A New Multistage Detector For Synchronous CDMA Communications

Zhen-Liang Shi, Weixiu Du, and Peter F. Driessen

Abstract—**In this paper, we develop a new multistage detector to approach the optimal solution of the detection problem in a synchronous code division multiple access (CDMA) system. The transformation modifies the diagonal elements of the Hessian matrix of the quadratic likelihood function and brings the continuous minimum of the transformed function as close to the optimal solution as possible. The computational complexity is essentially linear with the number of users, except that a few computations of a quadratic function are needed.**

I. Introduction

IN CODE-DIVISION multiple-access (CDMA) systems, the conventional detector is a filter matched to the spreading code of the desired user. The conventional detector suffers from the near–far effect due to the disparity in the received powers of users near to and far from the receiver. The optimal multiuser detector which eliminates the near–far effect, unfortunately, has an exponential computational complexity with the number of users. The linear decorrelating detector proposed by Schneider in [2] was proved to be optimal near–far resistant [3]. This detector is formulated without knowledge of the users' energies. When energies of all the users are known or can be estimated, the optimal detector will significantly outperform the decorrelating detector of [2].

A nonlinear sub-optimal detector called multistage detector was developed by Varanasi and Aazhang (Varanasi multistage detector) for the case of known user energies [4]. For this detector, the original K-dimensional integer minimization is approximated by K unidimensional integer minimizations. A significant improvement over the decorrelator of [2] is obtained by this detector of [4] with computational complexity growing linearly with the number of users. However, in many cases, the final decisions from the unidimensional optimizations cannot converge to the optimal solution.

In this paper, a new nonlinear multistage approach is proposed. Better performance can be obtained by the new scheme with only a slight increase in the computational complexity compared to the Varanasi detector.

Section II will review the optimal detector. The basic transformations into an equivalent zero–one optimization problem are presented in Section III. A continuous approach for the optimal zero–one solution is presented in Section IV, followed by two algorithms developed from this approach in Section V. Section VI will present the numerical results.

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II. Optimal Detector

For a K-user CDMA system, the received signal at a receiver in a symbol duration can be written in the discrete signal form as

$$r = S^T b + n \tag{1}$$

where $b = [b_1\ b_2\ \cdots\ b_K]^T \in \{-1,1\}^K$ contains all the users' bits for the symbol, i.e., $b_k \in \{-1,1\}$ represents the kth user's bit. $S = [s_1\ s_2\ \cdots\ s_K]^T \in R^{K\times m}$ with $s_k \in R^{m\times 1}$ denoting the kth user's spreading code vector. m is the number of chips in a symbol with $m \geq K$ to ensure that the spreading codes are linearly independent. $n = [n_1\ n_2\ \cdots n_m]^T \in R^{m\times 1}$ is a sample vector from the additive white Gaussian noise (AWGN) with zero mean and a power spectral density (PSD) N_0.

The optimal multiuser detector selects the most likely (maximum likelihood) hypothesis $\hat{b} = (\hat{b}_1, \cdots, \hat{b}_K)$ given the received signal r which corresponds to selecting the noise realization with minimum energy [3], i.e.,

$$\begin{aligned}\hat{b} &= \arg \min_{b\in\{-1,1\}^K} \|r - S^T b\|^2 \\ &= \arg \min_{b\in\{-1,1\}^K} \tfrac{1}{2} b^T H b - y^T b\end{aligned} \tag{2}$$

where the vector $y = Sr$, which is a vector of sufficient statistics for b, represents K decisions from K conventional receivers, and $H = SS^T$ is a $K \times K$ positive definite matrix. The above problem is known to be nondeterministic polynomial time hard (NP-hard) [3].

III. Basic Transformations

Two basic transformations are presented in this section. First, we transfer the problem of (2) into an equivalent zero–one optimization problem, in which zero and one in the transferred function correspond to -1 and 1, respectively, in the original problem. Letting $x = (b+e)/2$, where K-dimensional vector $e = [1\ 1\ \cdots\ 1]^T$, the ith element of x will be $x_i = 0$ for $b_i = -1$ and $x_i = 1$ for $b_i = 1$. It is easy to verify that the minimization problem of (2) is equivalent to

$$\begin{cases} \text{minimize } f(x) = \frac{1}{2} x^T Q x + c^T x \\ \text{subject to } x \in \{0,1\}^K \\ \text{minimize } f_b(b) = \frac{1}{2} b^T H b - y^T b + \frac{1}{2} e^T H e - y^T e \\ \text{subject to } b \in \{-1,1\}^K \end{cases} \tag{3}$$

where $c = -2(He + y)$ and $Q = 4H$, which is called the Hessian matrix [1].

Furthermore, for any given diagonal matrix $\Phi = \operatorname{diag}\{\phi_1\ \ \phi_2\ \ \cdots\ \ \phi_K\}$ and a vector made up of the diagonal elements $\phi = [\phi_1\ \phi_2\ \cdots\ \phi_K]^T$, it can be shown that the problem

$$= \begin{cases} \text{minimize } f(x) = \frac{1}{2}x^T\overline{Q}x + \overline{c}^T x \\ \text{subject to } x \in \{0,1\}^K \\ \text{minimize } f(x) = \frac{1}{2}x^T Q x + c^T x - \frac{1}{2}\sum x_i(x_i - 1)\phi_i \\ \text{subject to } x \in \{0,1\}^K \end{cases} \tag{4}$$

is equivalent to (3), where $\overline{Q} = Q - \Phi$ and $\overline{c} = c + \frac{1}{2}\phi$. The $f(x)$ in (4) is called the transformed function of $f(x)$ in (3). The equivalence holds because the transformation does not affect the function values at any of the zero–one solutions. Here we call a solution constrained to be a member of $\{0,1\}$ a zero–one solution. Noted that the continuous minimum of the transformed quadratic function is necessarily changed. The continuous minimization is unconstrained, with the solution given by $Q^{-1}c$ for (3) and $\overline{Q}^{-1}\overline{c}$ for (4).

IV. Continuous Approach to the Zero–One Solution

Carter [1] proposed an iterative method to find a basic transformation, such that the unconstrained continuous minimum solution of the transformed quadratic function can draw nearer to the zero–one minimum solution after each iteration. The idea is based on the fact that the transformed continuous function minimum $f(\hat{x})$ is upper bounded by the function value $f(x^I)$, where $\hat{x}$ and x^I are the solutions for the continuous and zero–one minima, provided that $\overline{Q}$ is positive definite. If the two minima are equal, the continuous solution will coincide with the zero–one solution. As the continuous minimum $f(\hat{x})$ closely approaches to the zero–one minimum $f(x^I)$, the continuous solution $\hat{x}$ should approach to x^I. The question raised here is how to find a transformation such that the continuous minimum function value increases as quickly as possible. The following theorem is the answer.

Theorem: Let $\hat{x}$ be the continuous minimum solution of the function f in (3). Suppose the ith diagonal element of Q is decreased by ϕ_i, then, the continuous minimum of the transformed function will change by [1]

$$\Delta f = \frac{\phi_i}{2}\left[\frac{(\frac{1}{2} - \hat{x}_i)^2}{(\phi_i g_{ii} - 1)} + \frac{1}{4}\right] \tag{5}$$

where g_{ii} is the ith diagonal element of Q^{-1}.

Using $j = 1, 2, \cdots$ to denote the time index of the modification with $Q(0)$ as the original matrix from (3), the parameters for the jth modified system become

$$Q(j) = Q(j-1) - \phi_i e_i e_i^T \tag{6}$$

$$Q(j)^{-1} = Q(j-1)^{-1} - \tau g_i g_i^T \tag{7}$$

$$c(j) = c(j-1) + \tfrac{1}{2}\phi_i e_i \tag{8}$$

$$\hat{x}(j) = \hat{x}(j-1) + r \tag{9}$$

where g_i is the ith column of $Q^{-1}(j-1)$, and the column vector e_i has a "one" in the ith position and zeros elsewhere. τ and r are computed by $\tau = \phi_i/(\phi_i g_{ii} - 1)$ and $r = \tau g_i(\frac{1}{2} - \hat{x}_i(j-1))$. The following corollary gives the condition for the new Hessian $Q(j)$ to be positive definite.

Corollary: In the theorem above, the new Hessian matrix $Q(j)$ will always be positive definite provided ϕ_i is less than $1/g_{ii}$.

One needs to maximize Δf while also ensuring that Δf is positive. The proof of the corollary is in [1]. By maximizing the Δf with respect to ϕ_i, the function minimum $f(\hat{x}(j))$ will approach to $f(x^I)$ at the fastest rate. The resulting values of ϕ_i and Δf, also from [1], are given by

$$\phi_i = \begin{cases} 2\hat{x}_i(j-1)/g_{ii}, & \text{for } \hat{x}_i(j-1) \le \frac{1}{2} \\ 2(1 - \hat{x}_i(j-1))/g_{ii}, & \text{for } \hat{x}_i(j-1) > \frac{1}{2} \end{cases} \tag{10}$$

$$\Delta f = \begin{cases} \hat{x}_i(j-1)^2/(2g_{ii}), & \text{for } \hat{x}_i(j-1) \le \frac{1}{2} \\ (1 - \hat{x}_i(j-1))^2/(2g_{ii}), & \text{for } \hat{x}_i(j-1) > \frac{1}{2} \end{cases} \tag{11}$$

and the ith component of the new function minimum will be

$$\hat{x}_i(j) = \begin{cases} 0, & \text{for } \hat{x}_i(j-1) < \frac{1}{2} \\ 1, & \text{for } \hat{x}_i(j-1) > \frac{1}{2} \\ 1/2, & \text{for } \hat{x}_i(j-1) = \frac{1}{2}. \end{cases} \tag{12}$$

The new function is shown to always be positive definite in [1].

V. Multistage Detector

Based on the theorem in the last section, a new multistage detection algorithm is proposed.

MD algorithm: Multistage Detection

1) For $i = 1$ to K, let
$$t_i = \begin{cases} \hat{x}_i(j-1), & \text{if } \hat{x}_i(j-1) \le \frac{1}{2} \\ (1 - \hat{x}_i(j-1)), & \text{if } \hat{x}_i(j-1) > \frac{1}{2} \end{cases}.$$
2) Find the index p such that $|t_p| = \max_{i=1}^K |t_i|$, i.e., $\hat{x}_p$ is the component of the current minimum which is furthest away from a zero–one value.
3) Compute ϕ_p by (10) and compute $Q(j), Q^{-1}(j), c(j)$ and $\hat{x}(j)$ by (6)–(9), respectively.
4) Compute $f(\hat{x}^I(j))$. $\hat{x}^I(j)$ is the quantized zero–one point from $\hat{x}(j)$ at the current stage.
Compare the $f(\hat{x}^I(j))$ with the smallest one obtained at previous stages and keep the smallest one.
5) $j = j + 1$; Repeat from 1) until $j > M$.

For each stage, the dominant operations are the computation of $f(\hat{x}^I(j))$ and the update of Q^{-1}. For a M-stage detector, the algorithm needs M computations of (7) and at most M computations of $f(\hat{x}^I)$. M is independent of K. For most cases, the algorithm will quickly converge to the x^I in three or four stages. However, there are still some cases for which the algorithm cannot bring the continuous minima close enough to the x^I. For these cases, the values of the continuous minimum $\hat{x}$ from each iteration of the algorithm will oscillate among a few points. Furthermore, the value of the index p, as determined by the second step of the MD algorithm, will also oscillate among a few fixed values after each iteration of the algorithm. The elements corresponding to these fixed values of the index are deemed the most unstable elements. For $K \le 10$, p often takes two fixed positions in the vector $\hat{x}$.

An observation in [5] can be used to explain why the algorithm does not always converge. In [5], it is shown that for any $x^* = \{x_1^*\ x_2^*\ \cdots\ x_K^*\} \in \{0,1\}^K$, we can construct a unique $\phi^* = \{\phi_1^*\ \phi_2^*\ \cdots\ \phi_K^*\} \in R^K$, such that

TABLE I
RESULTS FOR THE EXAMPLE (j: NUMBER OF STAGES)

j	$\hat{x}$					$f(\hat{x})$
0	(-0.45	0.02	1.79	1.06	-0.21)	-33.86
1	(0.48	0.55	1.00	0.27	0.58)	-30.86
2	(0.00	0.25	1.15	0.65	0.13)	-30.11
3	(-0.44	-0.03	1.28	1.00	-0.28)	-30.09
4	(0.00	0.25	1.15	0.65	0.13)	-30.08
5	(-0.43	-0.03	1.28	1.00	-0.28)	-30.07

$x^* = -(\overline{Q}^*)^{-1}\overline{c}^*$, where $\overline{Q}^* = Q - \Phi^*$, and $\overline{c}^* = c + \frac{1}{2}\phi^*$. ϕ^* is given by

$$\phi_i^* = 2(2x_i^* - 1)\left(c_i + \sum_{j \in I_1(x^*)} q_{ij}\right) \tag{13}$$

where $I_1(x^*) = \{i | x_i^* = 1\}$. We have found that the necessary condition for $\overline{Q}^*$ to be positive semidefinite (psd) is $x^* = x^I$. The MD algorithm will converge if and only if the $\overline{Q}^*$ for $x^* = x^I$ is psd. Whether or not $\overline{Q}^* \geq 0$ for $x^* = x^I$ will be sensitive to the noise components of y and therefore the noise components of c. Whether $\overline{Q}^* \geq 0$ for $x^* = x^I$ will also depend on the structure of Q or the quality of the cross-correlations of PN codes.

In the rest of this section, a modified algorithm is proposed to obtain successful convergence when $\overline{Q}^*$ for the zero–one minimum is indefinite. The objective is to single out some elements of x which cause the $\overline{Q}^*$ to be indefinite. Such elements may be excluded from the consideration in the algorithm, and the $\overline{Q}^*$ for the reduced problem will often become positive definite. Then the problem can be solved by the continuous approach again. To find such elements, we present the following example for a five-user CDMA system.

Example: For the problem defined by (3), let

$$Q = \begin{bmatrix} 50.48 & -5.11 & 15.32 & 15.32 & -25.53 \\ -5.11 & 25.30 & -3.61 & 10.84 & -3.61 \\ 15.32 & -3.61 & 25.30 & -3.61 & -3.61 \\ 15.32 & 10.84 & -3.61 & 25.30 & -3.61 \\ -25.53 & -3.61 & -3.61 & -3.61 & 25.30 \end{bmatrix}$$

and

$$c = [-26.18 \quad -8.69 \quad -35.20 \quad -14.54 \quad 4.18]^T$$

then $\overline{\phi}^*$ computed from (13) with the optimal zero–one solution $x^* = [1\ 1\ 1\ 0\ 1]$ is

$$\overline{\phi}^* = [17.97 \quad 8.56 \quad -3.63 \quad -8.79 \quad -6.55]^T$$

and

$$\overline{Q}^* = \begin{bmatrix} 32.50 & -5.11 & 15.32 & 15.32 & -25.53 \\ -5.11 & 16.74 & -3.61 & 10.84 & -3.61 \\ 15.32 & -3.61 & 28.93 & -3.61 & -3.61 \\ 15.32 & 10.84 & -3.61 & 34.08 & -3.61 \\ -25.53 & -3.61 & -3.61 & -3.61 & 31.85 \end{bmatrix}$$

which is indefinite with the eigenvalues as 40.43; 15.11; 24.19; −3.09; 67.46. The results for five iterations by Algorithm 1 are illustrated in Table I.

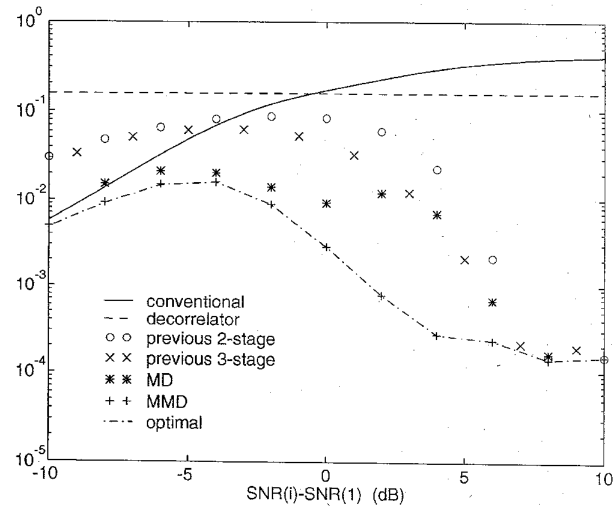

Fig. 1. BER of user one for a five-user system. The largest cross-correlation is 5/7, SNR (1) = 8 dB, $M = 4, N = 3$.

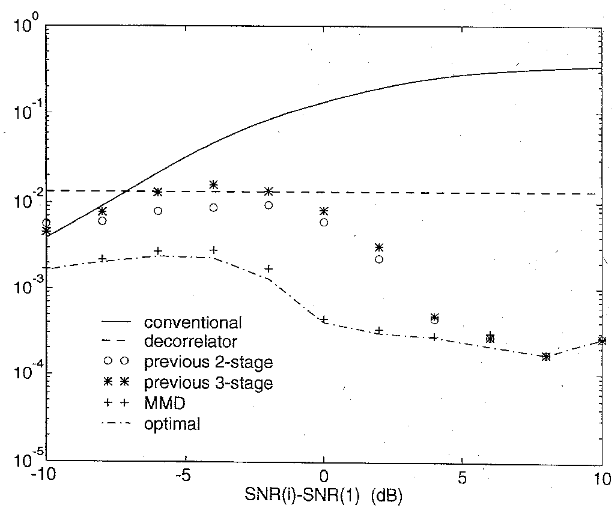

Fig. 2. BER of user one for a ten-user system. The largest cross-correlation is 7/15, SNR(1) = 8 dB, $M = 4, N = 3$.

The algorithm is not convergent, because $\hat{x}$, as the iteration goes on, will jump back and forth around two points [0.00 0.25 1.15 0.65 0.13] and [−0.43; −0.03; 1.28; 1.00; −0.28]. The elements one and four are the most unstable ones. Often, the correct decisions for the most unstable elements $\hat{x}_p$ are the opposites of their quantized values, i.e., the correct decisions are one, if $\hat{x}_p = 0$ and zero, if $\hat{x}_p = 1$. Accordingly, we propose a modified MD algorithm to increase the frequency of convergence when $\overline{Q}^*$ for x^I is indefinite.

Modified Multistage Detection (MMD) Algorithm:

1) Run the MD algorithm for M stages.
2) Find the two most unstable elements $\hat{x}_p$.
3) For one of the most unstable elements, substitute x_p of the original function f with the opposite of the quantized value of $\hat{x}_p$. The new function f has $K-1$ dimensions.
4) Run the MD algorithm for the reduced function for N stages, choose the best result for $\hat{x}_p$ from the comparison with the one obtained before.

5) For the other most unstable element, repeat 3) and 4).

This heuristic method is shown to be very effective for this particular communication problem.

VI. Numerical Results

In this section, bit-error rates (BER's) of the two algorithms are obtained by simulations. For comparison, simulation results from the Varanasi multistage algorithm [4], conventional, decorrelating, and optimal detectors are also presented.

First, we consider a five-user CDMA system. We use the same codes as were used in Fig. 4 of [4]. The largest correlation coefficient among the spreading codes is 5/7. Fig. 1 shows the BER of the first user with its $\mathrm{SNR}_1 = 8$ dB, versus the ratio of the user's signal strength to the strength of the other four signals. The improvement over all the other schemes by the new multistage detectors can be seen from this figure. Note that the MMD algorithm achieves near optimal performance. Fig. 2 illustrates the same performance comparison for a ten-user system. Gold sequence of length 15 are used with the largest correlation coefficient of 7/15. As the user cross-correlation values decrease, the performance of both the decorrelating detector and the conventional detector improve, but the modified multistage detector still achieves near optimal performance.

It should be noted that the number of stages in the two algorithms is insensitive to the number of users in the system, thus the computational complexity of the algorithms will not increase much as the number of users goes up.

VII. Conclusion

This paper proposes two new multistage detectors for synchronous CDMA systems. The new detectors are able to achieve better performance than that of the Varanasi multistage detector, especially in the case of high cross-correlation between the spreading codes of any pair of distinct users.

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