Design of Frequency-Response-Masking FIR Filters Using SOCP with Coefficient Sensitivity Constraint

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***Abstract*— In this paper, we present an analysis on the coefficient sensitivity (CS) of a second-order cone programming (SOCP) based method for the design of FRM filters and show that the method is guaranteed to produce FRM filters with low CS as long as the CS of the initial FRM filter is low. Moreover, we present an enhanced SOCP-based design method that incorporates a constraint on the CS measure S_1^2 recently introduced by Y. C. Lim et al. Our formulation shows that SOCP provides a suitable design setting for FRM filters, where the CS is taken into account. A design example illustrates the ability of the proposed method to produce FRM filters with very low CS without sacrificing the filter's other performance.**

I. INTRODUCTION

The frequency-response masking (FRM) technique has been a subject of research since the work of [1], see e.g. references [2]–[21] for the subsequent development in analysis and design of various types of FRM filters. The finite word-length (FWL) performance of FRM filters, which seems to have been overlooked in the past, has been examined recently in [22],[23]. In those papers, it is demonstrated that unless the coefficient sensitivity issue is taken into consideration by the design algorithm, FRM filters with excellent performance in terms of approximation accuracy but also with exceedingly high coefficient sensitivity (CS) may be produced. The papers propose several design methods for FRM filters with minimum CS and guaranteed performance in terms of minimax approximation of a desired frequency response.

The objectives of this paper are twofold. First, we present an analysis on the CS of the design technique proposed in [12] which is a method based on second-order cone programming (SOCP). Our examination shows that the SOCP-based design method is guaranteed to produce FRM filters with low CS as long as the CS of the initial FRM filter is low. Second, we present an enhanced SOCP-based design method that incorporates a constraint on the CS measure S_1^2 introduced in [22],[23]. Our formulation shows that SOCP provides a suitable design setting for FRM filters, where the CS of the FRM filter must be taken into account. A design example is presented to illustrate the ability of the proposed SOCP algorithm to produce FRM filters with very low CS without sacrificing the filter's performance in terms of approximation accuracy.

II. COEFFICIENT SENSITIVITY OF ALGORITHM IN [12]

Consider the standard structure of an FRM Filter illustrated in Fig. 1 where

$$H_a(z) = \sum_{k=0}^{N-1} h_k z^{-k}, \quad H_{ma}(z) = \sum_{k=0}^{N_a-1} h_{ak} z^{-k} \tag{1a}$$

$$H_{mc}(z) = \sum_{k=0}^{N_c-1} h_{ck} z^{-k} \tag{1b}$$

are linear-phase transfer functions. For simplicity, in the rest of the paper the filter lengths N, N_a, and N_c are assumed to be odd. With straightforward modifications, the subsequent analysis and design techniques also apply to other cases.

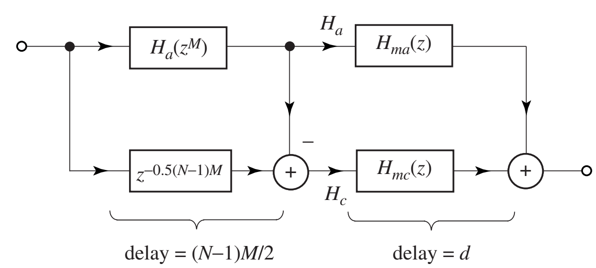

Fig. 1. A basic FRM filter structure.

The zero-phase frequency response of the FRM filter is given by

$$H(\omega, \boldsymbol{x}) = [\boldsymbol{a}^T \boldsymbol{c}(\omega)][\boldsymbol{a}_a^T \boldsymbol{c}_a(\omega)] + [1 - \boldsymbol{a}^T \boldsymbol{c}(\omega)][\boldsymbol{a}_c^T \boldsymbol{c}_c(\omega)] \tag{2a}$$

$$\boldsymbol{a} = [h_n \ 2h_{n+1} \ \cdots \ 2h_{N-1}]^T, \ n = \tfrac{N-1}{2} \tag{2b}$$

$$\boldsymbol{a}_a = [h_{an_a} \ 2h_{a(n_1+1)} \ \cdots \ 2h_{a(N_a-1)}]^T, \ n_a = \tfrac{N_a-1}{2} \tag{2c}$$

$$\boldsymbol{a}_c = [h_{cn_c} \ 2h_{c(n_c+1)} \ \cdots \ 2h_{c(N_c-1)}]^T, \ n_c = \tfrac{N_c-1}{2} \tag{2d}$$

$$\boldsymbol{c}(\omega) = [1 \ \cos M\omega \ \cdots \ \cos nM\omega]^T \tag{2e}$$

$$\boldsymbol{c}_a(\omega) = [1 \ \cos \omega \ \cdots \ \cos n_a\omega]^T \tag{2f}$$

$$\boldsymbol{c}_c(\omega) = [1 \ \cos \omega \ \cdots \ \cos n_c\omega]^T \tag{2g}$$

and

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{a}_a \\ \boldsymbol{a}_c \end{bmatrix} \tag{3}$$

The basic idea in the SOCP algorithm of [12] is to generate a sequence of increments $\{\boldsymbol{\delta}_k\}$ so that $H(\omega, \boldsymbol{x}_k)$ gradually approximates a desired frequency response $H_d(\omega)$ in a minimax sense, where

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{\delta}_k \quad \text{for } k = 0,\ 1,\ 2,\ \ldots,\ K-1 \tag{4}$$

and K denotes the number of iterations at convergence. Two techniques used in the implementation of this idea are to use a linear approximation of $H(\omega, \boldsymbol{x}_{k+1})$ as

$$H(\omega, \boldsymbol{x}_{k+1}) = H(\omega, \boldsymbol{x}_k + \boldsymbol{\delta}) \approx H(\omega, \boldsymbol{x}_k) + \boldsymbol{g}_k^T(\omega)\boldsymbol{\delta} \tag{5}$$

where $\boldsymbol{g}_k(\omega) = \nabla H(\omega, \boldsymbol{x}_k)$ and, in order to justify (5), to impose a norm constraint on increment $\boldsymbol{\delta} : \|\boldsymbol{\delta}\| \leq \beta$ for a small positive scalar β. If we denote the upper bound of the approximation error $|H(\omega, \boldsymbol{x}_k + \boldsymbol{\delta}) - H_d(\omega)|$ by η, then the SOCP for determining an optimal $\boldsymbol{\delta}$ assumes the form

$$\text{minimize} \quad \eta \tag{6a}$$

$$\text{subject to} \quad W(\omega)|H(\omega, \boldsymbol{x}_k) + \boldsymbol{g}_k^T(\omega)\boldsymbol{\delta} - H_d(\omega)| \leq \eta \tag{6b}$$

$$\|\boldsymbol{\delta}\| \leq \beta \tag{6c}$$

see [12] for the details.

We begin our examination of the CS of the above SOCP algorithm by recalling the definition of a sensitivity measure S_1^2 introduced in [22],[23], which is given in our notation as

$$S_1^2 = N\|\boldsymbol{h}_a - \hat{\boldsymbol{h}}_c\|^2 + N_a\|\boldsymbol{h}\|^2 + N_c\|\boldsymbol{e} - \boldsymbol{h}\|^2 \tag{7}$$

with

$$\boldsymbol{h} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{N-1} \end{bmatrix}_{N\times 1}, \quad \boldsymbol{h}_a = \begin{bmatrix} h_{a0} \\ h_{a1} \\ \vdots \\ h_{a(N_a-1)} \end{bmatrix}_{N_a\times 1}$$

$$\hat{\boldsymbol{h}}_c = \begin{bmatrix} \boldsymbol{0} \\ h_{c0} \\ \vdots \\ h_{c(N_c-1)} \\ \boldsymbol{0} \end{bmatrix}_{N_a\times 1}$$

where $N_a \geq N_c$ has been assumed for simplicity, $\hat{\boldsymbol{h}}_c$ is a vector of dimension N_a produced by padding $(N_a - N_c)/2$ zeros beyond each end of $\boldsymbol{h}_c$, and $\boldsymbol{e}$ is the $(N+1)/2$-th column of the identity matrix $\boldsymbol{I}_N$. From (7) it follows that as long as the coefficient magnitudes of the three subfilters $H_a(z)$, $H_{ma}(z)$, and $H_{mc}(z)$ are not exceedingly large, the CS of the FRM filter shall remain reasonably low. Furthermore, using (4) we can write

$$\boldsymbol{x}^* = \boldsymbol{x}_0 + \sum_{i=0}^{K-1} \boldsymbol{\delta}_i \tag{8}$$

where $\boldsymbol{x}^* = \boldsymbol{x}_K$ denotes that solution obtained after K SOCP iterations. Using (8) and (6c), we have

$$\|\boldsymbol{x}^* - \boldsymbol{x}_0\| = \|\sum_{i=0}^{K-1} \boldsymbol{\delta}_i\| \leq \sum_{i=0}^{K-1} \|\boldsymbol{\delta}_i\| \leq K\beta \tag{9a}$$

and

$$\|\boldsymbol{x}^*\| \leq \|\boldsymbol{x}_0\| + \sum_{i=0}^{K-1} \|\boldsymbol{\delta}_i\| \leq \|\boldsymbol{x}_0\| + K\beta \tag{9b}$$

With the design method in [12], typically the number of SOCP iterations at convergence is no more than 10 and β in (6c) is fairly small, thus (9a) indicates that the starting point $\boldsymbol{x}_0$ and solution point $\boldsymbol{x}^*$ are not far from each other when we observe them in the l-dimensional space X with $l = n + n_a + n_c$. In addition, (9b) suggests that if the length of the initial point $\boldsymbol{x}_0$ is not exceedingly large, then so is the length of the solution point $\boldsymbol{x}^*$. Since $\boldsymbol{x}^*$ is directly related to the impulse responses of the optimized subfilters (see (2b)–(2d) and (3)), by (7) this implies that the CS of the FRM filter obtained is always low as long as the initial FRM filter has a low CS. We demonstrate the validity of the analysis by applying the SOCP-based algorithm in [12] to design an FRM filter with $N = 45$, $N_a = 27$, $N_c = 19$, $M = 9$, $\omega_p = 0.3$, and $\omega_a = 0.305$ (here the design specifications are identical to those in the example described in [22],[23]). The initial $\boldsymbol{x}_0$ was generated using the method in [1], the value of β in (6c) was set to 0.1533, and no constraints on CS were imposed. The algorithm converges in 8 iterations, and the CS measure S_1^2 for the initial design and optimal designs were found to be 24.4532 and 38.9035, respectively, where the step size for coefficient quantization was set to 2^{-14}. The peak ripple magnitude in the passband was 0.009586 and minimum stopband attenuation was 40.4179dB. Fig. 2 compares the impulse responses of the three subfilters of the optimal FRM filter with those of the initial subfilters. From these figures one sees no drastic differences between the two sets of impulse responses. As a matter of fact, the norm differences between the two sets of filter coefficients were quite small:

$$\|\boldsymbol{h}^* - \boldsymbol{h}^{(0)}\| = 0.5394,\ \|\boldsymbol{h}_a^* - \boldsymbol{h}_a^{(0)}\| = 0.0816,\ \|\boldsymbol{h}_c^* - \boldsymbol{h}_c^{(0)}\| = 0.1052$$

This confirms that a satisfactory solution with low CS is within reach in a small vicinity surrounding a reasonable initial point $\boldsymbol{x}_0$ which can be secured using the method of [1].

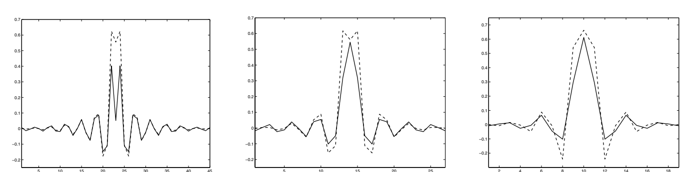

Figure 2: Impulse responses of optimal (solid lines) and initial (dashed lines) (a) $H_a(z)$, (b) $H_{ma}(z)$, and (c) $H_{mc}(z)$.

III. An Enhanced SOCP Algorithm with Coefficient Sensitivity Constraint

As argued in Sec. 2, the CS of the FRM filters designed using the method in [12] is inherently low. However, the design example presented in [22],[23] shows that with the same design specifications, the sensitivity measure S_1^2 can be made as low as 26.34. In this section, the SOCP-based algorithm in [12] is enhanced by imposing a constraint on S_1^2.

Rather than minimizing S_1^2 subject to other performance measures, the view point taken in developing the proposed algorithm is that the main goal of the algorithm remains to be the optimal performance in terms of peak ripple in the passband and stopband attenuation while the CS should be kept as low as possible. In so doing, the SOCP setting in (6a)–(6c) remains unchanged, and we impose an additional constraint to deal with CS.

Using (7), the sensitivity measure S_1^2 can be expressed as

$$S_1^2 = \left\| \begin{bmatrix} \sqrt{N_c}\boldsymbol{h} \\ \sqrt{N_a}\boldsymbol{h} \\ \sqrt{N}(\boldsymbol{h}_a - \hat{\boldsymbol{h}}_c) \end{bmatrix} - \hat{\boldsymbol{e}} \right\|^2 \tag{10}$$

where vectors $\boldsymbol{h}$, $\boldsymbol{h}_a$, and $\hat{\boldsymbol{h}}_c$ can be related to $\boldsymbol{x}$ in (3) as

$$\boldsymbol{h} = [\boldsymbol{J}\ \boldsymbol{0}\ \boldsymbol{0}]\boldsymbol{x} \tag{11a}$$

$$\boldsymbol{h}_a = [\boldsymbol{0}\ \boldsymbol{J}_a\ \boldsymbol{0}]\boldsymbol{x} \tag{11b}$$

$$\hat{\boldsymbol{h}}_c = [\boldsymbol{0}\ \boldsymbol{0}\ \hat{\boldsymbol{J}}_c]\boldsymbol{x} \tag{11c}$$

$$\boldsymbol{J} = \begin{bmatrix} 0 & & & \\ \vdots & & 0.5\hat{\boldsymbol{I}}_n & \\ 0 & & & \\ 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & 0.5\boldsymbol{I}_n & \\ 0 & & & \end{bmatrix}, \ \boldsymbol{J}_a = \begin{bmatrix} 0 & & & \\ \vdots & & 0.5\hat{\boldsymbol{I}}_{n_a} & \\ 0 & & & \\ 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & 0.5\boldsymbol{I}_{n_a} & \\ 0 & & & \end{bmatrix}$$

$$\hat{\boldsymbol{J}}_c = \begin{bmatrix} & & \boldsymbol{0} & \\ -- & -- & -- & -- \\ 0 & & & \\ \vdots & & 0.5\hat{\boldsymbol{I}}_{n_c} & \\ 0 & & & \\ 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & 0.5\boldsymbol{I}_{n_c} & \\ -- & -- & -- & -- \\ & & \boldsymbol{0} & \end{bmatrix}, \ \hat{\boldsymbol{e}} = \begin{bmatrix} \sqrt{N_c}\boldsymbol{e} \\ 0 \\ 0 \end{bmatrix} \tag{11d}$$

In (11d), $\boldsymbol{I}_n$, $\boldsymbol{I}_{n_a}$, and $\boldsymbol{I}_{n_c}$ are identity matrices of dimension n, n_a, and n_c, respectively, $\hat{\boldsymbol{I}}_n$, $\hat{\boldsymbol{I}}_{n_a}$, and $\hat{\boldsymbol{I}}_{n_c}$ are matrices generated by flipping their counterpart identity matrices upside down, and the zero matrix at top and buttom of $\hat{\boldsymbol{J}}_c$ are of size $(n_a - n_c) \times (n_c + 1)$. It follows that

$$\begin{bmatrix} \sqrt{N_c}\boldsymbol{h} \\ \sqrt{N_a}\boldsymbol{h} \\ \sqrt{N}(\boldsymbol{h}_a - \hat{\boldsymbol{h}}_c) \end{bmatrix} = \boldsymbol{A}^T\boldsymbol{x}$$

$$\text{with} \quad \boldsymbol{A}^T = \begin{bmatrix} \sqrt{N_c}\boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} \\ \sqrt{N_a}\boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \sqrt{N}\boldsymbol{J}_a & -\sqrt{N}\hat{\boldsymbol{J}}_c \end{bmatrix} \tag{12}$$

So if we let d_{FWL} be a desired upper bound for S_1, then imposing a constraint that S_1 be bounded from above by a desired level d_{FWL} can be cast as

$$\|\boldsymbol{A}^T\boldsymbol{x} - \hat{\boldsymbol{e}}\| \le d_{FWL} \tag{13}$$

In the kth iteration, $\boldsymbol{x} = \boldsymbol{x}_k + \boldsymbol{\delta}$ and (13) becomes

$$\|\boldsymbol{A}^T\boldsymbol{\delta} + \boldsymbol{b}_k\| \le d_{FWL}$$

where $\boldsymbol{b}_k = \boldsymbol{A}\boldsymbol{x}_k - \hat{\mathbf{e}}$. Incorporating this constraint into (6), we formulate an enhanced SOCP problem as

$$\begin{aligned} &\text{minimize} && \eta && \text{(14a)} \\ &\text{subject to} && W(\omega)|H(\omega,\boldsymbol{x}_k) + \boldsymbol{g}_k^T(\omega)\boldsymbol{\delta} - H_d(\omega)| \le \eta && \text{(14b)} \\ & && \text{for } \omega \in \Omega_d && \\ & && \|\boldsymbol{\delta}\| \le \beta && \text{(14c)} \\ & && \|\boldsymbol{A}^T\boldsymbol{\delta} + \boldsymbol{b}_k\| \le d_{FWL} && \text{(14d)} \end{aligned}$$

Problem (14) can be solved using an efficient SOCP solver like SeDuMi [24].

IV. A Design Example

The proposed method was applied to design an FRM filter with $N = 45$, $N_a = 27$, $N_a = 19$, $M = 9$, $\omega_p = 0.3$ and $\omega_a = 0.305$. We note that the design specifications are the same as those in the example presented in [22],[23]. The weighting function $W(\omega)$ in (14b) was piecewise constant with value 1 in the passbnad and 1.07 in the stopband and the values of β in (14c) and d_{FWL} in (14d) were set to 0.2168 and 5.4 respectively. Note that setting $d_{FWL} = 5.4$ means imposing a bound $5.4^2 = 29.16$ on the sensitivity measure S_1^2. If a smaller d_{FWL} is used, our simulations indicate that the performance of the FRM filter in terms of its approximation accuracy will start to deteriorate. On the other hand, this bound appears to be satisfactory as it is pretty close to the smallest possible value of S_1^2 while the performance of FRM is still acceptable [22],[23]. The set Ω_d in (14b) contains 2000 frequency grids for the passband and stopband. The SOCP solver SeDuMi [24] was used to solve the sequential SOCP problem (14) and it took the proposed algorithm 8 iterations to converge to a solution on a PC (with a 3.4 GHz Pentium 4 processor) with 26.91 seconds of CPU time. The quantization step size was set to 2^{-14}. The quantized filter coefficients of $H_a(z)$, $H_{ma}(z)$, and $H_{mc}(z)$ are given in Table I and its performance in terms of peak passband ripple, minimum stopband attenuation, and CS measure S_1^2 as compared with those obtained in [22],[23] are given in Table II. The amplitude response and passband behavior of the FRM filter obtained are depicted in Fig. 3a and b, respectively. We see that SOCP provides a suitable framework for joint optimization subject to a CS constraint for FRM filters with satisfactory approximation accuracy and very low CS.

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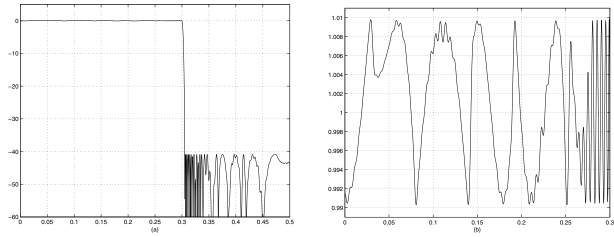

Figure 3: (a) Frequency response and (b) passband magnitude of the quantized FRM filter with CS constraint.

TABLE I

COEFFICIENTS OF QUANTIZED $H_a(z)$, $H_{ma}(z)$ AND $H_{mc}(z)$.

$h(1:23)$	$h_a(1:14)$
0.00341796875000	-0.03460693359375
-0.00708007812500	0.00823974609375
-0.00207519531250	0.04162597656250
0.00439453125000	-0.04437255859375
0.00006103515625	-0.01544189453125
-0.00848388671875	0.07073974609375
0.00292968750000	-0.02264404296875
0.00909423828125	-0.08013916015625
-0.00823974609375	0.08190917968750
-0.00994873046875	0.04412841796875
0.01422119140625	-0.15454101562500
0.00677490234375	0.00018310546875
-0.02239990234375	0.34057617187500
-0.00054931640625	0.48156738281250
0.03039550781250	$h_c(1:10)$
-0.01147460937500	-0.00781250000000
-0.03869628906250	0.00512695312500
0.03308105468750	0.01348876953125
0.04467773437500	-0.02520751953125
-0.07550048828125	-0.00500488281250
-0.05645751953125	0.06585693359375
0.20782470703125	-0.04492187500000
0.02935791015625	-0.10144042968750
	0.29644775390625
	0.61218261718750

TABLE II

PERFORMANCE COMPARISONS

FRM Filter	Peak passband ripple	Minimum stopband attenuation (dB)	S_1^2
Filter in Table I of [23]	0.009949	40.0674	6.7797×10^9
Filter in Table II of [23]	0.010041	39.9628	26.4288
SOCP-based design without CS constraint	0.009586	40.4187	38.9035
SOCP-based design with CS constraint	0.009874	40.6479	28.2468

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