Attainable Throughput of an Interference-Limited Multiple-Input Multiple-Output (MIMO) Cellular System

S. Catreux, P. F. Driessen, and L. J. Greenstein

***Abstract*—We investigate the high spectral efficiency capabilities of a cellular data system that combines the following: 1) multiple transmit signals, each using a separately adaptive modulation; 2) adaptive array processing at the receiver; and 3) aggressive frequency reuse (reuse in every cell). We focus on the link capacity between one user and its serving base station, for both uncoded and ideally coded transmissions. System performance is measured in terms of *average* data throughput, where the average is over user location, shadow fading, and fast fading. We normalize this average by the total bandwidth, call it the *mean spectral efficiency*, and show why this metric is a useful representation of system capability. We then quantify it, using simulations, to characterize multiple-input multiple-output systems performance for a wide variety of channel conditions and system design options.**

***Index Terms*—Adaptive arrays, adaptive modulation, antenna diversity, cellular mobile communications.**

I. Introduction

MULTIPLE transmit antennas, adaptive modulation, and adaptive receiver arrays are all wireless communications techniques that can be used to increase spectral efficiency. Adaptive array processing at the receiver has long been used to increase the capacity of wireless systems [1], [2]. With multiple antennas at *both* the receiver and transmitter, forming a multiple-input multiple-output (MIMO) system with n transmitting antennas and $m \geq n$ receiving antennas, it is possible to achieve an n-fold increase in capacity, provided there is significant decorrelation of the complex path gains to the receive array elements [3]–[10]. Adaptive modulation, in which the transmission parameters (e.g., power, constellation size) are adapted to exploit prevailing channel conditions, also yields significant increases in capacity [11]–[14].

Previous results on MIMO systems were all obtained for a single link with no external interference [3]–[10]. Here, we quantify attainable MIMO performance in interference-limited cellular systems and compare it to that of more traditional approaches (i.e., those that use receive-diversity only, or no diversity) under the same conditions. Note that we use adaptive modulation rate in conjunction with the MIMO technique, i.e., each transmit signal uses a separately adaptive modulation, matched to the instantaneous channel condition.[1] Using a general-purpose system-level simulation platform developed for this study, we define a metric for the system performance and quantify it over a broad range of channel conditions and system design options. We outline our models and assumptions in Section II, present our numerical findings in Section III, and cite some further areas worthy of research in Section IV.

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[1]This is in contrast to V-BLAST, which imposes the same data rate on all transmitters.

II. System Model

The general configuration for a communication link that employs multiple transmitting and receiving antennas is illustrated in Fig. 1. A single user's data stream is demultiplexed among the n transmitting antennas, each of which conveys a distinct substream. Each substream is encoded into symbols drawn from an M-QAM [2] family of modulations. Note that the QAM constellation size may differ from one substream to another and is chosen via an adaptive modulation algorithm. This system is spectrally efficient because all the signal components are sent out in parallel, at the same time and over the same frequency bandwidth. Therefore, they share a common wireless channel. These signal components are received by an antenna array whose sensor outputs are processed such that the original data substreams can be recovered.

We consider an MIMO system in a cellular data environment, where a given cell (comprising a serving base station and one mobile user on every frequency channel) is surrounded by one contiguous tier of six cells, with full frequency reuse in every cell. The assumption of only one tier of interferers (made to simplify the simulations) is optimistic; we offset it, at least partially, with the pessimistic assumption that all cochannel interferers are transmitting all the time.

We study the performance of MIMO systems by investigating various system sizes, denoted by (n, m), where n (m) indicates the number of transmit (receive) antennas. Performance comparisons are made with more conventional approaches, such as single-input single-output (SISO) system (1,1) (no diversity) and single-input multiple-output (SIMO) systems $(1, m)$ (diversity only at the receiver end). We will focus on the downlink but present findings for the uplink as well.

The simulation approach is as follows.

Terminal Location: Each user is randomly located, with uniform probability over the cell.

Complex Path Gains: We simulate the complex path gains to the serving and interfering bases by considering the inverse distance law, Rayleigh (complex Gaussian) fading, lognormal shadow fading, and the antenna pattern when sectoring is used. We use the following parameters values: path

[2]M-QAM is quadrature amplitude modulation, where M is the constellation size.

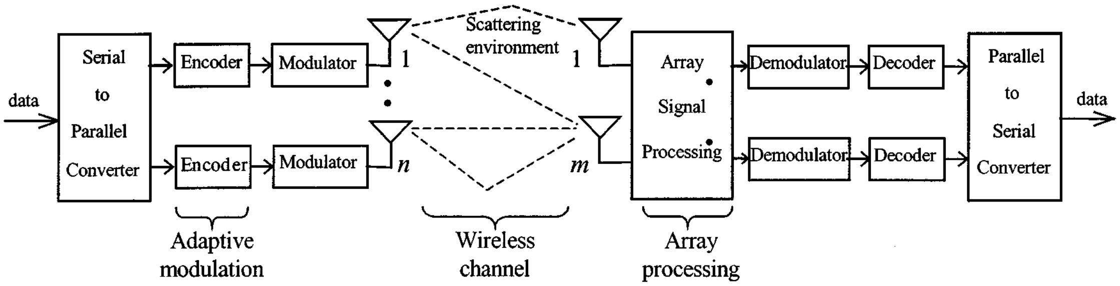

Fig. 1. Model of digital communication system with multiple transmitting and receiving antennas.

loss exponent $\gamma = 3.7$, shadow fading standard deviation $\sigma = 0, 4$, or 8 dB and Ricean K-factor $= 0$ or 10. When sectoring is used, we assume three sector antennas per cell, each with a 3-dB beamwidth of 90°.

Array Processing: We consider two alternative schemes for separating at the receiver the n signals transmitted from the base. One scheme linearly combines the received signals using a set of weights that yields the *minimum mean square error* between the estimate and the true signal (MMSE scheme). The second scheme, called *ordered successive interference cancellation* (OSIC-MMSE) is an improved version of MMSE suggested in [9] and [15]. It is a recursive procedure that sequentially detects the different signal components in an optimal order. First, MMSE combining is applied to the received vector signal. Then the substream with the highest output signal-to-(interference-plus-noise) ratio (SINR) is detected first, and its contribution is subtracted from the total received vector signal. The same process is repeated until all n substreams are detected.

Adaptive Modulation Rate: We assume an algorithm that perfectly adapts the transmission rate on each transmit antenna (via the number of modulation levels), according to the radio channel and interference conditions. The per-user data throughput Y is the sum of the throughputs of the n decoupled subchannels, where the throughput T_i of subchannel i is determined for two extremes cases: 1) *ideally coded signals* (perfect error correction), where throughput is given by the Shannon capacity $T_i = \log_2(1 + \mathrm{SINR}_i)$, SINR_i being the SINR at the ith output of the combiner, and 2) *uncoded signals*, with perfect error detection in each block. In this case, the throughput is given by $T_i = \log_2 M_i(1 - \mathrm{BLER}(\mathrm{SINR}_i))$, where $\log_2 M_i$ is the number of bits per symbol, and BLER is the block error rate for L-bit blocks. By plotting this term for an uncoded M-QAM family of modulations, we found that the envelope of the throughput versus SINR is closely approximated by the Shannon capacity, shifted by a factor of about 8 dB [18]. In other words, for our purposes, T_i can be expressed generally as $T_i = \log_2(1 + \mathrm{SINR}_i/a)$, where $a = 1$ with ideal coding and $a \approx 0.16$ with no coding. The results for any specific practical coding scheme can be expected to fall between these two cases. Finally, the per-user throughput Y is expressed as $Y = \sum_{i=1}^{n} T_i$.

Throughput Metric: We average the per-user throughput Y over the short-term Rayleigh fadings of the path gains. The result (called the user's *spectral efficiency*) is a function of user position (distance from its serving base) and shadow fading. Its average over all user locations and shadow fadings is the *mean spectral efficiency*, in b/s/Hz, which is our primary metric for making comparisons. In a cell with many users, the total information rate delivered divided by the total bandwidth is a random variable narrowly distributed about this value. It is thus a very good measure of data system capacity.

The key assumptions in this study are that the channel has a flat frequency response (delay spread is negligible), it is static during the packet duration (slow fading), and its complex path gains are uncorrelated. We also assume perfect adaptation of the modulation rate to the channel state. Additionally, the total power transmitted on each link is the same, regardless of n, i.e., the power is P/n per transmit antenna. The multipath-averaged signal-to-noise ratio (SNR) is the same at each receiver branch for a given location and is a random variable over the shadow fading. The median of this random variable when the mobile is at the cell boundary is a chosen parameter in our simulations, denoted by ρ. The median of the multipath-averaged SNR at a distance d is written as med (SNR) $= \rho(D/d)^{\gamma}$, where D is the cell radius. Finally, we assume no cell site diversity in most of our computations, i.e., users communicate with the base that is *nearest*, not strongest. However, we also examine the possible benefits of cell site diversity.

III. Results

A. $m, n \leq 3$ and Unlimited Modulations

Fig. 2 presents a first set of results that show the range of mean spectral efficiencies attainable using systems (1,1), (1,3), and (3,3), in the presence of cochannel interferers (CCI). The figure additionally reveals the influence of two design choices namely, (1) *power control* (either no power control or signal-level-based power control); (2) *base antenna beam pattern* (either all base antennas are omnidirectional or they are directional in each of three sectors, with a 90° half-power beamwidth per antenna). The parameters, γ, σ, and ρ are set at 3.7, 8, and 20 dB, respectively, and the reuse factor is $R = 1$. The Ricean K-factor is equal to 0. The main findings are summarized below.

We first recall that these results were partially shown in [18], where it was seen that MIMO system (3,3) suffers more performance degradation that SIMO system (1,3), when going from a noise-limited environment to an interference-limited environment. This degradation is due to the lack of degrees of freedom

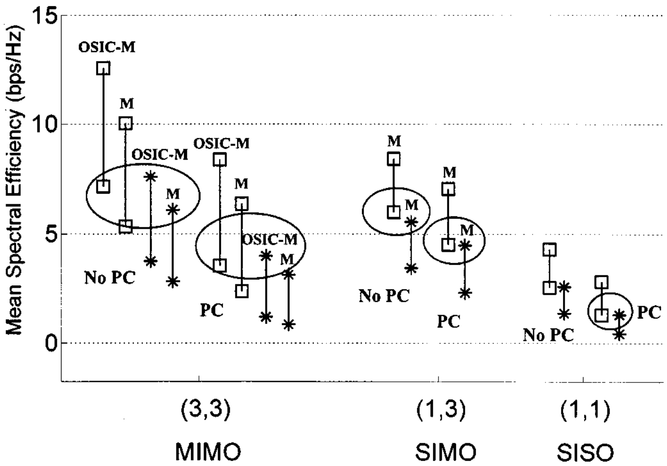

Fig. 2. Mean spectral efficiency (b/s/Hz) for systems (1,1), (1,3) and (3,3) in multi-cell environment. Downlink, $K = 0$, $R = 1$, $\sigma = 8$ dB, $\rho = 20$ dB. □: Sectoring, $*$: Omni, M: MMSE combining, OSIC-M: OSIC-MMSE combining.

to combat CCI. We will see in Section III-B that increasing the number of receivers brings back the throughput advantage of MIMO systems over SIMO systems.

The influence of the choice of the base antenna beam pattern is clear: the throughput benefit in using sectored over omnidirectional antennas is about twofold. This is consistent with results from conventional systems (using three-sector antennas enables cellular planners to bring down the reuse factor from 12 to 7, which amounts to an increase in capacity close to twofold [17]).

Next, we compare signal-based power control with no power control. We see that the mean spectral efficiencies of systems (1,1) and (3,3) drop by about 50% when using power control. System (1,3) is more robust and undergoes only half that loss. This finding is consistent with [12], which reports that using adaptive modulation without power control provides a significantly higher throughput than using adaptive modulation with SINR-based power control. We present further power control results in Section III-C.

Finally, when considering the combining technique used for MIMO system (3,3), we see that in all cases, OSIC-MMSE outperforms MMSE. This is not surprising, since OSIC takes advantage of successive interference cancellation from prior detections.

B. $m, n \leq 6$ and Limited Modulations

Increasing the number of receivers beyond the number of transmitters will add more degrees of freedom for interference cancellation and thus will improve the performance of MIMO systems by providing diversity against fading and CCI. Fig. 3 shows how the mean spectral efficiency is affected by system size, specifically targeting systems (1,1), (1,3), (1,6), (3,3), and (3,6). It also shows the effect of limiting the number of modulation levels.

Table I presents the mean spectral efficiency for all systems considered, for the case of uncoded signals, unlimited modulation, and with OSIC-MMSE applied to the MIMO systems, (3,3) and (3,6).

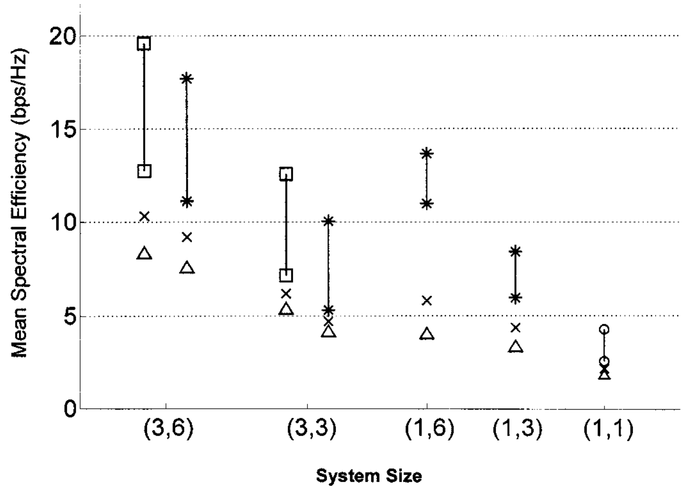

Fig. 3. Mean spectral efficiency (b/s/Hz) for systems (1,1), (1,3) (1,6), (3,3) and (3,6) in multi-cell environment, for unlimited and limited modulations. Downlink, $K = 0$, $R = 1$, $\sigma = 8$ dB, $\rho = 20$ dB, sectored antennas. □: OSIC-MMSE combining. $*$: MMSE combining. o: no processing, $\times$: Adaptive modulation limited to 64QAM. $\triangle$: Adaptive modulation limited to 16QAM.

There is almost a two-to-one improvement in performance when using MIMO system (3,6) instead of MIMO system (3,3). This enhancement is provided by the extra degrees of freedom used by the receiver to cancel cochannel interferers and reduce the effect of multipath fading. However, increasing the number of receivers in an SIMO system also improves the performance significantly. In fact, the SIMO system (1,6) is close in performance to the MIMO system (3,6). However, the mean spectral efficiency of 11 b/s/Hz shown for SIMO system (1,6) implies a constellation with 2048 points, whereas MIMO system (3,6) can achieve this same capacity level with three data streams using $2^4 = 16$ constellation points (16-QAM) each.

In Fig. 3, the symbol $\times$ indicates the mean spectral efficiency for uncoded signals, when the adaptive modulation is limited to a maximum constellation size of 64 points. The triangle symbol represents the mean spectral efficiency when the maximum is set at 16 points. The system (1,6) undergoes a dramatic degradation, losing 50%–60% of its performance, while system (1,3) suffers from 25%–45% of degradation. MIMO systems have a more temperate loss. The advantage of MIMO systems over SIMO systems is now more evident: The gain of MIMO system (3,3) over SIMO system (1,3) is now about 1.6, and the gain of MIMO system (3,6) over SIMO system (1,6) is a little over 2. We conclude that MIMO links outperform SIMO links when practical modulations are used, though less so than in a single-cell environment [19].

C. Other Findings

Beyond the results presented above, we examined a number of other conditions, parameters and spectral efficiency properties. In each case reported here, our findings are distilled from a large number of examined combinations of system size (n, m), coding (none versus Shannon limit), MIMO receiver processing, power control (none versus signal-based), base station antenna pattern (omni versus sector) and modulation (limited versus unlimited).

TABLE I
UNCODED MEAN SPECTRAL EFFICIENCIES IN MULTICELL ENVIRONMENT (B/S/HZ), UNLIMITED MODULATION

System	(1,1)	(1,3)	(1,6)	(3,3)	(3,6)
Mean Spectral Efficiency	2.57	5.98	11.02	7.15	12.76

Reuse Factor, R—All results presented here have been for frequency reuse in every cell $(R = 1)$. For larger reuse factors, CCI is reduced, leading to more throughput per user. At the same time, the number of active users per cell is reduced by a factor R. In virtually every case examined, the gain in per-user throughput was more than offset by this reduction, so that $R = 1$ is the optimal reuse factor in our all-data scenario.

Median SNR, ρ—For all results presented here, the median SNR at the cell boundary has been $\rho = 100$ (20 dB). For the noise-limited case, the mean spectral efficiency grows monotonically with the logarithm of ρ, as shown in [19]. In the multicell environment, however, there is a ρ beyond which mean spectral efficiency becomes CCI-limited, and there is no further increase with ρ. Over the many cases we examined, the plateau began somewhere in the range 15–20 dB.

Propagation Parameters (γ, σ, K)—We have used a distance exponent of $\gamma = 3.7$ throughout our calculations. This value is typical and appropriate, though we know that it can range from 3.0 to 5.0 or more from cell to cell [16] and that CCI decreases monotonically with increasing γ. Thus, we can predict that mean spectral efficiency will increase (decrease) somewhat as γ increases (decreases) from 3.7.

In the case of shadow fading standard deviation, σ, we explored values other than 8 dB, and we found a minor impact, with a generally decreasing trend. Specifically, over a large set of cases, the mean spectral efficiency increased moderately as σ decreased from 8 dB to 0, the increase never exceeding 50%.

The Ricean K-factor also showed a minor impact, with a generally decreasing trend for MIMO systems. Specifically, over many MIMO cases treated, mean spectral efficiency decreased moderately as K increased from 0 to 10 (which is at the high end of practical K-factors). The decrease was never more than 50% and was substantially lower in most cases.

Uplink Versus Downlink—All results here are for the downlink, but we have observed, over a large set of cases, a strong tracking of results for the uplink and downlink. The uplink has higher capacity in almost every case, but most often by less than 10% and never by more than 24%.

Variability of Throughput—The variability of user spectral efficiency about the mean was also examined, for numerous cases and for several values of σ. A typical example is given in Fig. 4 for an SISO system (1,1), with sector antennas and no power control. For $\sigma = 0$, the distribution is approximately log-normal (as indicated by the nearly straight line), but it becomes less so as σ increases. Also, the spread of values increases with increasing σ in the region of low throughput. Thus, there is a moderate decrease in the distribution mean with increasing σ, a trend cited earlier. Similar results were obtained in all other cases.

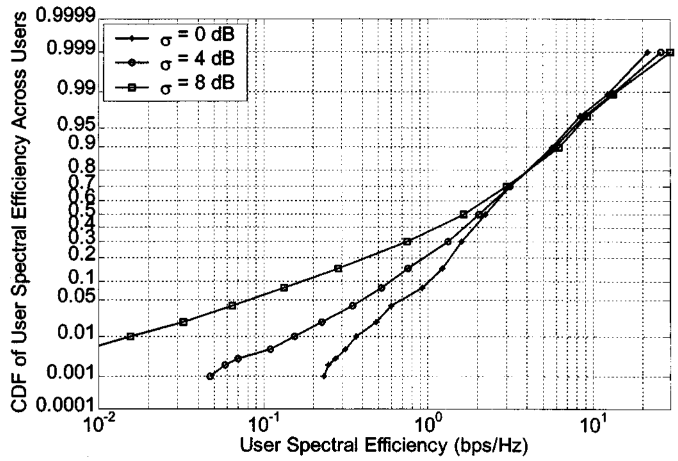

Fig. 4. CDF of user spectral efficiency across users, over the cell, for SISO system (1,1), in the single-cell environment. Uncoded case, $K = 0, \rho = 20$ dB, downlink, no power control.

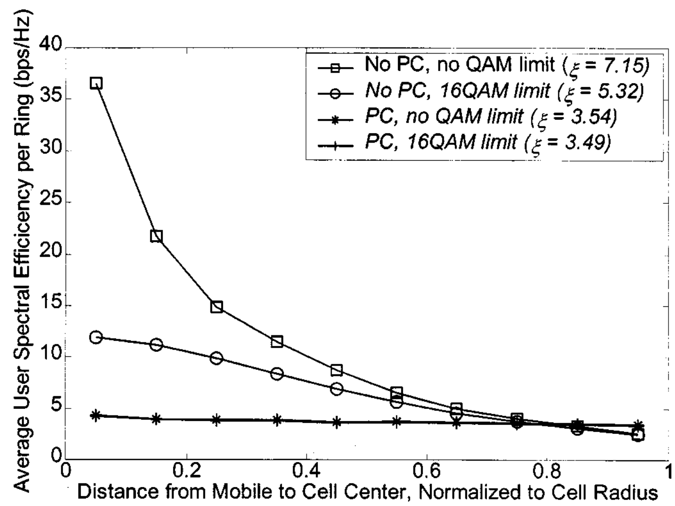

Fig. 5. Average user spectral efficiency as a function of user position, where ξ is the mean spectral efficiency in b/s/Hz.

Throughput Versus Distance—The averaging of user spectral efficiency over a cell obscures the sometimes strong influence of user position (distance). To evaluate distance effects, we divided the cell into 10 annular rings, each of width equal to 10% of the radius, and averaged spectral efficiency over shadow fading and position for users located uniformly in each ring. A typical set of results is given in Fig. 5 for an MIMO system (3,3) with sector antennas. With unlimited modulation and no power control, unrealistically large values are computed close to the base, due to very high SINR. Introducing a limited constellation size (16-QAM) leads to a dramatic leveling. This effect is even stronger using power control, with or without limitations on constellation size, at a cost in total cell throughput.

Cell Site Diversity—In all of our results, we have assumed each user communicates with the nearest cell site, not necessarily the "best" cell site. However, we also examined the benefits of signal-based cell site selection, wherein each user selects the base to which the mean path gain is highest. We found that this process can raise the throughput by around 20% for users at the cell boundary, with the average over the cell increasing by no more than a few percent.

IV. Conclusions

We have shown that the combination of adaptive modulation, aggressive frequency reuse, efficient array processing and multiple antenna transmission can significantly increase the data throughput in a cellular system. These techniques should find rewarding application to future generations of wireless systems.

There are many possible avenues for further work in this interesting and promising research area. Other impairments, in addition to CCI, should be examined to make the results more realistic. Among them are adaptive algorithm implementation errors, dispersion, time variations, correlated path gains, and control/overhead issues associated with using rapidly adapted data rates.

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