# A Regularized Least Squares Approach for Ultra-Wideband Time-of-Arrival Estimation with Wavelet Denoising

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***Abstract*— Wireless localization and ranging are challenging tasks which demand high signal-to-noise ratio (SNR) for an increase in accuracy. Impulsive ultra-wideband (UWB) technology is a promising signaling alternative that is capable of high-resolution ranging with minimal cost on SNR. Unfortunately, typical time-of-arrival (ToA) estimation algorithms are complicated and perform poorly in the low SNR environment. In this paper, we propose a *regularized* least squares approach with wavelet denoising to improve the estimator accuracy at low SNR. Our approach estimates the ToA as a by-product of the channel estimation; furthermore, it is simple and enables fast, on-the-fly, accurate ToA estimation applicable to real-time application. We demonstrate the robustness of our algorithm by simulation where it is shown to outperform other high-resolution algorithms in ToA estimation.**

## I. Introduction

Since the spectral allocation approved by the U.S. Federal Communications Committee in 2002, there have been many potential applications envisioned for ultra-wideband (UWB) technology. Particularly in ranging and localization, UWB impulse radio (UWB-IR) is a promising candidate that can enable centimeter accuracy with minimum cost on the signal-to-noise ratio (SNR) [1].

Locating a node in a wireless sensor network involves recording the range information from radio signals traveling between a target node (TN) and a group of reference nodes (RNs) [1]. The range information can be retrieved by several techniques, e.g., with signal strength, or time-of-arrival (ToA) estimation. Due to its inherent wideband characteristic, UWB-IR with time-based positioning technique is a promising solution for cost-effective, high-resolution ranging and localization.

Impulse-based ranging technique with ToA estimation was first considered in [2], where it utilizes generalized maximum likelihood estimator to detect the direct path arrival. Although it shows a promising result on the measured data, it complicates the matter with the statistical modeling of several parameters from measurement, which are subject to change depending on the environment, and requires Nyquist rate samples that is costly to obtain. To reduce the sampling rate requirement, [3] proposed the use of symbol rate samples after an energy detector to estimate the ToA. Unfortunately, because of a square-law device its performance degrades significantly at low SNR. An inverse problem approach to ToA estimation was proposed by [4], where the authors estimate the ToA by treating it as a by-product of the large scale linear least squares (LS) solution. Although this algorithm is simple, the problem is *ill-posed* and suffers output instability, attributable to both noise and a dense multipath, inherent of an impulsive wideband channel. Moreover, its performance is not well documented and the relationship to channel sampling rate is not examined. In [5], ToA estimation based on high-resolution, peak-detection-based (PDB) iterative algorithms was proposed, where variants of the suboptimal maximum likelihood (ML) channel estimator are shown to provide good result while being efficient at energy capture.

To improve the estimator accuracy at low SNR while retaining its simplicity, we propose a *regularized* LS (RLS) approach with wavelet denoising (WD) to the problem of ToA estimation. Pioneered by Donoho and Johnstone [6], WD has been successfully applied to boost the low SNR performance of time-delay estimator [7] and several direction finding algorithms, e.g., [8], [9]. Our technique utilizes discrete wavelet transform (DWT), and hyperbolic shrinkage of [10] with the threshold developed by Donoho [11] to effectively enhance the SNR prior to RLS channel estimation; thereafter, the final retrieval of accurate ToA information. Our approach is simple and enables fast, on-the-fly, high-resolution ToA estimation applicable to real-time ranging system. From simulation, our algorithm is shown to outperform the PDB algorithms of [5] when Nyquist sampling rate is available. To the best of authors' knowledge, this approach has yet to be adopted for UWB-IR ToA estimation. Note that higher range resolution can be achieved by interpolating from the Nyquist rate samples.

The rest of the paper is organized as follows: Section II presents the overall system model, including a description of both LS and RLS channel estimators. To examine the benefits of denoising, Section III describes the critical components of WD and how they contribute to the SNR enhancement. Finally, we propose our ToA estimator in Section IV, and compare its performance against the PDB algorithms in Section V. Concluding remarks are given in Section VI.

## II. System Model

The position of a sensor node is directly related to the ToA of the first multipath component. To estimate the ToA, a UWB ranging system periodically transmits sub-nanosecond pulses between the RNs and a TN of unknown distance. For a single pulse transmitted through free-space, the received signal at the TN under multipath can be modeled as

$$r(t) = \sum_{l=0}^{L-1} \alpha_l w(t - \tau_l) + n(t)\,, \tag{1}$$

where $w(t)$ is the received pulse template of duration $T_p$, $\alpha_l$ and $\tau_l$ are amplitude and time delay of $l$-th multipath, $L$ is an unknown *a priori* which presents the number of propagation paths, $n(t)$ is the additive white Gaussian noise (AWGN) with double-sided spectral density $N_0/2$. The purpose of ToA estimation is to accurately predict $\tau_0$ over an observation interval $[0, T)$.

To locate the TN, a series of measurements is first recorded between itself and the RNs. Assuming the observation interval consists of $K$ equally spaced delays for $k = 0, 1, \ldots, K-1$, each associating with a sparse channel tap $a_k$. We can then simplify (1) by associating the sparse set $\{a_k\}_{k=0}^{K-1}$ of channel coefficients with uniformly delayed received pulse template $w(t - k\Delta)$ for $k = 0, 1, \ldots, K-1$, as

$$r(t) = \sum_{k=0}^{K-1} a_k w(t - k\Delta) + n(t)\,. \tag{2}$$

Suppose the received signal is sampled at sampling time $T_s$. Given time instant samples $t_i = (i-1)T_s$ for $i = 1, 2, \ldots, M$, (2) can be written as

$$r(t_i) = \sum_{k=0}^{K-1} a_k w(t_i - k\Delta) + n(t_i)\,, \quad i = 1, 2, \ldots, M\,, \tag{3}$$

which in matrix notation is given by

$$\mathbf{r} = \mathbf{W}\mathbf{a} + \mathbf{n} = \mathbf{s} + \mathbf{n}\,, \tag{4}$$

where $\mathbf{a} = [a_0, a_1, \ldots, a_{K-1}]^T$ is the vector of sparse channel coefficients, and $\mathbf{n} = [n(t_1), n(t_2), \ldots, n(t_M)]^T$ is the noise samples vector, $\mathbf{s} = \mathbf{W}\mathbf{a}$ is the signal portion of $\mathbf{r}$, and

$$\mathbf{W} = \begin{bmatrix} w(t_1) & w(t_1 - \Delta) & \ldots & w(t_1 - (K-1)\Delta) \\ w(t_2) & w(t_2 - \Delta) & \ldots & w(t_2 - (K-1)\Delta) \\ \vdots & \vdots & \ddots & \vdots \\ w(t_M) & w(t_M - \Delta) & \ldots & w(t_M - (K-1)\Delta) \end{bmatrix}, \tag{5}$$

represents a $M \times K$ matrix which comprises of delayed and sampled version of $w(t)$. In contrast to the matrix representations of [4] and [5], the step-size in (5) can be varied for the sake of estimation accuracy. Consequently, depending on $\Delta$, the linear system in (4) can be either over or underdetermined.

### A. LS Solution

We treat the ToA estimation as a by-product of the LS channel estimator by solving the solution to (4). For an overdetermined noiseless system there exists a unique solution which solves the problem

$$\min ||\mathbf{W}\mathbf{a} - \mathbf{r}||^2\,, \tag{6}$$

which yields the LS solution given by

$$\hat{\mathbf{a}}_{\mathrm{LS}} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\mathbf{r} = \mathbf{W}^{\perp}\mathbf{r}\,, \tag{7}$$

where $|| \cdot ||^2$ is the Euclidean norm, $(\cdot)^T$ denotes matrix transpose, $(\cdot)^{-1}$ is the matrix inverse, $\hat{\mathbf{a}}$ denotes an estimate of $\mathbf{a}$ and $\mathbf{W}^{\perp}$ is the Moore-Penrose inverse of $\mathbf{W}$. Unlike [5], $\mathbf{W}^{\perp}$ can be *pre-computed* and *stored* when a desired resolution $\Delta$ is given. Unfortunately, (7) is often sensitive to noise in the received signal and can be quite unstable when $\mathbf{W}$ is ill-posed.

### B. Regularized LS Solution

To find a meaningful result when the solution to (6) becomes unstable, we apply the technique of *regularization*. *Regularization* is a well-known technique for dealing with instability in the inverse problem [12] by forcing an ill-posed problem into a well-posed one with some *a priori* information. The RLS solution solves the problem

$$\min\{||\mathbf{W}\mathbf{a} - \mathbf{r}||^2 + \lambda||\mathbf{a}||^2\}\,, \tag{8}$$

where $\lambda \geq 0$ is the *regularization parameter* which controls the solution's energy. Note that with $\lambda = 0$ the solution to (8) reduces to the LS one. With $\lambda > 0$, it is straightforward to show that the unique global solution to (8) is given by

$$\hat{\mathbf{a}}_{\mathrm{RLS}} = (\mathbf{W}^T\mathbf{W} + \lambda\mathbf{I})^{-1}\mathbf{W}^T\mathbf{r} = \mathbf{W}_{\lambda}^{\perp}\mathbf{r}\,, \tag{9}$$

where $\mathbf{W}_{\lambda}^{\perp}$ is called the regularized pseudo-inverse, which can also be *pre-calculated* and *stored* for fast, on-the-fly processing.

## III. Wavelet Denoising

To realize a stable LS solution for accurate ToA estimation, we apply the well-established technique of WD. Since its introduction in [6], denoising with DWT has become a powerful tool to recover noise corrupted data. To recover $M$ samples of a known data sequence $\mathbf{s}$ from the noise-corrupted observation $\mathbf{r} = \mathbf{s} + \mathbf{n}$, where $\mathbf{n}$ denotes a $M \times 1$ vector samples of AWGN with variance $\sigma^2$, the purpose of WD is to differentiate the wavelet coefficients of $\mathbf{s}$ from those of $\mathbf{n}$, assuming the coefficients of $\mathbf{s}$ resides mostly in the low frequency region and can be compressed into a few large values in the wavelet domain. The compression is carried out by multiplying $\mathbf{r}$ with a $M \times M$ orthonormal wavelet matrix $\mathbf{W}_W$, as

$$\begin{aligned} \mathbf{r}_W &= \mathbf{W}_W\mathbf{r} = \mathbf{W}_W\mathbf{s} + \mathbf{W}_W\mathbf{n} \\ &= \mathbf{s}_W + \mathbf{n}_W\,, \end{aligned} \tag{10}$$

where the matrix $\mathbf{W}_W$ can be *pre-determined* by knowing the wavelet filter order $F$ and decomposition level $J$. Moreover, due to the orthonormal property of $\mathbf{W}_W$, the noise is similarly mapped to $\mathbf{n}_W$ with identical statistics. However, because of its wideband nature, coefficients of noise are usually small and can be discarded; whereas, the large coefficients of the desired signal are retained [6], [11]. Differentiating amongst these coefficients is identical to the filtering operation, where (10) is multiplied by a matrix $\mathbf{H}$ modeled as

$$\mathbf{H} = \mathrm{diag}[h(1), h(2), \ldots, h(M)]\,. \tag{11}$$

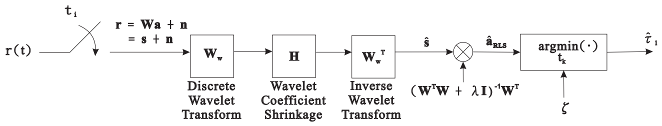



Fig. 1. The RLS-WD ToA estimator.

The elements of $\mathbf{H}$ are set according to the thresholding criterion, with hard and soft thresholds from [11] being the most common, or the hyperbolic shrinkage proposed by Vidakovic [10] defined as

$$h(i) = \begin{cases} \sqrt{\left(1 - \frac{\delta^2}{|r_W(i)|^2}\right)} & \text{if } |r_W(i)| > \delta \\ 0 & \text{otherwise} \end{cases} , \tag{12}$$

where $r_W(i)$ denotes the $i$-th element of $\mathbf{r}_W$, and $\delta$ is the threshold from [6], [11], given by

$$\delta = \sqrt{2\sigma^2 \log(M)} \,. \tag{13}$$

The recovery of the desired signal $\mathbf{s}$ is now given by

$$\hat{\mathbf{s}} = \mathbf{W}_W^T \mathbf{H} \mathbf{r}_W = \mathbf{W}_W^T \mathbf{H} \mathbf{W}_W \mathbf{r} \,, \tag{14}$$

where $\hat{\mathbf{s}}$ represents an estimate of $\mathbf{s}$, but with the noise being significantly reduced. The process of discarding and retaining the wavelet coefficients results in the overall SNR enhancement.

## IV. RLS-WD ToA Estimation

The distinct advantage of UWB in ranging is its high precision with minimal penalty on SNR. Many existing ToA estimators, however, do not work well at the low SNR region, thus are limited to only short distance ranging. To improve the estimator accuracy under low SNR, we adopt WD with RLS channel estimation as shown in Fig. 1 for simple, yet accurate, ToA estimation. We name this the RLS-WD ToA estimator.

One drawback of denoising with DWT is the requirement of noise information, where its ability to remove noise depends entirely on how accurate the noise variance can be estimated. For narrowband signals and images, which map to a few large, low-frequency coefficients in the wavelet domain, noise variance can be estimated from the finest scale wavelet coefficients [11]. However, due to the wideband characteristic of UWB, estimating variance from the first level decomposition is often incorrect. For that, assuming a large distance between nodes and a large sample size $M$, the variance can be estimated from the first few hundred noise samples as

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (r(t_i) - \hat{\mu})^2 \,, \tag{15}$$

where $\hat{\mu}$ is the sample mean and $N$ a subset of $M$. Now, the RLS-WD ToA estimator can be summarized as

1) Receive $M$ samples of observation $\mathbf{r}$ at sampling rate $T_s$ over the interval $[0, T)$.
2) Estimate the noise variance $\hat{\sigma}^2$ according to (15).
3) Select the wavelet filter order $F$ and WD decomposition level $J$, apply Daubechies DWT and Vidakovic hyperbolic shrinkage to $\mathbf{r}$, and estimate the desired signal according to (14).
4) Choose the channel tap estimator resolution $\Delta$, and $K = T/\Delta$, construct $\mathbf{W}$ according to (5).
5) Estimate the channel $\hat{\mathbf{a}}$ using either LS solution in (7), or RLS algorithm in (9), with a pre-determined $\lambda$.
6) Estimate the ToA as
$$\hat{\tau}_0 = \arg\min_{t_k} |\hat{\mathbf{a}}| > (1 - \zeta) \,,$$
where $\zeta$ is the threshold set as a percentage of the maximum estimated amplitude.

## V. Simulation Results

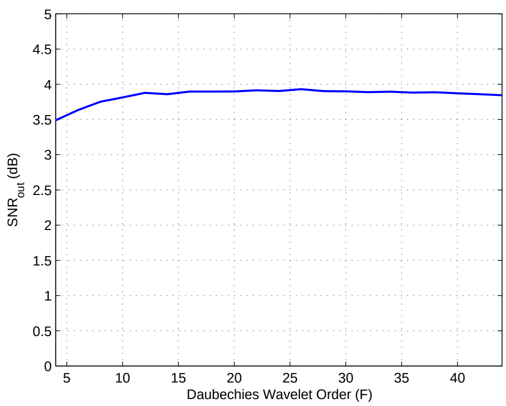
To show the advantage of our algorithm over its counterparts, we evaluate their performance by computer simulation in MATLAB$^{\text{TM}}$ with the Uvi_Wave software package [13] for Daubechies DWT. To accurately examine the performance under multipath, we use the CM3 channel model from IEEE $802.15.3$a [14], which models a severe office non line-of-sight environment. The received template $w(t)$ is assumed to be the typical Gaussian doublet with pulse parameter $\tau_m = 0.6$ ns, which has a zero-to-zero pulse width of $2$ ns. The pulse is sampled at $T_s = 0.1$ ns with the observation interval $T = 50$ ns, representing a medium distance ranging application. To study the performance of RLS-WD ToA estimator, we vary $\Delta$ as a multiple of $T_s$ when constructing $\mathbf{W}$. A thousand different channel realizations are simulated prior to the final performance evaluation.

To determine the most suitable $F$ for WD, we plot the output SNR as $F$ varies for a fixed input SNR[1] of $0$ dB in Fig. 2(a). As shown, by applying WD to the received signal we can have close to $4$ dB of gain across all $F$. Since increasing $F$ has no effect on the output SNR, we may further reduce the denoising complexity by selecting the smallest $F$ before performance tapers off, which is $F = 8$ in our case. For that value of $F$, Fig. 2(b) illustrates the effectiveness of WD[2] as input SNR varies. We see that WD results in substantial gain at low SNR before diminishing return at high SNR. However, when considering long distance ranging, the performance at low SNR is often of great interest. For the ease of simulation, the variance in (13) is assumed to be perfectly estimated, hence the results shown act as a lower bound.

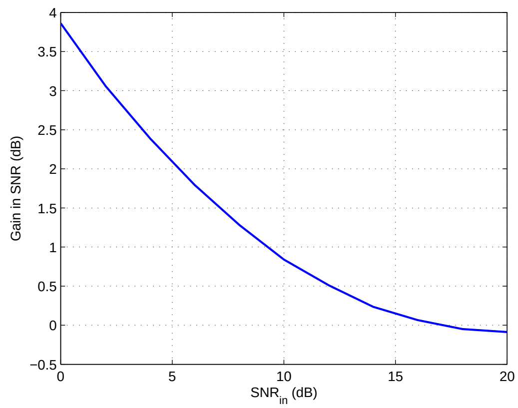
We compare the performance of our estimator with the suboptimal ML PDB estimators in [5], namely, single search (PDB-SS1), search and subtract (PDB-SS2), search subtract and readjust (PDB-SSR). In general, these estimators first compute the discrete match filter (MF) output between $\mathbf{r}$ and a sampled pulse template; thereafter, the selection of the maximum MF peaks with or without iteratively canceling

[1] The output and input SNR are defined according to $\text{SNR}_{\text{out}} = 20\log_{10}(||\mathbf{s}||/||\hat{\mathbf{s}} - \mathbf{s}||)$ and $\text{SNR}_{\text{in}} = 20\log_{10}(||\mathbf{s}||/||\mathbf{n}||)$, respectively, where $||\cdot||$ denotes the Frobenius matrix norm.

[2] The effectiveness of WD can be measured from the gain in SNR [8], defined as $\text{SNR}_{\text{out}} - \text{SNR}_{\text{in}}$.




(a) SNR of denoised signal as $F$ varies for 0 dB input SNR.




(b) Gain in SNR as a function of input SNR for $F = 8$.

Fig. 2. Denoising performance of Daubechies DWT for UWB-IR.

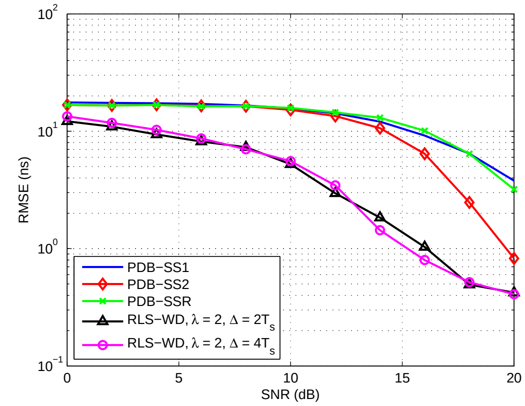



Fig. 3. RMSE of ToA estimation as SNR varies for different algorithms with $\zeta = 95\%$.

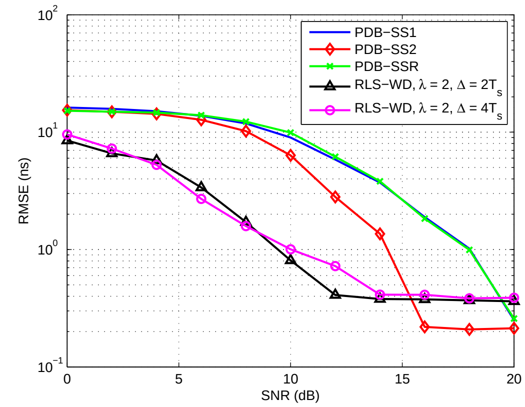



Fig. 4. RMSE of ToA estimation as a function of SNR for different algorithms with $\zeta = 90\%$.

the peaks from $\mathbf{r}$, depending on the algorithm. For these algorithms, a similar ToA estimation criterion to the RLS-WD is used and the number of peaks to detect $Z$ is set to $100$. Fig. 3 shows the behavior of root mean-squared error (RMSE) versus SNR when $\zeta = 95\%$, with the RLS-WD evaluated at $\lambda = 2$ in (9), and $\Delta = 2T_s$ and $4T_s$. Clearly, the RLS-WD algorithm outperforms others under all SNR. At low SNR, it performs better due to the input SNR enhancement from WD; whereas, at high SNR, the contribution from WD lessens. An interesting observation is the identical performance for $\Delta = 2T_s$ and $4T_s$, which depending on the system requirement we may choose either one without compromising the overall performance.

We observe a similar improvement on the RMSE when $\zeta = 90\%$, as shown in Fig. 4. Specifically, the RLS-WD outperforms PDB algorithms when SNR $< 15$ dB. For SNR $\geq 15$ dB, all algorithms exhibit an error floor, particularly a higher floor for RLS-WD than PDB-SSR, which we suspect is due to the limitation of the RLS. Once again, we stress on the importance of performance gain at low SNR that is more critical when considering long distance ranging application.

Apart from the RMSE behavior, we are also interested in the energy capture, as in [5]. For that, Fig. 5 illustrates the mean energy capture as a function of SNR for all algorithms, and the RLS-WD is computed with $\lambda = 0.1$. The energy capture is computed between the received signal and its estimate. From Fig. 5 we note that the RLS-WD loses energy initially due to denoising, but it quickly recovers at high SNR when the received signal is less noisy. Also, a spacing of $2T_s$ on $\Delta$ captures significantly more energy than for $\Delta = 4T_s$ since $K$ decreases as we increase $\Delta$ in the signal model. Note that if all algorithms undergo denoising before ToA estimation, the energy captured by RLS-WD would outperform all PDB estimators.

Another parameter of interest is the choice of $\zeta$, which inherently affects the estimator performance. For that, Fig. 6 shows the RMSE behavior for a varying $\zeta$ at $\Delta = 2T_s$ for SNR = $\{0, 4, 8, 12, 16, 20\}$ dB. Generally, a large $\zeta$ does not produce the best result, especially in a low SNR environment where noise can often be mistakenly identified as the direct path. However, at high SNR, a large $\zeta$ often produces the best result since the noise is either small or has been mostly removed by WD.

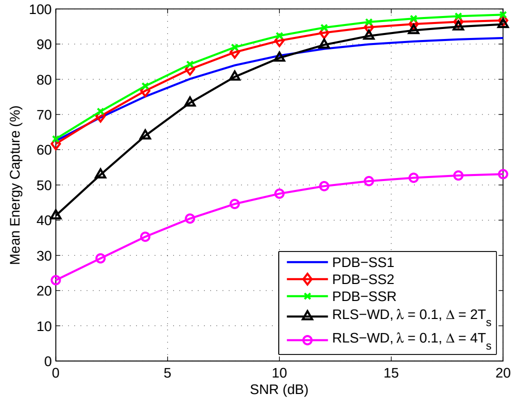



Fig. 5. Mean energy capture as SNR varies for different algorithms. The energy capture is computed between the received signal and its estimate.

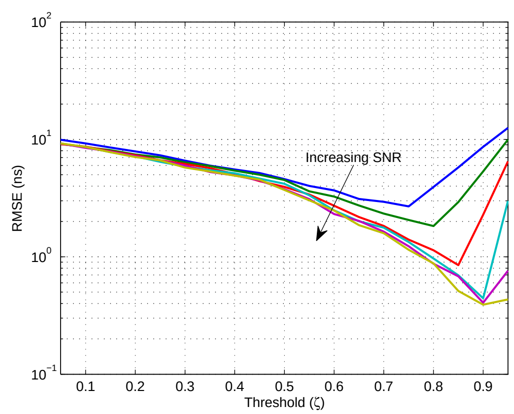



Fig. 6. RMSE versus $\zeta$ for the RLS-WD ToA estimator at $\Delta = 2T_s$ for SNR = $\{0, 4, 8, 12, 16, 20\}$ dB, top to bottom.

In terms of complexity, the computational load of the RLS-WD is mostly constant. Specifically, the RLS is equivalent to LS over a sphere and its complexity is about $4M^2K + 22K^3$ flops [15] with the remaining load going to DWT, which is of $\mathcal{O}(M)$ per WD process [16]. However, when recognizing the WD as a series of matrix multiplications in (14) with pre-computed matrices further indicates a constant processing time for our approach. In contrast, the complexity of the PDB estimators depends entirely on the channel condition and the number of iterations $Z$. Table I compares the flop count amongst the algorithms. For the PDB algorithms, they require $4M^2$ flops per MF computation, $5M$ and $2z^2M + 2z^3/3 + 2M$ flops to compute the channel gain per iteration for PDB-SS2 and PDB-SSR, respectively, where $z$ denotes the iteration index. Hence, in a dense multipath channel, the RLS-WD would require less processing time than its counterparts.

## VI. Conclusion

In this paper, we have proposed the RLS-WD ToA estimator, which estimates the ToA as a by-product of the channel estimation. Our approach is simple and can provide fast, on-the-fly, accurate ToA estimation applicable to real-time ranging system. From simulation, we have demonstrated the robustness of our algorithm where it outperformed other high-resolution algorithms. Apart from being a better estimator, the RLS-WD is also capable of accurate channel estimation; however, its threshold $\zeta$ must be set according to the SNR for the best performance. Finally, we have shown that the RLS-WD has lower computational complexity than its counterparts especially when processing a dense multipath channel.

TABLE I
Comparison of Computational Complexity

| | Flop Count |
|---|---|
| PDB-SS1 | $4M^2$ |
| PDB-SS2 | $4ZM^2 + 5ZM$ |
| PDB-SSR | $4ZM^2 + 4ZM + \mathcal{O}(Z^4)$ |
| RLS-WD | $(4M^2K + 22K^3) + 4FJM$ |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