CHAPTER 7

AMPLITUDE MODULATION

Transmit information-bearing (message) or baseband signal (voice-music)

through a Communications Channel

Baseband = band of frequencies representing the original signal

for music 20 Hz - 20,000 Hz, for voice 300 - 3,400 Hz

write the baseband (message) signal $m(t) \leftrightarrow M(f)$

Communications Channel

Typical radio frequencies 10 KHz $\rightarrow$ 300 GHz

write $c(t) = A_c \cos(2\pi fct)$

$c(t)$	=	Radio Frequency Carrier Wave
A_c	=	Carrier Amplitude
fc	=	Carrier Frequency

Amplitude Modulation (AM)

$\rightarrow$ Amplitude of carrier wave varies $\pm$ a mean value in step with the baseband signal $m(t)$

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

Mean value A_c.

page 7.2

Recall a general signal

$$s(t) = a(t)\cos[2\pi f_c t + \phi(t)]$$

For AM

$$\begin{aligned} a(t) &= A_c[1 + k_a m(t)] \\ \phi(t) &= 0 \text{ or constant} \end{aligned}$$

k_a = Amplitude Sensitivity

Note 1 $|k_a m(t)| < 1$or $[1 + k_a m(t)] > 0$
2 $f_c \gg w$ = bandwidth of $m(t)$

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AM Signal In Time and Frequency Domain

$$s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$$

$$s(t) = A_c[1 + k_a m(t)]\frac{(e^{j2\pi f_c t} + e^{-j2\pi f_c t})}{2}$$

$$\begin{aligned} s(t) = {} & \frac{A_c}{2}e^{j2\pi f_c t} + \frac{A_c}{2}e^{-j2\pi f_c t} \\ & + \frac{A_c k_a}{2}m(t)e^{j2\pi f_c t} \\ & + \frac{A_c k_a}{2}m(t)e^{-j2\pi f_c t} \end{aligned}$$

To find $S(f)$ use:

$m(t) \leftrightarrow M(f)$

$e^{j2\pi f_c t} \leftrightarrow \delta(f - f_c)$

$e^{-j2\pi f_c t} \leftrightarrow \delta(f + f_c)$

$\exp(j2\pi f_c t)m(t) \leftrightarrow M(f - f_c)$

$\exp(-j2\pi f_c t)m(t) \leftrightarrow M(f + f_c)$

$$\begin{aligned} S(f) = {} & \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] \\ & + \frac{A_c k_a}{2}[M(f - fc) + M(f + f_c)] \end{aligned}$$

page 7.4

$$
\begin{aligned}
s(t) &= A_c[1 + k_a m(t)] \cos 2\pi f_c t \\
&= \frac{A_c}{2}[1 + k_a m(t)][exp(j2\pi f_c t) + exp(-j2\pi f_c t)]
\end{aligned}
$$

If $k_a m(t) > 1$, then

$\rightarrow$ Overmodulation

$\rightarrow$ Envelope Distortion

see Text p. 262 - Fig. 7.1

max value of $k_a m(t)$ = Percent Modulation

Look at AM signal in frequency domain

$$
\begin{aligned}
S(f) &= \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] \\
&\quad + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]
\end{aligned}
$$

where $M(f) = FT$ of $m(t)$

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Example 1 p. 265 - Single Tone Modulation

$$\begin{aligned} m(t) &= A_m \cos 2\pi f_m t \qquad f_m = 1 \text{ KHz (message frequency)} \\ s(t) &= A_c(1 + k_a m(t)] \cos 2\pi f_c t \\ s(t) &= A_c(1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t \\ &\mu = k_a A_m < 1 \\ &= \text{Modulation Factor (Percentage)} \end{aligned}$$

Draw $s(t)$ for $\mu < 1$

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Rewrite $s(t)$ using trig identity

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta) \text{ with } \alpha = f_c, \quad \beta = f_m$$

$$\begin{aligned} s(t) &= A_c[\cos 2\pi f_c t + \frac{\mu}{2}\cos 2\pi(f_c + f_m)t \\ &\qquad + \frac{\mu}{2}\cos 2\pi(f_c - f_m)t] \\ S(f) &= \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] \\ &+ \frac{A_c\mu}{4}[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \\ &+ \frac{A_c\mu}{4}(\delta(f - f_c + f_m) + \delta(f + f_c - f_m))] \end{aligned}$$

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$$[s(t)]^2 = \text{ power into } 1\Omega \sim |S(f)|^2$$

Carrier Power $\quad 2\left(\frac{A_c}{2}\right)^2 = \frac{A_c^2}{2}$

Positive and
Negative Frequencies $\nearrow$

Upper Sideband Power $\quad 2\left(\frac{A_c\mu}{4}\right)^2 = \frac{A_c^2\mu^2}{8}$

Lower Sideband Power $\quad 2\left(\frac{A_c\mu}{4}\right)^2 = \frac{A_c^2\mu^2}{8}$

$$\frac{\text{USB + LSB Power}}{\text{Total Power}} = \frac{\frac{A_c^2\mu^2}{8} + \frac{A_c^2\mu^2}{8}}{\frac{A_c^2}{2} + \frac{A_c^2\mu^2}{8} + \frac{A_c^2\mu^2}{8}} = \frac{\mu^2}{2+\mu}$$

<u>Example</u>

Carrier Power $\frac{A_c^2}{2}$	% Modulation $\mu \cdot 100$	Modulation (Audio) Power USB + LSB $\frac{A_c^2}{4}\mu^2$	Total Power $(2+\mu^2)\frac{A_c^2}{4}$	Audio Power Total $\frac{\mu^2}{2+\mu^2}$
100	100	50	150	0.33
100	50	12.5	112.5	0.11
100	20	4	104	0.04
100	0	0	100	0

page 7.8

Generation of AM Waves - P. 267

Square Law Modulator -

3 stages: Adder, Non-Linearity (NL), Bandpass Filter (BPF)

For this example, we use non-linearity $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$

$$\begin{aligned} \text{Here } v_1(t) &= A_c \cos 2\pi f_c t + m(t) \\ \rightarrow v_2(t) &= \overbrace{a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t} \\ &\quad + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t \end{aligned}$$

The first (bracketed) term is desired AM wave at carrier f_c with

$$k_a = \frac{2a_2}{a_1}$$

Remaining terms are at baseband or $2f_c$ and are filtered out by BPF. BPF has center frequency f_c bandwidth $2w$. The $\cos^2$ term has components at baseband and at $2f_c$ because

$$\cos^2 \alpha = \frac{1}{2}(1 + 2\cos 2\alpha) \text{ with} \alpha = 2\pi f_c t$$

page 7.10

Given $v_2(t)$ find $V_2(f)$ Curve

$$
\begin{aligned}
v_2(t) &= a_1 A_c \cos 2\pi f_c t && \text{a} \\
& \quad 2a_2 A_c m(t) \cos 2\pi f_c t && \text{b} \\
& \quad +a_1 m(t) && \text{c} \\
& \quad +a_2 m^2(t) && \text{d} \\
& \quad +a_2 A_c^2 \underbrace{\frac{1}{2}(1 + \cos 2 \cdot 2\pi f_c t)}_{\cos^2 2\pi f_c t} && \text{e}
\end{aligned}
$$

$$
\begin{aligned}
V_2(f) &= a_1 \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] \\
& \quad +2a_2 A_c [M(f - f_c) + M(f + f_c)] \\
& \quad +a_1 M(f) \\
& \quad +a_2 FT[m^2(t)] \\
& \quad +\frac{a_2 A_c^2}{2}[\delta(f) + \delta(f - 2f_c) + \delta(f + 2f_c)]
\end{aligned}
$$

Assignment

Text: p. 384, p. 7.1, p. 7.3, p. 269 Ex. 2

page 7.11

Generation of AM Waves Continued - P. 269-270

Switching Modulator

Same as square law except non linearity is now: $\rightarrow$

$$v_2(t) = \begin{cases} v_1(t) & v_1(t) > 0 \\ 0 & v_1(t) < 0 \end{cases}$$

$$\begin{aligned} v_1(t) &= A_c \cos 2\pi f_c t + m(t) \\ &= c(t) + m(t) \end{aligned}$$

Assume $|m(t)| \ll A_c$

Thus $v_2(t) = \begin{cases} v_1(t) & c(t) > 0 \\ 0 & c(t) < 0 \end{cases}$

Thus switch diode on and off at rate f_c

diode on only when $c(t) > 0$

$$v_2(t) = [A_c cos 2\pi f_c t + m(t)] g_p(t)$$

where $g_p(t)$ is a unipolar square wave at f_c.

page 7.12

Recall Fourier Series ELEC 260 p. 106

$f(t)$ with period T

$$f(t) = a_o + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}$$

$$a_o = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\frac{2\pi nt}{T}dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\frac{2\pi nt}{T}dt$$

Consider $g_p(t)$ with period T_0 and frequency $f_c = 1/T_0$

$$a_o = \frac{1}{T_o}\int_{-T_o/4}^{T_o/4} 1\cdot dt = \frac{1}{T_o}\,\frac{T_o}{2} = \frac{1}{2}$$

$$a_n = \frac{2}{T_o}\int_{-T_o/4}^{T_o/4} \cos\frac{2\pi nt}{T_o}dt = \frac{2}{T_o}\,\frac{T_o}{2\pi n}\sin\left.\frac{2\pi nt}{T_o}\right|_{\frac{-T_o}{4}}^{\frac{T_o}{4}}$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{2\pi nT_o/4}{T_o}\right) - \sin\left(-\frac{2\pi nT_o/4}{T_o}\right)\right]$$

$$= \frac{1}{\pi n}2\sin\left(\frac{\pi n}{2}\right) = \left\{\begin{array}{ll} \frac{2}{\pi n} & n = 1,\ 5,\ 9\ \ldots \\ \frac{-2}{\pi n} & n = 3,\ 7,\ \ldots \end{array}\right\} = \frac{(-1)^{n-1}}{2n-1}\cdot\frac{2}{\pi}$$

$n = 2m-1, \quad m = 1, 2, 3, \ldots$

page 7.13

Relabel $m \to n$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left[\frac{2\pi(2n-1)f_c t}{T_o}\right] \text{ with } f_c = 1/T_o$$

write $g_p(t)$ as Fourier cosine series (see ELEC 260)

$$\begin{aligned}
g_p(t) &= \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)] \\
v_2(t) &= [A_c \cos 2\pi f_c t + m(t)]g_p(t) \\
&= [A_c \cos 2\pi f_c t + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t + \ldots\right] \\
&= \frac{A_c}{2}\cos 2\pi f_c t + m(t)\frac{2}{\pi}\cos 2\pi f_c t + \frac{m(t)}{2} + \ldots \\
&= \underbrace{\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c}m(t)\right] A_c \cos 2\pi f_c t}_{\text{AM wave with } k_a = \frac{4}{\pi A_c}} + \underbrace{\ldots}_{\text{unwanted components at frequencies removed from } f_c}
\end{aligned}$$

= AM wave with $k_a = \frac{4}{\pi A_c}$ unwanted components

at frequencies removed from f_c

page 7.14

Demodulation of AM Waves

Square Law Demodulator

No good unless $m(t) \ll A_c$ i.e. % modulation very low see P. 271 - Skip this

Envelope Detector - P. 272

page 7.15

7.2 P. 274 - Double Sideband Suppressed Carrier Modulation (DSBSC)

Carrier $c(t) = A_c \cos 2\pi f_c t$ does not carry information $\rightarrow$ waste of power

Recall $\dfrac{\text{Power in } m(t)}{\text{Power in } c(t)} = \dfrac{1}{3}$ max at 100% modulation

$\rightarrow$ Suppress the Carrier

$$
\begin{aligned}
\text{AM:}\quad s_{AM}(t) &= [1 + \quad m(t)]c(t) \\
&= c(t) + m(t)\ c(t) \qquad k_a = 1 \\
\text{DSB:}\quad s(t) &= \qquad m(t)c(t) \\
&= A_c \cos 2\pi f_c t\ m(t) \\
S(f) &= \frac{1}{2} A_c [M(f - f_c) + M(f - f_c)]
\end{aligned}
$$

Phase reversal when $m(t) < 0$.

page 7.16

DSBSC

$$
\begin{aligned}
s(t) &= c(t)m(t) \\
&= A_c \cos 2\pi f_c t \quad m(t) \\
&= \frac{A_c}{2}[\exp(-j2\pi f_c t) + \exp(j2\pi f_c t)]m(t)
\end{aligned}
$$

Recall shifting property

$$m(t) \leftrightarrow M(f)$$

$$\exp(j2\pi f_c t)m(t) \leftrightarrow M(f - f_c)$$

$$S(f) = \frac{A_c}{2}[M(f + f_c) + M(f - f_c)]$$

Single tone modulation

$$
\begin{aligned}
m(t) &= A_m \cos 2\pi f_m t \\
M(f) &= \frac{A_m}{2}(\delta(f - f_m) + \delta(f + f_m)] \\
M(f - f_c) &= \frac{A_m}{2}[\delta(f - f_m - f_c) + \delta(f + f_m - f_c)] \\
M(f + f_c) &= \frac{A_m}{2}[\delta(f - f_m + f_c) + \delta(f + f_m + f_c)]
\end{aligned}
$$

page 7.18

3.5 Generation of DSBSC Waves - P. 276

$$\begin{aligned} DSBs(t) &= c(t)m(t) \\ &= A_c \cos 2\pi f_c t \quad m(t) \end{aligned}$$

Balanced modulator

$$\begin{aligned} s_1(t) &= A_c[1 + k_a m(t)] \cos 2\pi f_c t \\ s_2(t) &= A_c[1 - k_a m(t)] \cos 2\pi f_c t \\ s(t) &= s_1(t) - s_2(t) \\ &= 2k_a A_c \cos 2\pi f_c t \quad m(t) \\ &= 2k_a c(t)m(t) \end{aligned}$$

page 7.19 P. 277 To produce DSB signal $s(t) = m(t)c(t)$

Ring modulator (mixer) multiplies $m(t)$ with $c(t)$.

For positive half-cycles of $c(t)$

D1 and D3 conduct, D2 and D4 open

point a connected to point b

point c connected to point d

current through T2 primary proportional to $m(t)$.

For negative half-cycles of $c(t)$

D2 and D4 conduct, D1 and D3 open

point a connected to point d

point b connected to point c

current through T2 primary proportional to $-m(t)$.

Use square wave $c(t)$ to avoid problems with diode offsets.

page 7.20

Ring Modulator

$$
\begin{aligned}
s(t) &= c(t)m(t) \\
&= \underbrace{\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]}_{\text{Square Wave Carrier}}\underbrace{m(t)}_{\text{mod}}
\end{aligned}
$$

Compare $c(t)$ here with $g_p(t)$ on page 7.12.

Here $c(t)$ range -1 to $+1$, no DC term

$g_p(t)$ range 0 to 1, with DC term.

$$
S(f) = C(f) \otimes M(f) = \int_{-\infty}^{\infty} C(\lambda)M(f-\lambda)d\lambda
$$

$$
S(f) = \overbrace{\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\ \frac{[\delta(f-f_c(2n-1))+\delta(f+f_c(2n-1))]}{2}}^{C(f)} \otimes M(f)
$$

For $n = 1$

$$
\begin{aligned}
S(f) &= \frac{4}{2\pi}[\delta(f-f_c)+\delta(f+f_c)] \otimes M(f) \\
&= \int_{-\infty}^{\infty}\frac{4}{2\pi}(\delta(\lambda-f_c)+\delta(\lambda+f_c)]M(f-\lambda)d\lambda \\
&= \frac{4}{2\pi}[M(f-f_c)+M(f+f_c)]
\end{aligned}
$$

For $n = 2\ldots$

$n = 3\ldots$

page 7.20A

Superheterodyne Receiver

Since f_c varies from one signal to the next, the receiver is designed to convert all f_c to a fixed frequency f_{IF}. This way we can use the same filter and detector for any signal at any f_c.

RF	540-1600	KHz	AM Broadcast
LO	995-2055	KHz	
IF = LO - RF	455	KHz	

Example   Consider CFAX at 1070 KHz

Here LO = 1525 KHz	SUM = 2595
IF = LO = RF = 455 KHz	DIF = 455

Mixer outputs sum and difference frequencies

thus another station at 1980 KHz (image frequency) will also be received since

LO = 1525 KHz

RF = 1980 KHz

RF-LO = 455 KHz

Image is filtered out by RF AMP/filter

Homodyne receiver uses zero frequency IF, i.e. LO=RF, IF=0 page 7.20X

General I-Q receiver - demodulator

applies for any signal, AM, DSB and others.

Input signal

$$\begin{aligned} s(t) &= a(t)cos[2\pi f_{IF}t + \theta(t)] \\ &= x(t)\ \cos 2\pi f_{IF}t - y(t)\ \sin 2\pi f_{IF}t \end{aligned}$$

Output signals $I = x(t)$ and $Q = y(t)$.

Exercise - show this using algebra

page 7.23

DSBSC Demodulation

Costas Receiver

same as general I-Q receiver with extra feedback signal

Two coherent detectors	Output
One LO $\cos(2\pi f_c t + \phi)$	$\frac{1}{2}A_c \cos\phi \;\; m(t)$
Other LO $\sin(2\pi f_c t + \phi)$	$\frac{1}{2}A_c \sin\phi \;\; m(t)$

Here $f_c = f = f_{IF}$.

Phase discriminator output $e(t)$ is a DC control signal

$$
\begin{aligned}
e(t) &= \left(\frac{1}{2}A_c \cos\phi \;\; m(t)\right) \cdot \left(\frac{1}{2}A_c \sin\phi \;\; m(t)\right) \\
&= \frac{A_c^2}{4} m^2(t) \cos\phi \sin\phi = \frac{A_c^2}{8} m^2(t) \sin 2\phi
\end{aligned}
$$

The DC control signal is used as a feedback signal to reduce ϕ

$$
\begin{aligned}
TP11 &= \cos 2\pi f_c t \cos(2\pi f_c t + \phi) \\
TP19 &= \cos 2\pi f_c t \sin(2\pi f_c t + \phi) \\
TP13 &=
\end{aligned}
$$

page 7.24

Costas receiver in more detail

Costas receiver is same as general I-Q receiver with extra feedback signal $e(t) = I \cdot Q$ used to adjust oscillator frequency f to be close to f_{IF}. In general $f_{IF} \neq f$.

IF input signal $s_{IF}(t) = m(t)\cos(2\pi f_{IF} t + \phi_{in})$.

From table 3 page 626,

$$2\cos\alpha\sin\beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$\begin{aligned}
TP11 &= m(t)\cos(2\pi f_{IF} t + \phi_{in})\cos(2\pi f t + \phi_\ell) \\
&= \frac{1}{2}m(t)[\cos(2\pi(f_{IF} - f)t + \phi_{in} - \phi_\ell) + \cos(2\pi(f_{IF} + f)t + \phi_{in} + \phi_\ell)] \\
TP19 &= m(t)\cos(2\pi f_{IF} t + \phi_{in})\sin(2\pi f t + \phi_\ell) \\
&= \frac{1}{2}m(t)[\sin(2\pi(f_{IF} - f)t + \phi_{in} - \phi_\ell) + \sin(2\pi(f_{IF} + f)t + \phi_\ell + \phi_{in})] \\
TP13 &= \frac{1}{2}m(t)\cos[2\pi(f_{IF} - f)t + \phi_{in} - \phi_\ell) \\
TP12 &= \frac{1}{2}m(t)\sin(2\pi(f_{IF} - f)t + \phi_{in} - \phi_\ell)
\end{aligned}$$

page 7.25

Costas receiver continued

$$\text{Define } \triangle f = f_{IF} - f$$
$$\theta = \phi_\ell - \phi_{in}$$

$$\begin{aligned} e(t) &= (TP12)\cdot(TP13) \\ &= \frac{1}{4}m^2(t)\cos(2\pi\triangle ft-\theta)\sin(2\pi\triangle ft-\theta) \\ &= \frac{1}{8}m^2(t)\sin(4\pi\triangle ft-2\theta) \end{aligned}$$

Using $\sin\alpha\cos\alpha = \frac{1}{2}\sin\cdot 2\alpha$

$$e(t) = 0 \text{ if } \triangle f = 0 \text{ and } \theta = 0$$

If $e(t) \neq 0$, then VCO will be adjusted so that $e(t) = 0$

After LPF 5Hz

$$\begin{aligned} e_{LP}(t) &= \frac{1}{8}\sin(4\pi\triangle ft-2\theta)\ \underbrace{<m^2(t)>}_{\text{Average Value}} \\ &= \text{DC Control Signal} \end{aligned}$$

For small $(4\pi\triangle ft - 2\theta) = x,\quad \sin x \sim x.$

Thus $e_{LP}(t) \simeq 4\pi\triangle ft - 2\theta$

P. 283 - Quadrature Carrier Multiplexing

- Combine two independent DSBSC modulated waves on the same frequency f_c with two modulations $m_1(t)$ $m_2(t)$
- To achieve this we use two carrier waves at f_c but 90^o out of phase

$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

To receive the two independent modulations $m_1(t)$ $m_2(t)$ use the general I-Q receiver. The I-Q receiver contains two separate DSBSC demodulators with local oscillators (carriers) 90^o out of phase, $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$.

For this to work, we need to have the local oscillators to have frequency f_c exactly the same as the frequency of $s(t)$.

Note that $s(t)$ is in the general format with $x(t) = m_1(t)$ and $y(t) = -m_2(t)$.

Quadrature Carrier Multiplexing - continued

We will show that there is no mixing of the two modulations, even though the carrier frequency is the same

$$
\begin{aligned}
v(t) &= s(t)\cos 2\pi f_c t \\
&= A_c m_1(t)\cos 2\pi f_c t \cos 2\pi f_c t \\
&+ A_c m_2(t)\sin 2\pi f_c t \cos 2\pi f_c t \\
\cos^2\alpha &= \frac{1}{2}(1-\cos 2\alpha) \\
\cos\alpha\sin\alpha &= \frac{1}{2}\sin 2\alpha \\
v(t) &= \frac{1}{2}A_c m_1(t)[1-\cos 4\pi f_c t] \\
&+ \frac{1}{2}A_c m_2(t)\sin 4\pi f_c t
\end{aligned}
$$

<u>After LPF</u>

$$v_o(t) = \frac{1}{2}A_c m_1(t)$$

page 7.29

P. 284 - Single Sideband Modulation

In DSBSC, the two sidebands carry redundant information thus we can eliminate one sideband to get <u>SSB-SC</u> usually referred to as <u>SSB</u>.
SSB with carrier is called <u>SSB-AM</u>
We begin with frequency domain description of SSB

1. Single tone modulation $m(t) = \cos 2\pi f_m t$
2. General modulation $m(t)$

Looking only at positive frequencies

$m(t)$

AM

DSB-SC

SSB

page 7.31

3.9 Generation of SSB Waves

1 - Filtering (Frequency Discrimination)

2 - Phase Discrimination (general I-Q transmitter)

1 Filter Method

Produce Modulator + Filter

Need sharp filter to pass USB and reject LSB
e.g 8-pole crystal filter at 10.7 MHz not practical

Upconvert to desired carrier frequency

Assignment #4 P. 388 P. 7.4 # 16, 19a, 20a page 7.31A

2 General I-Q transmitter

$$s(t) = x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t$$

page 7.32

text P. 288

Time Domain Description of SSB

$$\boxed{s(t) = \frac{A_c}{2}[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t]}$$

$-$ For upper sideband

$+$ For lower sideband

$\hat{m}(t)$ = Hilbert transform of $m(t)$ see notes p. 3.3-3.4

-90^o Phase shift for all positive frequencies

$+90^o$ Phase shift for all negative frequencies

$H(f)$ special kind of "filter"

$$\begin{aligned}
\text{Recall } H(f) &= -j \text{ sgn } f \text{ p.3.3-3.4} \\
h(t) &= 1/\pi t \\
\hat{m}(t) &= m(t) \otimes h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau \\
\hat{M}(f) &= M(f)H(f) \\
&= -j \text{ sgn } f \; M(f)
\end{aligned}$$

page 7.33

Single tone modulation for SSB

Example 3 - P. 291

$$\begin{aligned}\text{If } m(t) &= A_m \cos 2\pi f_m t \\ \hat{m}(t) &= A_m \sin 2\pi f_m t \\ s(t) &= \frac{A_c}{2}[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t] \\ &= \frac{A_c A_m}{2}[\cos 2\pi f_m t \cos 2\pi f_c t \mp \sin 2\pi f_m t \sin 2\pi f_c t]\end{aligned}$$

Recall $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$, thus

$$s(t) = \frac{A_c A_m}{2}\cos[2\pi(f_c \pm f_m)t]$$

$s(t)$ is a single tone at frequency $f_c \pm f_m$. See also text figure 7.3 1 p. 266

page 7.34

SSB - Time $\leftrightarrow$ Frequency Domain

$$
\begin{aligned}
s(t) &= \frac{A_c}{2}[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t] \\
\hat{M}(f) &= M(f)H(f) = -j \text{ sgn } f \; M(f) \quad \text{p. } 7.32
\end{aligned}
$$

$$
\begin{aligned}
S(f) &= \frac{A_c}{2}[M(f) \otimes \frac{1}{2}\{\delta(f-f_c)+\delta(f+f_c)\} \\
&\quad \pm -j \text{ sgn } f \; M(f) \otimes \frac{1}{2j}\{\delta(f-f_c)-\delta(f+f_c)\}] \\
&= \frac{A_c}{4}[M(f) \otimes \delta(f-f_c) \\
&\qquad + M(f) \otimes \delta(f+f_c) \\
&\qquad \pm \text{ sgn } f \; M(f) \otimes \delta(f-f_c) \\
&\qquad \mp \text{ sgn } f \; M(f) \otimes \delta(f+f_c)] \\
&= \frac{A_c}{4}[M(f-f_c)+M(f+f_c) \\
&\qquad \pm \text{sgn}(f-f_c) \; M(f-f_c) \mp \text{sgn}(f+f_c) \; M(f+f_c)] \\
&= \frac{A_c}{2}M(f-f_c), \;\; f-f_c>0, f>f_c
\end{aligned}
$$

similarly

$$
S(f) = \frac{A_c}{2}M(f+f_c), \;\; f+f_< > 0, f < -f_c
$$

P. 293 Demodulation of SSB

$$
\begin{aligned}
c_{LO}(t) &= \cos 2\pi f_c t \\
s(t) &= \frac{A_c}{2}[m(t)\cos 2\pi f_c t \mp \hat{m}(t)\sin 2\pi f_c t] \\
v(t) &= s(t)\cos 2\pi f_c t \\
&= \frac{A_c}{2}\; m(t)\cos 2\pi f_c t \cos 2\pi f_c t \\
&\qquad \mp \frac{A_c}{2}\; \hat{m}(t)\cos 2\pi f_c t \sin 2\pi f_c t \\
\cos^2\alpha &= \frac{1}{2}(1+\cos 2\alpha) \\
\cos\alpha\sin\alpha &= \frac{1}{2}\sin 2\alpha \\
v(t) &= \underbrace{\frac{A_c}{4}m(t)}_{\text{Message Signal}} + \underbrace{\frac{A_c}{4}m(t)\cos 4\pi f_c t \mp \frac{A_c}{4}\hat{m}(t)\sin 4\pi f_c t}_{\text{Unwanted Terms}}
\end{aligned}
$$

This assumes no phase or frequency error in local oscillator $C_{LO}(t)$

Demodulation of SSB with Frequency Error

$$
\begin{aligned}
C_{LO}(t) &= \cos 2\pi(f_c + \triangle f)t \\
s(t) &= \frac{A_c}{2}[m(t)\cos 2\pi f_c t \mp \hat{m}(t)\sin 2\pi f_c t] \\
v(t) &= s(t)\cos 2\pi(f_c + \triangle f)t \\
&= \frac{A_c}{2} m(t)\cos 2\pi(f_c + \triangle f)t \cos 2\pi f_c t \\
&\mp \frac{A_c}{2}\hat{m}(t)\cos 2\pi(f_c + \triangle f)t \sin 2\pi f_c t \\
\cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)] \\
\cos\alpha\sin\beta &= \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)] \\
\alpha &= 2\pi(f_c + \triangle f)t \quad \beta = 2\pi f_c t \quad \alpha - \beta = 2\pi\triangle f t \\
v(t) &= \frac{A_c}{2} m(t)\frac{1}{2}\cos 2\pi\triangle f t \\
&\mp \frac{A_c}{2}\frac{\hat{m}(t)}{2}\sin 2\pi\triangle f t + \text{ Double Frequency Terms}
\end{aligned}
$$

After LPF, $v_0(t)$ is an SSB signal on a low frequency carrier $\triangle f$.

page 7.38

Carrier Plus SSB Wave

$$
\begin{aligned}
s(t) &= \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier}} + \underbrace{m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t}_{\text{USB}} \\
&= (A_c + m(t)] \cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t \\
&= a(t)\cos(2\pi f_c t + \phi) \\
& \quad \text{where } a(t) = \sqrt{[A_c + m(t)]^2 + [\hat{m}(t)]^2} \\
a(t) &= \text{output of envelope detector} \\
&= \sqrt{A_c^2 + 2A_c m(t) + m^2(t) + \hat{m}^2(t)} \\
& \quad \text{If } A_c \gg |m(t)| \text{ and } A_c \gg |\hat{m}(t)| \\
a(t) &= \sqrt{A_c^2 + 2A_c m(t)} \qquad\qquad \text{recall } (1+x)^{1/2} \simeq 1 + \frac{1}{2}x \\
&= A_c \sqrt{1 + \frac{2}{A_c} m(t)} \\
&= A_c \left[1 + \frac{1}{A_c} m(t)\right] = A_c + m(t)
\end{aligned}
$$

Thus $a(t) = m(t) +$ DC bias term as long as $\begin{matrix} A_c \gg |m(t)| \\ A_c \gg |\hat{m}(t)| \end{matrix}$

page 7.39 **Superheterooyne Receiver for USB**

To tune in #2 change LO to 10.559 MHz

SIG -LO = 10.559 - 10.104 = 455 Khz

Tunable filter to eliminate image frequency

could also choose LO = 9.645 MHz for Station #1.

Question: Can we demodulate LSB with this receiver?

page 7.40

In practice it is not cost effective to filter at frequencies much greater than 10.7 MHz. This is because the filter skirts cannot be made steep enough .

Typical Crystal Filter Spec.

Typical Voice Audio signal 300 Hz - 2600 Hz

If 2300 Hz passband centered on audio

then -60 dB attenuation will be at 400 Hz in opposite sideband.

page 7.50

Summary of modulation types, assuming the message $|m(t)| < 1$

$$
\begin{aligned}
&\text{AM}\\
s(t) &= [1 + k_a m(t)] \cos(2\pi f_c t + \phi)\\
&\text{DSB}\\
s(t) &= m(t) \cos(2\pi f_c t + \phi)\\
&\text{USB}\\
s(t) &= m(t) \cos(2\pi f_c t + \phi) - \hat{m}(t) \sin(2\pi f_c t + \phi)\\
&\text{FM}\\
s(t) &= \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\alpha) d\alpha]\\
&= \cos[2\pi f_c t + \phi(t)] = \cos[\theta_i(t)]\\
&\text{PM}\\
s(t) &= \cos[2\pi f_c t + k_p m(t)]
\end{aligned}
$$

$$
\begin{aligned}
\text{For } m(t) &= A_m \cos 2\pi f_m t \text{ with } A_m = 1\\
\text{AM } m(t) &= [1 + k_a \cos 2\pi f_m t] \cos(2\pi f_c t + \phi)\\
\text{DSB } s(t) &= \cos 2\pi f_m t \cos(2\pi f_c t + \phi)\\
\text{USB } s(t) &= \cos 2\pi f_m t \cos(2\pi f_c t + \phi)\\
&\quad - \sin 2\pi f_m t \sin(2\pi f_c t + \phi)\\
\text{FM } s(t) &= \cos(2\pi f_c t + \frac{k_f}{f_m} \sin 2\pi f_m t]\\
\text{PM } s(t) &= \cos[2\pi f_c t + k_p \cos 2\pi f_m t
\end{aligned}
$$