Improved Frame Synchronization Performance for CCITT Algorithms Using Bit Erasures

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Abstract—**For the CCITT recommendations on frame synchronization (as enhanced in [2]), performance in the presence of jamming, interference or other disturbance is improved by using bit erasure information. The acquisition time and misalignment detection times are reduced and the false exit time is increased, with only a minor reduction in the already large false entry time.**

I. Introduction

Digital communication systems in which the data is organized into frames or blocks require reliable frame alignment or frame synchronization [1]–[3]. Frame synchronization performance for signals which are subject to jamming, interference, impulsive noise, or other disturbance, can be improved by using bit erasure information [4], [5]. In this paper, we consider hard bit decisions and erasures with a threshold test. Frame synchronization may also be done using soft inputs [6] centered around a maximum search (not a threshold test), or using a more optimal likelihood function (rather than correlation) with a threshold test [8], or using full channel state information [7]. However, soft inputs may not be available in high speed systems, or where hardlimiting is used.

An error and erasure tolerance is chosen to meet performance criteria depending on the application. For time division multiplexed (TDM) systems where alignment is performed according to the CCITT algorithms as modified by [2] (Fig. 1), the performance is determined by the transition times between three principal states: LA (locked in alignment), LO (falsely locked out of alignment), and S (searching for alignment). The movement between the three states in Fig. 1 is restricted; alignment is assumed when μ_1 successive acceptable frame alignment words are detected, and a search for a new alignment position is initiated after μ_2 successive unacceptable frame alignment words are detected. The CCITT algorithms are modified as in [2] by allowing some errors and erasures in the frame alignment word. One performance criterion is to minimize the acquisition time $T_{\mathrm{S:LA}}$ and misalignment detection time $T_{\mathrm{LO:S}}$ while simultaneously maximizing the false exit time $T_{\mathrm{LA:S}}$ and the false entry time $T_{\mathrm{S:LO}}$.[1]

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[1] An alternate criterion [1] is to maximize the probability of correct frame alignment (sync acquisition) P_{SYNC} in one pass through a frame of time (marker plus data), with a specified upper bound on the total probability of a simulated alignment (false sync) $P_{t,\mathrm{sim}}$ in an incorrect position in the overlap region.

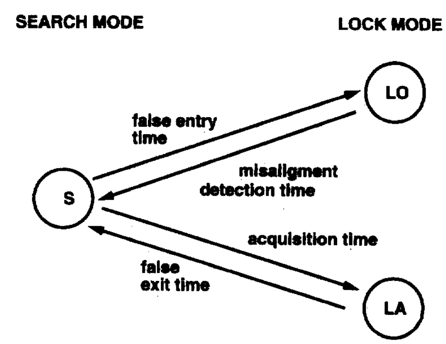

Fig. 1. State transition diagram for frame synchronization (from [2]).

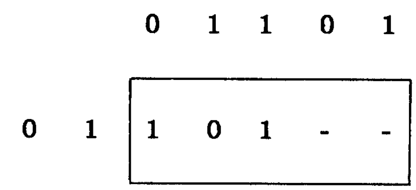

Fig. 2. Marker overlap $n = 3, h_0(n) = 2$.

The performance of frame sync in the presence of errors and erasures is considered in [4] assuming that the probability of erasure is 0.25. Results are not available for systems using CCITT algorithms, or for lower probabilities of erasure, and the probability of acquisition in the overlap region is not determined. Frame synchronization in packet transmission (as opposed to the continuous TDM transmission considered here) is analyzed in [5].

The paper is organized as follows. In Section II, the probability of frame alignment as a function of the overlap n between the incoming data stream and the marker is determined, and the assumptions made in comparing the performance with and without erasures are stated. Numerical results are presented in Section III.

II. Probabilities of Frame Alignment

Fig. 2 illustrates the overlap n between a marker of length M and the incoming data stream, with the corresponding Hamming distance (number of disagreements) $h_o(n)$ between the overlapping portions of the marker. We define the bit error probability p, bit erasure probability s, with $q = 1-p-s$, the probability p_r of mismatch, and bit erasure probability s_r in random data bits with $q_r = 1 - p_r - s_r$. The total probability $P_{t,\mathrm{sim}}$ of simulated alignment in an incorrect position in terms of $M, n\epsilon\{0..M\}, p, s, p_r, s_r$ the error tolerance h and erasure

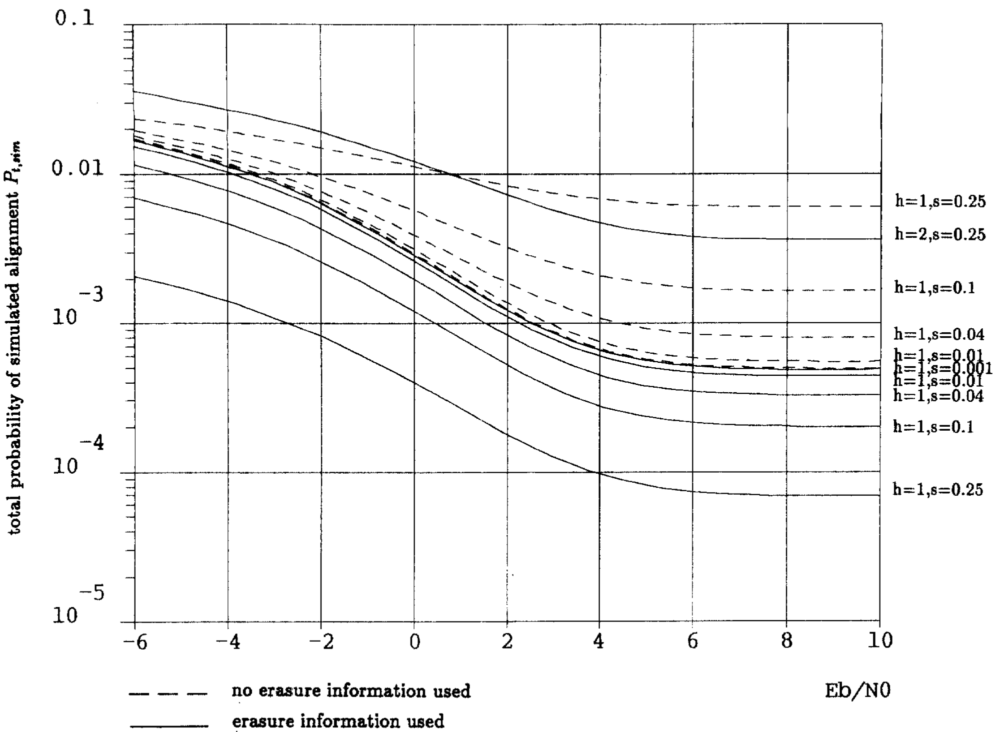

Fig. 3. $P_{t,\mathrm{sim}}$ versus E_b/N_0 with parameter s.

tolerance h_x is given by

$$\begin{aligned} P_{t,\mathrm{sim}} &= 1 - \prod_{n=1}^{M-1} [1 - P_B(h, h_x, n, h_o(n))] \\ &\simeq \sum_{n=1}^{M-1} P_B(h, h_x, n, h_o(n)) \\ &= \sum_{n=1}^{M-1} \sum_{j=0}^{h} \sum_{k=0}^{h_x} g_{jkn} \end{aligned} \tag{1}$$

where $P_B(h, h_x, n, h_o(n))$ is the probability of h or fewer disagreements and h_x or fewer erasures with overlap n. It is assumed in (1) that the marker is preceded by random data or noise, the acquisition tests at each n are independent [2], and $P_B(h, h_x, n, h_o(n)) \ll 1$. Following [5], the g_{jkn} used to evaluate P_B are obtained from

$$\begin{aligned} \sum_{j=0}^{M} \sum_{k=0}^{M-j} g_{jkn} z_1^j z_2^k &= (p + qz_1 + sz_2)^{h_o} (q + pz_1 + sz_2)^{n-h_o} \\ &\quad \cdot (q_r + p_r z_1 + s_r z_2)^{M-n} \end{aligned} \tag{2}$$

The probability of acquisition in the correct position $P_{\mathrm{SYNC}} = P_B(h, h_x, M, 0)$ and the probability of false acquisition (simulated alignment) at a particular position in random data $P_{\mathrm{FA}} = P_B(h, h_x, 0, 0)$.

Bit erasures are declared when jamming, impulsive noise or other external interference is detected by the receiver. Thus erasures are assumed to occur with fixed probability $s = s_r$ in the marker bits and random data bits independent of the signal-to-noise ratio. This assumption may not be true if erasures are caused by fading or AWGN. The erasure events are assumed to be statistically independent, but this may not be true for systems where the jamming impulse is long compared to a bit period. Following [4], [5], we consider a binary erasure channel with crossover probability $p = p_b(1 - s)$ where $p_b = Q[\sqrt{2E_b/N_o}]$ for binary antipodal signalling over an additive white Gaussian noise channel with signal energy per bit E_b and two-sided noise power spectral density $N_0/2$. Thus, p is the residual error rate for those bits for which the receiver did not detect a disturbance. In the random data bits, the probability p_r of a mismatch is the same as the probability q_r of a match, so that $p_r = q_r = (1 - s)/2$.

To determine the improvements in the transition times of [2] achieved by using erasure information, we assume that the results of [2] are for an 'errors only' system, i.e. the bit error rate P_e in [2] is partly caused by interference, but the available erasure information is ignored. We further assume that any bit which could have been erased is equally likely to be correct or incorrect. Thus we compare the 'errors only' results of [2] with the 'errors-and-erasures' results where p and s are chosen so that $P_e = p + s/2$. We then choose $s = 2\alpha P_e$ and $p = (1 - \alpha)P_e$, where α is the fraction of the bit errors in [2] caused by interference rather than noise.

III. Numerical Results

Fig. 3 shows $P_{t,\mathrm{sim}}$ versus E_b/N_o with s as a parameter. The 12 b marker 0000 0110 1011 [2] was used for the

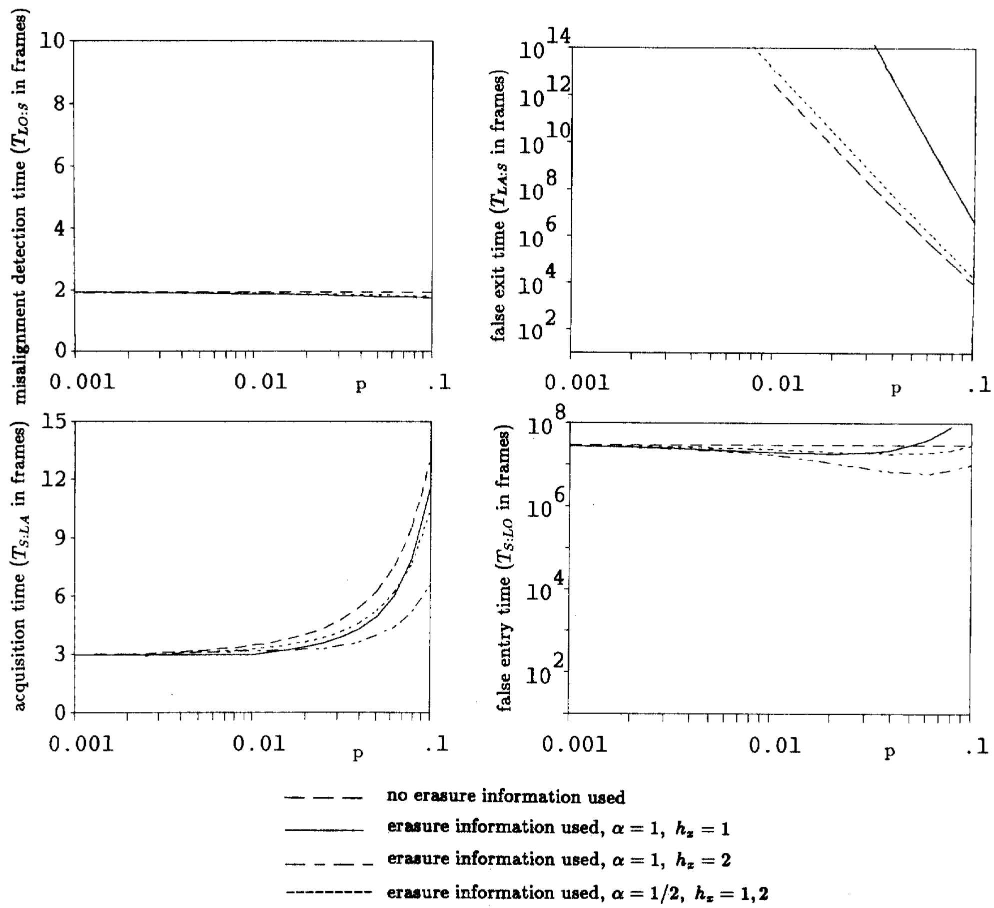

Fig. 4. Mean transition times for modified CCITT algorithms. No erasure information used. Erasure information used, $\alpha = 1, h_x = 1$, Erasure information used, $\alpha = 1, h_x = 2$. Erasure information used, $\alpha = 0.5, h_x = 1, 2$.

calculations of $h_o(n)$. For a given s and h, with $h_x = \min(2h - 2j, M - j)$ in (1) as in [4], $P_{t,\text{sim}}$ is reduced if erasure information is used. For $E_b/N_0 \to -\infty\,\text{dB}, P_{t,\text{sim}} \to (M-1)P_{\text{FA}}$.

Fig. 4 shows the acquisition time, misalignment detection time, false exit time and false entry time calculated according to the formulas in [2] with and without using erasure information. The results apply to CCITT Recommendation G751, Fourth multiplex level, frame length $K = 2928$, marker length $M = 12$, as modified in [2] with $\mu_1 = 3$ and $\mu_2 = 2$. For $h_x = 1$ and $h = 0$, a modest improvement in three of the transition times is obtained, with only a slight reduction in $T_{\text{L:SO}}$. The amount of improvement depends on the choice of α. Even at $\alpha = 1$, the acquisition time $T_{\text{S:LA}}$ is improved by using erasure information, in spite of an erasure probability which is assumed to be twice the error probability ($s = 2P_e$). If h_x is increased to 2 and $h = 0, T_{\text{S:LA}}$ is further improved (by a factor of 2) for high bit error rates near $P_e = 0.1$, at the cost of a further small reduction in $T_{\text{L:SO}}$. These improvements may be significant in some systems.

In the calculation of $T_{\text{S:LA}}$ in [2], it is assumed that $P_{t,\text{sim}} \leq P_{\text{FA}}$ for a sensible frame alignment word, i.e. the total probability of simulation when scanning in the overlap region ($M - 1$ tests) is of the order of the probability of simulation in just one test in the random data region[2] . For $M = 12$ and $h = 1$, Fig. 3 can be used to determine the lowest value of E_b/N_o for which this assumption is valid. For example, if $s = 0$ so that $P_{\text{FA}} = 3.17 \times 10^{-3}$, then this assumption is true for $E_b/N_o > -0.3$ dB.

IV. Conclusions

Digital communications systems which require frame alignment and where the bits are subjected to jamming, interference, or other disturbance which can be reliably detected and erased by the receiver have been considered. For such systems, the use of erasure information will improve frame alignment performance by increasing P_{SYNC} and reducing $P_{t,\text{sim}}$ in the overlap region. As a result, improvement is obtained in the acquisition and other transition times when the CCITT algorithms as modified in [2] are used. The analysis presented here

[2] A more relaxed criterion for a sensible frame alignment word [1] is that $P_B(h, n, h_o(n)) < P_{\text{FA}}$ for all $n \epsilon \{1, M-1\}$.

enables the system designer to determine the improvement available in the context of his particular system.

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REFERENCES

[1] R. A. Scholtz, "Frame synchronization techniques", *IEEE Trans. Commun.*, vol. COM-28, pp. 1204–1212, Aug. 1980.

[2] E. V. Jones and M. N. Al-Subbagh, "Algorithms for frame alignment—Some comparisons," *IEE Proc.*, vol. 135, pt. F, pp. 529–536, Dec. 1985.

[3] M. N. Al-Subbagh and E. V. Jones, "Optimum patterns for frame alignment," *IEE Proc.*, vol. 135, pt. F, pp. 594–603, Dec. 1988.

[4] C. K. Rushforth and R. J. Currie, "Frame synchronization in the presence of errors and erasures," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-19, pp. 498–505, 1983.

[5] P. F. Driessen, "Performance of frame synchronization in packet transmission using bit erasure information," *IEEE Trans. Commun.*, vol. 39, pp. 567–573, Apr. 1991.

[6] J. L. Massey, "Optimum Frame Synchronization," *IEEE Trans. Commun.*, vol. COM-20, pp. 115–119, Apr. 1972.

[7] P. Robertson, "Maximum likelihood frame synchronization for flat fading channels," in *Proc. Int. Conf. Commun. (ICC'92)*, June 1992, pp. 1426–1430.

[8] G.-G. Bi, "Performance of frame sync aquisition algorithms on the AWGN channel," *IEEE Trans. Commun.*, vol. 31, pp. 1196–1201, Oct. 1983.