Towards Global Design of Orthogonal Filter Banks and Wavelets

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Abstract

This paper investigates several design issues concerning two-channel conjugate quadrature (CQ) filter banks and orthogonal wavelets. It is well known that optimal designs of CQ filters and wavelets in least squares or minimax sense are nonconvex problems and to date only local solutions can be claimed. By virtue of recent progress in global polynomial optimization and direct design techniques for CQ filters, we in this paper present a design strategy that may be viewed as our endeavors towards global solutions for CQ filters. Two design scenarios are considered, namely the least squares designs with vanishing moment (VM) requirement, and equiripple (i.e. minimax) designs with VM requirement. Simulation studies are presented to verify our design concept for both LS and minimax designs of low-order CQ filters; and to evaluate and compare the proposed algorithms with existing design algorithms for high-order CQ filters.

1. INTRODUCTION

The class of two-channel conjugate quadrature (CQ) filter banks, also known as power-symmetric filter banks [14], is one of the most well-known building blocks for multirate systems and wavelet-based coding systems as it offers perfect reconstruction (PR) and other desirable properties. Despite the fact that many algorithms for the design of CQ filters have been proposed since 1980's, see e.g. references [1], [2], [10]-[14] and the work cited therein, to date only locally optimal designs can be claimed. From a mathematical point of view, this is primarily because the design problems are inherently nonconvex, admitting many local solutions. In this regard, this paper is an attempt to develop feasible strategies towards global designs of CQ filters.

The design methods proposed in this paper are made possible by virtue of recent progress in global polynomial optimization [6], [9] and direct design techniques for the CQ filters [1] in conjunction with our observations on a common pattern shared among globally optimal impulse responses of low-order CQ filters and a progressive design procedure in terms of filter length. Two design scenarios are considered, namely the least squares designs with vanishing moment (VM) requirement, and equiripple (i.e. minimax) designs with VM requirement. Simulation results for both LS and minimax CQ filters

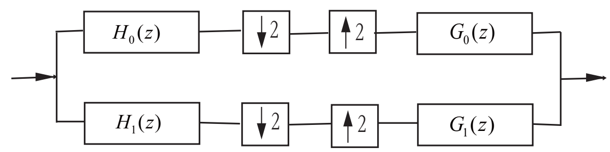

Figure 1. A 2-channel CQ filter bank.

are presented to verify our design concept for low-order CQ filters, and to evaluate and compare the performance of the proposed algorithms with existing design algorithms for high-order CQ filters.

2. NOTATION AND BACKGROUND

2.1. Two-channel orthogonal filter banks

A two-channel causal FIR CQ filter bank consists of a pair of analysis filters H_0, H_1 and a pair of synthesis filters G_0 and G_1 as shown in Fig. 1, where the four filters are related by [2]

$$\begin{aligned} H_1(z) &= -z^{-(N-1)}H_0(-z^{-1}) \\ G_0(z) &= H_1(-z) \\ G_1(z) &= -H_0(-z) \end{aligned} \tag{1}$$

where $H_0(z) = \sum_{n=0}^{N-1} h_n z^{-n}$ is a lowpass FIR transfer function of length-N with N even. With (1), the aliasing is eliminated, and the PR is achieved if $H_0(z)$ satisfies

$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 2 \tag{2}$$

Eq. (2) is equivalent to a set of $N/2$ equality constraints as

$$\sum_{n=0}^{N-1-2m} h_n \cdot h_{n+2m} = \delta_m \quad \text{for } m = 0, 1, ..., (N-2)/2 \tag{3}$$

where δ_m is the Dirac sequence with $\delta_0 = 1$ and $\delta_m = 0$ for nonzero m. Eq. (3) is known as the double shift orthogonality in the wavelet literature. In addition to the PR conditions, CQ filters may be required to meet other constraints such as possessing certain number of VMs for constructing wavelets [10]-[13]. It is known that the number of VMs of a CQ filter bank is equal to the number of zeros of $H_0(z)$ at $\omega = \pi$. Because

$$\frac{d^l H_0(e^{j\omega})}{d\omega^l}\Big|_{\omega=\pi} = (-j)^l \sum_{n=0}^{N-1} (-1)^n \cdot n^l \cdot h_n$$

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a CQ filter has L vanishing moments if

$$\sum_{n=0}^{N-1}(-1)^n \cdot n^l \cdot h_n = 0 \text{ for } l = 0, 1, ..., L-1 \quad (4)$$

Thus a least squares (LS) design of CQ lowpass filter $H_0(z)$ having L VMs can be cast as

$$\text{minimize} \quad \int_{\omega_a}^{\pi} |H_0(e^{j\omega})|^2 d\omega \quad (5a)$$

$$\text{subject to:} \quad \text{constraints (3) and (4)} \quad (5b)$$

where ω_a is the normalized stopband edge of $H_0(z)$.

In this paper, we also consider the minimization of maximum instantaneous power of lowpass filter $H_0(z)$ over its stopband subject to PR and VM constraints. Thus the minimax design can be formulated as

$$\text{minimize} \quad \underset{\omega_a \le \omega \le \pi}{\text{maximize}} |H_0(e^{j\omega})| \quad (6a)$$

$$\text{subject to:} \quad \text{constraints (3) and (4)} \quad (6b)$$

2.2. Polynomial optimization problems

2.2.1. Polynomial Optimization Problems

A real-valued polynomial $f(\mathbf{x})$ in n-dimensional space R^n can be expressed as

$$f(\boldsymbol{x}) = \sum_{\boldsymbol{\alpha} \in \mathcal{F}} c(\boldsymbol{\alpha}) \boldsymbol{x}^{\boldsymbol{\alpha}} \quad (7)$$

where $c(\boldsymbol{\alpha}) \in R$, $\boldsymbol{x} = [x_1\ x_2\ ...\ x_n]$, $\boldsymbol{\alpha} = [\alpha_1\ \alpha_2\ ...\ \alpha_n] \in \mathcal{F} \subset \mathcal{Z}_+^n -$ the set of all vectors in R^n whose components are nonnegative integers, and $\boldsymbol{x}^{\boldsymbol{\alpha}} = x_1^{\alpha_1}\ x_2^{\alpha_2}\ ...\ x_n^{\alpha_n}$. The order (degree) of $f(\boldsymbol{x})$ is defined as the largest $\sum_i \alpha_i$.

A general polynomial optimization problem (POP) has the form

$$\text{minimize} \quad f_0(\boldsymbol{x}) \quad (8a)$$

$$\text{subject to:} \quad f_k(\boldsymbol{x}) \ge 0 \text{ for } k = 1, ..., L \quad (8b)$$

$$f_k(\boldsymbol{x}) = 0 \text{ for } k = L+1, ..., K \quad (8c)$$

where $f_k(\boldsymbol{x})$ for $k = 0,\ 1,\ ...,\ K$ are real-valued polynomials. POPs include linear programming (LP), convex quadratic programming (QP), semidefinite programming (SDP), and second-order cone programming (SOCP) problems as its special cases. More importantly, POPs stand for a substantially broader class that covers many nonconvex optimization problems [6].

2.2.2. Global Optimization of Problem (8)

A recent breakthrough in the field is made by Lasserre [6] in which it is proved that when the feasible region in (8) is compact (not necessarily convex), the global solution of (8) can be approximated as closely as desired (and often can be obtained exactly) by solving a finite sequence of SDP problems. A technical difficulty with the method of [6] is that the size of the SDP problems involved in a POP usually grows very quickly that may cause numerical difficulties even for POPs of moderate scales.

More recently, sparse SDP relaxation [7] is proposed for global solution of POPs of relatively larger scales with improved efficiency. The method is supported by MATLAB toolbox SparsePOP2000 [4][5]. Another MATLAB toolbox for POPs is GloptiPoly 3.4 [8] which is intended to solve the generalized problems of moments (GPM) that can be viewed as an extension of the classical problem of moments [9].

3. LEAST SQUARES DESIGNS

3.1. Global LS design of low-order filter banks

The design formulation in (5) can be expressed as

$$\text{minimize} \quad \boldsymbol{h}^T \boldsymbol{Q} \boldsymbol{h} \quad (9a)$$

$$\text{subject to:} \quad \sum_{n=0}^{N-1-2m} h_n \cdot h_{n+2m} = \delta_m \quad (9b)$$

$$\sum_{n=0}^{N-1} (-1)^n \cdot n^l \cdot h_n = 0 \quad (9c)$$

where $m = 0,\ 1,\ ...,\ (N-2)/2$, $l = 0,\ 1,\ ...,\ L-1$, $\boldsymbol{h} = [h_0\ h_1\ ...\ h_{N-1}]^T$ and $\boldsymbol{Q}$ is a constant symmetric positive definite Toeplitz matrix characterized by its first row $[\pi - \omega_a\ \ -\sin\omega_a\ ...\ -\sin(N-1)\omega_a/(N-1)]$. Evidently,this is a POP with $N/2 + L$ constraints, and the maximum order of all the polynomials involved is two.

For low-order filter banks, toolbox GloptiPoly 3.4 was found to work well. For example, with $N = 6$, $L = 2$ and $\omega_a = 0.56\pi$, the software produces four globally optimal impulse responses as

$$\boldsymbol{h}_{\text{LS}}^{(6,2)} = \begin{bmatrix} 0.33268098788629 \\ 0.80689591454849 \\ 0.45986215652386 \\ -0.13501431772967 \\ -0.08543638600240 \\ 0.03522516035714 \end{bmatrix}$$

$-\boldsymbol{h}_{\text{LS}}^{(6,2)}$, $\texttt{flipud}(\boldsymbol{h}_{\text{LS}}^{(6,2)})$ and $-\texttt{flipud}(\boldsymbol{h}_{\text{LS}}^{(6,2)})$ where $\texttt{flipud}(\boldsymbol{h})$ denotes a vector generated by flipping vector $\boldsymbol{h}$ upside down. We remark that the above four impulse responses satisfy constraints (9b) and (9c) and they yield the same minimum objective function value as $\boldsymbol{h}_{\text{LS}}^{(6,2)^T} \boldsymbol{Q} \boldsymbol{h}_{\text{LS}}^{(6,2)} = 0.173458$. Also note that $\boldsymbol{h}_{\text{LS}}^{(6,2)}$ (and $-\boldsymbol{h}_{\text{LS}}^{(6,2)}$) possess minimum phase as no zeros of their corresponding transfer functions are outside the unit circle. Unfortunately, the software fails to work as long as the filter length N is greater than or equal to 10. On the other hand, toolbox SparsePOP2000 was found to work for global design of filter banks up to $N = 16$. A technical problem with SparsePOP2000 is that, unlike GloptiPoly 3.4 being able to produce multiple global solutions, it requires to set a lower bound and an upper bound for the impulse response and only one global solution that falls within the bounds will be generated. Our design experiences suggest that the following bounds work well:

$$\boldsymbol{h_d} - 0.5\boldsymbol{e} \le \boldsymbol{h} \le \boldsymbol{h_d} + 0.5\boldsymbol{e} \quad (10)$$

where $\boldsymbol{h_d}$ is the impulse response of the length-N Daubechies filter [14] and $\boldsymbol{e}$ is an $N \times 1$ all-one vector.

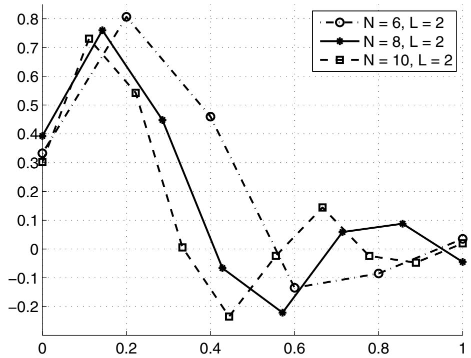

Figure 2. Pattern of LS impulse responses with different length N.

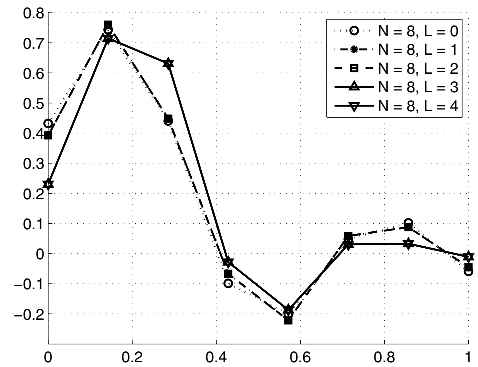

Figure 3. Pattern of impulse response with various VMs L.

3.2. LS design of high-order filter banks

In this section, we address LS designs with length N too high for the above mentioned software to handle.

3.2.1. Pattern of impulse responses of globally optimal filter banks

Although the current versions of the software examined earlier are of limited use, it turns out that observations made on the pattern of the impulse responses of low-order designs do provide useful clues for tackling the design of high-order CQ filters. Our observations are illustrated in Figs. 2 and 3. Shown in Fig. 2 are the impulse responses of globally optimal minimum-phase lowpass CQ filters of lengths $N = 6,\ 8$ and 10 (all with $L = 2$) obtained using SparsePOP2000 where the impulse responses are plotted over normalized interval $[0,\ 1]$ for better comparison. From the figure, it is clear that these impulse responses are distinctly different from each other. Nevertheless, it is equally clear that they exhibit a similar pattern: it starts with a short uphill to peak, then goes down to components of small values. In addition, viewing each impulse response as a curve (function), we see that the nearest neighbor to a given curve associated with filter length N is the curve associated with length $N + 2$. Furthermore, for a fixed filter length N the impulse responses of globally optimal CQ filters with various VMs are clustered and exhibit a pattern similar to that in Fig. 2. As an example, Fig. 3 shows the impulse responses of lowpass CQ filters with $N = 8$ and $L = 0,\ 1,\ 2,\ 3,\ 4$ obtained using either GloptiPoly 3.4 or SparsePOP2000.

3.2.2. A design strategy

Both the LS and minimax designs of CQ filters as formulated in (5) and (6) are nonconvex problems that possess multiple local solutions, and several (local) design techniques for CQ filters are available in the literature [2], [10]-[14]. A recent addition to this rich field of research is a direct design method [1] that deals with problems (5) and (6) by local convex approximations in a sequential manner, and the method is shown to produce satisfactory results.

Taking the above analysis into account, the situation facing the designer may be summarized as follows: (i) global designs of CQ filters are possible by using the methods of [6][7], but only for short filter lengths; (ii) a common pattern exists among the impulse responses of globally optimal lowpass minimum-phase CQ filters of short lengths and, the optimal impulse response of length $N + 2$ falls within a small vicinity of the optimal impulse response of length N; and (iii) a sequential design method that requires a reasonable initial design for producing a locally optimal design is within reach. A strategy for the design of minimum-phase CQ filters of a long (even) length N is developed based on these observations, and can be described in steps as follows:

1. Set an initial working filter length N_w, say, to 4, and design a globally optimal, minimum-phase, CQ filter of length N_w using e.g. GloptiPoly 3.4. Denote the impulse response obtained by $\boldsymbol{h}_w$.

2. Generate a length-($N_w + 2$) interpolated version of $\boldsymbol{h}_w$ by e.g. linear interpolation. Denote the interpolated vector by $\boldsymbol{h}_{wi}$.

3. Apply the method of [1] with $\boldsymbol{h}_{wi}$ as its initial point (impulse response) to design an optimal CQ filter of length $N_w + 2$. Denote the impulse response obtained by $\boldsymbol{h}_w$.

4. If $N = N_w + 2$, output $\boldsymbol{h}_w$ as the optimal design and terminate; otherwise, set $N_w = N_w + 2$ and repeat from Step 2.

Although no claim about the global optimality of the above design methodology can be made for large N, the designs obtained are quite likely to be globally optimal because in each round of iteration the initial point is sufficiently close to the global minimizer and the algorithm in [1] is known to converge to a nearby minimizer. In the next section, we provide experimental evidences that support this analysis.

3.3. Design examples and performance evaluation

3.3.1. Performance of the proposed method for low-order designs

The method described in Sec. 3.2.2 was applied to design lowpass minimum-phase CQ filters of length $N = 6, 8, ..., 16$. For all designs, the normalized stopband edge was set to $\omega_a = 0.56\pi$ and the number of VMs was set to $L = 1$. In each design, toolbox GloptiPoly 3.4 was applied only once to (9) to generate a globally optimal minimum-phase impulse response with $N = 4$ and $L = 1$, denoted by $\boldsymbol{h}_{\text{LS}}^{(4,1)}$. In the case of $N = 6$, $\boldsymbol{h}_{\text{LS}}^{(4,1)}$ was linearly interpolated to length 6 and then used as the initial point to run the LS design algorithm in [1], and the impulse response obtained is denoted by $\hat{\boldsymbol{h}}_{\text{LS}}^{(6,1)}$. In the case of $N = 8$, we first obtain impulse response $\hat{\boldsymbol{h}}_{\text{LS}}^{(6,1)}$ as above; then linearly interpolate $\hat{\boldsymbol{h}}_{\text{LS}}^{(6,1)}$ to length 8 and use it as the initial point to run LS the algorithm in [1] to generate impulse response $\hat{\boldsymbol{h}}_{\text{LS}}^{(8,1)}$. The designs for $N = 10, ..., 16$ were carried out in a similar manner to produce impulse responses $\hat{\boldsymbol{h}}_{\text{LS}}^{(N,1)}$. For comparison purposes, globally optimal impulse responses $\boldsymbol{h}_{\text{LS}}^{(N,1)}$ for $N = 6, 8, ..., 16$ were obtained by using GloptiPoly 3.4 or SparsePOP2000. It was found that $\hat{\boldsymbol{h}}_{\text{LS}}^{(N,1)}$ and $\boldsymbol{h}_{\text{LS}}^{(N,1)}$ were practically identical for all even N from 6 to 16. We also remark that with the starting impulse response $\boldsymbol{h}_{\text{LS}}^{(4,1)}$ having minimum phase, the CQ filters so designed all have minimum phase, a desirable property for digital filters to be of practical use.

3.3.2. Performance of the proposed method for high-order designs

Supported by the design concept verification in Sec. 3.3.1, we proceed to apply the proposed method to design high-order lowpass CQ filters with length N up to 96. As an example, Fig. 4 shows the magnitude response of the CQ lowpass filter designed by the proposed method with $N = 96$, $L = 3$, and $\omega_a = 0.56\pi$. The energy of the filter over stopband, i.e. the value of the objective function $\boldsymbol{h}^T\boldsymbol{Q}\boldsymbol{h}$ in (9a), was found to be $E_{\text{LS}}^{(96,3)} = 1.185993 \times 10^{-9}$. For comparison, a CQ filter with the same design specifications, i.e. $N = 96$, $L = 3$, and $\omega_a = 0.56\pi$ was designed using the LS algorithm of [1]. The initial point used in the design was a linear-phase lowpass filter obtained by the conventional window-based technique. The stopband energy of the CQ filter obtained was found to be $\tilde{E}_{\text{LS}}^{(96,3)} = 1.309040 \times 10^{-9}$– a 10% increase compared with $E_{\text{LS}}^{(96,3)}$. Like the low-order designs, it was found that all high-order CQ filters produced possess minimum phase. The additional cost of the proposed method is that it requires the algorithm in [1] to be executed repeatedly until the desired filter order is reached.

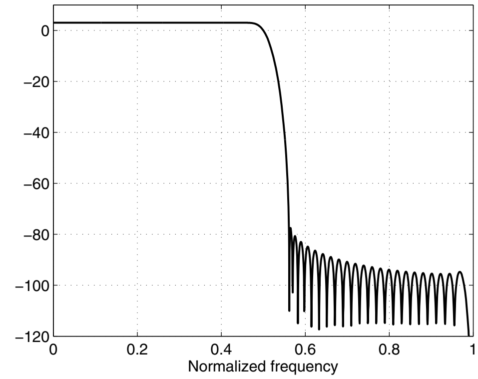

Figure 4. Magnitude response of LS $H_0(z)$ with $N = 96$, $L = 3$ and $\omega_a = 0.56\pi$.

4. MINIMAX DESIGNS

4.1. Global minimax design of low-order filter banks

The problem in (6) can be formulated as

$$\begin{aligned} &\text{minimize} && \eta && \text{(11a)}\\ &\text{subject to:} && \eta^2 - |H_0(e^{j\omega})|^2 \geq 0 \ \text{ for } \omega \in \Omega && \text{(11b)}\\ & && \sum_{n=0}^{N-1-2m} h_n \cdot h_{n+2m} = \delta_m && \text{(11c)}\\ & && \sum_{n=0}^{N-1} (-1)^n \cdot n^l \cdot h_n = 0 && \text{(11d)} \end{aligned}$$

where $m = 0, 1, ..., (N-2)/2$, $l = 0, 1, ..., L-1$ and $\Omega = \{\omega_a = \omega_1, \omega_2, ..., \omega_K = \pi\}$ is a set of K uniformly distributed frequency grids over stopband $[\omega_a, \pi]$. Define $\boldsymbol{c}(\omega) = [1 \ \cos\omega \ ... \ \cos(N-1)\omega]^T$ and $\boldsymbol{s}(\omega) = [0 \ \sin\omega \ ... \ \sin(N-1)\omega]^T$, we can write $|H_0(e^{j\omega})|^2$ in (11b) as

$$|H_0(e^{j\omega})|^2 = [\boldsymbol{h}^T\boldsymbol{c}(\omega)]^2 + [\boldsymbol{h}^T\boldsymbol{s}(\omega)]^2 \qquad (12)$$

Thus, this is a POP with $K + N/2 + L$ constraints and the maximum order of all the polynomials involved is two.

Toolbox GloptiPoly 3.4 was found to work for CQ filter of order 4. With $N = 4$, $L = 1$, $\omega_a = 0.56\pi$, and $\Omega = \{\omega_a, \omega_a + 0.025\pi, \omega_a + 0.05\pi, ..., \pi\}$ (which gives $K = 18$), the toolbox was able to produce four globally optimal impulse responses as

$$\boldsymbol{h}_{\text{minimax}}^{(4,1)} = \begin{bmatrix} 0.48296282173531 \\ 0.83651623138234 \\ 0.22414405492402 \\ -0.12940935473280 \end{bmatrix}$$

$-\boldsymbol{h}_{\text{minimax}}^{(4,1)}$, $\texttt{flipud}(\boldsymbol{h}_{\text{minimax}}^{(4,1)})$ and $-\texttt{flipud}(\boldsymbol{h}_{\text{minimax}}^{(4,1)})$. The maximum instantaneous energy over stopband for the above four impulse responses was found to be the same value $\eta^2 = 0.722218$. We also observed that $\boldsymbol{h}_{\text{minimax}}^{(4,1)}$ (and $-\boldsymbol{h}_{\text{minimax}}^{(4,1)}$) possess minimum phase. However, GloptiPoly 3.4 failed to

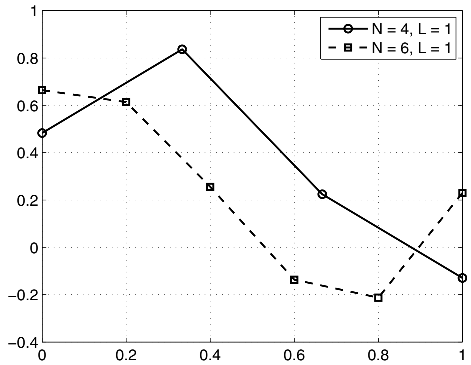

Figure 5. Pattern of minimax impulse responses with different length N.

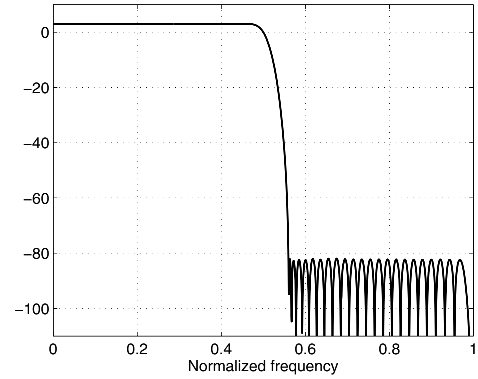

Figure 6. Magnitude response of minimax $H_0(z)$ with $N = 96, L = 3$ and $\omega_a = 0.56\pi$.

work for N as small as 6 because of the relatively large number of constraints in the minimax design. On the other hand, with the same bounds set for $\boldsymbol{h}$ as in (10) and $0 \leq \eta \leq 2$, SparsePOP2000 was able to produce global minimax designs for $N = 4$ and 6.

4.2. Minimax design of high-order filter banks

Like the observations made on the LS designs (see Sec. 3.2.1), globally optimal (in minimax sense) impulse responses obtained in Sec. 4.1 appear to exhibit a pattern similar to that in the LS case, as can be seen in Fig. 5. It is therefore natural to follow the strategy described in Sec. 3.2.2 for minimax designs of high-order filter banks.

4.3. Design examples and performance evaluation

4.3.1. Performance of the proposed method for a low-order design

The design strategy in Sec. 3.2.2 was applied to design lowpass minimum-phase minimax CQ filters. We set $\omega_a = 0.56\pi$, $L = 1$, and Ω contains $K = 110$ frequency grids. GloptiPoly 3.4 was applied to (11) to generate a globally optimal minimum-phase impulse response with $N = 4$ and $L = 1$, denoted by $\boldsymbol{h}_{\text{minimax}}^{(4,1)}$. Impulse response $\boldsymbol{h}_{\text{minimax}}^{(4,1)}$ was then linearly interpolated to length 6 and used as the initial point to run the minimax algorithm of [1]. The impulse response obtained is denoted by $\hat{\boldsymbol{h}}_{\text{minimax}}^{(6,1)}$. For comparison, SparsePOP2000 was applied to (11) to generate the globally optimal impulse response $\boldsymbol{h}_{\text{minimax}}^{(6,1)}$. The two impulse responses, $\hat{\boldsymbol{h}}_{\text{minimax}}^{(6,1)}$ and $\boldsymbol{h}_{\text{minimax}}^{(6,1)}$ were found to be practically identical, giving a support to our design concept for the minimax designs. We also note that with the starting impulse response $\boldsymbol{h}_{\text{minimax}}^{(4,1)}$ having minimum phase, the CQ filter $\hat{\boldsymbol{h}}_{\text{minimax}}^{(6,1)}$ obtained possesses minimum phase as well.

4.3.2. Performance of the proposed method for high-order designs

Following the design approach in Sec. 4.3.1, $\hat{\boldsymbol{h}}_{\text{minimax}}^{(6,1)}$ was interpolated to length 8 and used as the initial point for the minimax algorithm of [1] to generate $\hat{\boldsymbol{h}}_{\text{minimax}}^{(8,1)}$. Such process was repeated and high-order lowpass minimax CQ filters with length N up to 96 were designed. The filters obtained tend to be equiripple. As an example, Fig. 6 shows the magnitude response of the lowpass minimax CQ filter designed by the proposed method with $N = 96$, $L = 3$, $\omega_a = 0.56\pi$ and Ω containing $K = 110$ uniformly distributed frequency grids. The maximum instantaneous energy η^2 over stopband was found to be $E_{\text{minimax}}^{(96,3)} = 6.362729 \times 10^{-9}$. For comparison, a minimax CQ filter with the same specifications was designed using the minimax algorithm of [1]. The initial point used in the design was a linear-phase lowpass filter obtained by the conventional window-based technique. The maximum instantaneous energy over stopband of the CQ filter was found to be $\tilde{E}_{\text{minimax}}^{(96,3)} = 7.265100 \times 10^{-9}$– a 14% increase compared with $E_{\text{minimax}}^{(96,3)}$. As in the low-order designs, we also found that all high-order minimax CQ filters produced by the proposed method possess minimum phase. The additional cost of the method is due to the repetitive executions of the algorithm in [1] before the desired filter order is reached.

5. CONCLUSION

A new method for the design of two-channel orthogonal filter banks and wavelets has been proposed. Attempting to develop a methodology for global design of CQ filters, the proposed method is built on some recent progress in global polynomial optimization and direct design techniques for CQ filters, in conjunction with several critical observations on the globally optimal impulse responses and a progressive design procedure in terms of filter length. Several design examples have been presented to verify the design concept and evaluate the performance of the proposed algorithms.

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