ELEC 310

Quiz #2

Que. For the following LTI system with difference equation:

$$y(n) = 0.5\,y(n-1) + x(n-1)$$

01. Find $H(z)$
02. Find an expression for $h(n)$ versus n
03. Find $h(0)$, $h(1)$, $h(2)$

Solution:

01. $y(n) = 0.5\,y(n-1) + x(n-1)$

$$\Rightarrow Y(z) = 0.5\,z^{-1}Y(z) + z^{-1}X(z)$$

$$\Rightarrow Y(z)\left[1 - 0.5z^{-1}\right] = z^{-1}X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-0.5z^{-1}} = \frac{1}{z-0.5}$$

$$\Rightarrow H(z) = \frac{1}{z-0.5} \leftarrow$$

02. $$\frac{H(z)}{z} = \frac{1}{z(z-0.5)} = \frac{A}{z} + \frac{B}{z-0.5} = -\frac{2}{z} + \frac{2}{z-0.5}$$

$$\Rightarrow H(z) = -2 + \frac{2z}{z-0.5} = -2 + \frac{2}{1-0.5z^{-1}}$$

$$\Rightarrow h(n) = -2\delta(n) + 2\,(0.5)^n u(n) \leftarrow$$

03. $n=0 \Rightarrow h(0) = -2.\delta(0) + 2.(0.5)^0 u(0) = -2.1 + 2.1.1 = 0$

$n=1 \Rightarrow h(1) = -2.\delta(1) + 2\,(0.5)^1 u(1) = -2.0 + 2(0.5).1 = 1$

$n=2 \Rightarrow h(2) = -2.\delta(2) + 2\,(0.5)^2 u(2) = -2.0 + 2\times 0.25.1 = 0.5$

Alternative method to find $h(n)$: $H(z) = z^{-1}\frac{z}{z-0.5}$

Using properties $F(z) = \frac{cz}{z-p} \leftrightarrow f(n) = cp^n u(n)$ & $z^{-m}F(z) \leftrightarrow f(n-m)$

$c=1, p=0.5, m=1 \Rightarrow h(n) = (0.5)^{n-1}u(n-1) \leftarrow$

$\therefore h(0) = 0, \quad h(1) = 1, \quad h(2) = 0.5 \leftarrow$