**37.16**	33.84
	2.00	**39.01**	35.72
	3.00	**39.82**	37.63
	4.00	**40.36**	38.48
lena	0.50	**21.82**	17.76
	1.00	**25.92**	21.50
	2.00	**29.99**	26.38
	3.00	**31.62**	28.50
	4.00	**32.84**	29.83

- proposed method outperforms ED method in all 15 cases by at least 1.88 dB
- for same 200 test cases, proposed method beats ED method by 1.5 to 6.7 dB, averaging approximately 3 dB, having significantly improved ED method

Proposed Method vs. ED Method: Example (lena image, sampling density 2%)

Proposed		ED	
			
Approximation (29.99 dB)	Triangulation	Approximation (26.38 dB)	Triangulation

Proposed Method vs. ED Method: Example (animal image, sampling density 0.5%)

Proposed		ED	
			
Approximation (37.70 dB)	Triangulation	Approximation (33.86 dB)	Triangulation

Conclusions

- proposed improved method for generating mesh models of images derived from ED scheme
- makes use of DDTs in order to better exploit triangulation connectivity for improved approximation quality
- proposed method yields meshes of significantly higher quality than those obtained with ED scheme, both in terms of PSNR and visual quality
- comes at relatively modest cost in terms of computation time (e.g., only 1.5 seconds for lena image at sampling density of 2%)
- proposed method of great value to many applications that require mesh models of images

References

1. ED method: Y. Yang, M. N. Wernick, and J. G. Brankov. A fast approach for accurate content-adaptive mesh generation. *IEEE Trans. on Image Processing*, 12(8):866–881, Aug. 2003.
2. JNDSE criterion: P. Li and M. D. Adams. A tuned mesh-generation strategy for image representation based on data-dependent triangulation. *IEEE Trans. on Image Processing*, 22(5):2004–2018, May 2013.
3. C. L. Lawson. Software for C^1 surface interpolation. In J. R. Rice, editor, *Mathematical Software III*, pages 161–194. Academic Press, New York, NY, USA, 1977.
4. N. Dyn, D. Levin, and S. Rippa. Data dependent triangulations for piecewise liner interpolations. *IMA Journal of Numerical Analysis*, 10(1):137–154, 1990.
5. C. Dyken and M. S. Floater. Preferred directions for resolving the non-uniqueness of Delaunay triangulations. *Computational Geometry—Theory and Applications*, 34:96–101, 2006.
6. M. Garland and P. S. Heckbert. Fast polygonal approximation of terrains and height fields. Technical Report CMU-CS-95-181, School of Computer Science, Carnegie Mellon University, Pittsburgh, PA, USA, Sept. 1995.