MINIMIZATION OF L_2-SENSITIVITY FOR 2-D STATE-SPACE DIGITAL FILTERS SUBJECT TO L_2-SCALING CONSTRAINTS

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ABSTRACT

A new approach to the problem of minimizing L_2-sensitivity subject to L_2-norm scaling constraints for two-dimensional (2-D) state-space digital filters is proposed. Using linear-algebraic techniques, the problem at hand is converted into an unconstrained optimization problem, and the unconstrained problem obtained is then solved by applying an efficient quasi-Newton algorithm. Computer simulation results are presented to illustrate the effectiveness of the proposed technique.

I. INTRODUCTION

In the implementation of a fixed-point state-space digital filter with finite word length (FWL), the efficiency and performance of the filter are directly affected by the choice of its state-space filter structure. When a transfer function with infinite accuracy coefficients is designed and realized by a state-space model, the coefficients in the state-space model must be truncated or rounded to fit the FWL constraints. This coefficient quantization usually alters the characteristics of the filter. This motivates the study of the coefficient sensitivity minimization problem. For 2-D state-space digital filters, the L_1/L_2-sensitivity minimization problem [1]-[5] and L_2-sensitivity minimization problem [6]-[10] have been investigated. However, to the best knowledge of the authors, there is no study for the minimization of the L_2-sensitivity subject to the L_2-norm dynamic-range scaling constraints for 2-D state-space digital filters, although it has been known that the use of scaling constraints can be beneficial for suppressing overflow oscillation [11],[12].

This paper investigates the problem of minimizing L_2-sensitivity subject to L_2-norm dynamic-range scaling constraints for 2-D state-space digital filters. To this end, we introduce an expression for evaluating the L_2-sensitivity and formulates the L_2-sensitivity minimization problem subject to the scaling constraints. The constrained optimization problems at hand is then converted into an unconstrained optimization problem by using linear-algebraic techniques, and the unconstrained optimization problems obtained is solved using an efficient quasi-Newton algorithm [13].

II. L_2-SENSITIVITY ANALYSIS

Consider the following local state-space (LSS) model $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)_{m,n}$ for 2-D IIR digital filters [14]:

$$\begin{bmatrix} \boldsymbol{x}_{11}(i,j) \\ y(i,j) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c} & d \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(i,j) \\ u(i,j) \end{bmatrix} \tag{1}$$

where

$$\boldsymbol{x}_{11}(i,j) = \begin{bmatrix} \boldsymbol{x}^h(i+1,j) \\ \boldsymbol{x}^v(i,j+1) \end{bmatrix}, \quad \boldsymbol{x}(i,j) = \begin{bmatrix} \boldsymbol{x}^h(i,j) \\ \boldsymbol{x}^v(i,j) \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{A}_3 & \boldsymbol{A}_4 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix}, \quad \boldsymbol{c} = [\ \boldsymbol{c}_1 \quad \boldsymbol{c}_2\ ]$$

and $\boldsymbol{x}^h(i,j)$ is an $m \times 1$ horizontal state vector, $\boldsymbol{x}^v(i,j)$ is an $n \times 1$ vertical state vector, $u(i,j)$ is a scalar input, $y(i,j)$ is a scalar output, and $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{A}_3$, $\boldsymbol{A}_4$, $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{c}_1$, $\boldsymbol{c}_2$, and d are real constant matrices of appropriate dimensions. The LSS model (1) is assumed to be BIBO stable, separately locally controllable and observable [15]. Taking the 2-D z-transform on the LSS model (1) yields

$$\begin{aligned} \boldsymbol{F}(z_1,z_2) &= (\boldsymbol{Z}-\boldsymbol{A})^{-1}\boldsymbol{b}, \quad \boldsymbol{G}(z_1,z_2) = \boldsymbol{c}(\boldsymbol{Z}-\boldsymbol{A})^{-1} \\ H(z_1,z_2) &= \boldsymbol{c}(\boldsymbol{Z}-\boldsymbol{A})^{-1}\boldsymbol{b} + d, \qquad \boldsymbol{Z} = z_1\boldsymbol{I}_m \oplus z_2\boldsymbol{I}_n. \end{aligned} \tag{2}$$

Definition 1: Let $\boldsymbol{X}$ be an $m \times n$ real matrix and let $f(\boldsymbol{X})$ be a scalar complex function of $\boldsymbol{X}$, differentiable with respect to all the entries of $\boldsymbol{X}$. The sensitivity function of f with respect to $\boldsymbol{X}$ is then defined as

$$\boldsymbol{S}_{\boldsymbol{X}} = \frac{\partial f}{\partial \boldsymbol{X}}, \qquad (\boldsymbol{S}_{\boldsymbol{X}})_{ij} = \frac{\partial f}{\partial x_{ij}} \tag{3}$$

where x_{ij} denotes the (i,j)th entry of matrix $\boldsymbol{X}$.

From *Definition 1*, the sensitivities of $H(z_1, z_2)$ with respect to the coefficient matrices in (1) are computed as

$$\begin{aligned} \frac{\partial H(z_1,z_2)}{\partial \boldsymbol{A}} &= [\boldsymbol{F}(z_1,z_2)\boldsymbol{G}(z_1,z_2)]^T \\ \frac{\partial H(z_1,z_2)}{\partial \boldsymbol{b}} &= \boldsymbol{G}^T(z_1,z_2) \\ \frac{\partial H(z_1,z_2)}{\partial \boldsymbol{c}^T} &= \boldsymbol{F}(z_1,z_2)\ . \end{aligned} \tag{4}$$

Definition 2: Let $\boldsymbol{X}(z_1, z_2)$ be an $m \times n$ complex matrix valued function of the complex variables z_1 and z_2. The L_2 norm of $\boldsymbol{X}(z_1, z_2)$ is then defined as

$$\begin{aligned} &\|\boldsymbol{X}(z_1,z_2)\|_2 \\ &= \left(\mathrm{tr}\left[\frac{1}{(2\pi j)^2}\oint_{\Gamma_1}\oint_{\Gamma_2}\boldsymbol{X}(z_1,z_2)\boldsymbol{X}^*(z_1,z_2)\frac{dz_1dz_2}{z_1z_2}\right]\right)^{\frac{1}{2}} \end{aligned} \tag{5}$$

where $\Gamma_i = \{z_i : |z_i| = 1\}$ for $i = 1, 2$.

From (4) and *Definition 2*, the overall L_2-sensitivity measure for the LSS model in (1) is defined by

$$\begin{aligned} S_{2D} &= \left\|\frac{\partial H(z_1,z_2)}{\partial \boldsymbol{A}}\right\|_2^2 + \left\|\frac{\partial H(z_1,z_2)}{\partial \boldsymbol{b}}\right\|_2^2 \\ &\quad + \left\|\frac{\partial H(z_1,z_2)}{\partial \boldsymbol{c}^T}\right\|_2^2 \\ &= \left\|[\boldsymbol{F}(z_1,z_2)\boldsymbol{G}(z_1,z_2)]^T\right\|_2^2 + \left\|\boldsymbol{G}^T(z_1,z_2)\right\|_2^2 \\ &\quad + \|\boldsymbol{F}(z_1,z_2)\|_2^2\,. \end{aligned} \tag{6}$$

The L_2-sensitivity measure in (6) can be written as

$$S_{2D} = \mathrm{tr}[\boldsymbol{M}_A] + \mathrm{tr}[\boldsymbol{W}_o] + \mathrm{tr}[\boldsymbol{K}_c] \tag{7}$$

where

$$\begin{aligned} \boldsymbol{M}_A &= \frac{1}{(2\pi j)^2}\oint_{\Gamma_1}\oint_{\Gamma_2}[\boldsymbol{F}(z_1,z_2)\boldsymbol{G}(z_1,z_2)]^T \\ &\quad \cdot\boldsymbol{F}(z_1^{-1},z_2^{-1})\boldsymbol{G}(z_1^{-1},z_2^{-1})\frac{dz_1dz_2}{z_1z_2} \\ \boldsymbol{W}_o &= \frac{1}{(2\pi j)^2}\oint_{\Gamma_1}\oint_{\Gamma_2}\boldsymbol{G}^T(z_1,z_2)\boldsymbol{G}(z_1^{-1},z_2^{-1})\frac{dz_1dz_2}{z_1z_2} \\ \boldsymbol{K}_c &= \frac{1}{(2\pi j)^2}\oint_{\Gamma_1}\oint_{\Gamma_2}\boldsymbol{F}(z_1,z_2)\boldsymbol{F}^T(z_1^{-1},z_2^{-1})\frac{dz_1dz_2}{z_1z_2}. \end{aligned}$$

Letting

$$\begin{aligned} \boldsymbol{F}(z_1,z_2) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\boldsymbol{f}(i,j)z_1^{-i}z_2^{-j} \\ \boldsymbol{G}(z_1,z_2) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\boldsymbol{g}(i,j)z_1^{-i}z_2^{-j}, \end{aligned} \tag{8}$$

2-D Gramians $\boldsymbol{M}_A$, $\boldsymbol{K}_c$, and $\boldsymbol{W}_o$ can be written as

$$\begin{aligned} \boldsymbol{M}_A &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\boldsymbol{H}^T(i,j)\boldsymbol{H}(i,j) \\ \boldsymbol{K}_c &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\boldsymbol{f}(i,j)\boldsymbol{f}^T(i,j) \\ \boldsymbol{W}_o &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\boldsymbol{g}^T(i,j)\boldsymbol{g}(i,j) \end{aligned} \tag{9}$$

where

$$\boldsymbol{H}(i,j) = \sum\sum_{(0,0)\le(k,r)<(i,j)}\boldsymbol{f}(k,r)\boldsymbol{g}(i-k,j-r).$$

Let a 2-D coordinate transformation be defined by

$$\overline{\boldsymbol{x}}(i,j) = \boldsymbol{T}^{-1}\boldsymbol{x}(i,j) \tag{10}$$

where $\boldsymbol{T} = \boldsymbol{T}_1 \oplus \boldsymbol{T}_4$ with nonsingular $m \times m$ $\boldsymbol{T}_1$ and $n \times n$ $\boldsymbol{T}_4$ matrices. Then the LSS model in (1) is e-quivalent to the new realization $(\overline{\boldsymbol{A}}, \overline{\boldsymbol{b}}, \overline{\boldsymbol{c}}, d)_{m,n}$ which is characterized by

$$\overline{\boldsymbol{A}} = \boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T}, \quad \overline{\boldsymbol{b}} = \boldsymbol{T}^{-1}\boldsymbol{b}, \quad \overline{\boldsymbol{c}} = \boldsymbol{c}\boldsymbol{T} \tag{11}$$

in the sense that the transfer function $H(z_1, z_2)$ remain-s invariant under such a transformation.

Applying the coordinate transformation in (10) to the LSS model in (1), we can change (7) to

$$S_{2D}(\boldsymbol{T}) = \mathrm{tr}[\overline{\boldsymbol{M}}_A] + \mathrm{tr}[\overline{\boldsymbol{W}}_o] + \mathrm{tr}[\overline{\boldsymbol{K}}_c] \tag{12}$$

where

$$\begin{aligned} \overline{\boldsymbol{M}}_A &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\boldsymbol{T}^T\boldsymbol{H}^T(i,j)\boldsymbol{T}^{-T}\boldsymbol{T}^{-1}\boldsymbol{H}(i,j)\boldsymbol{T} \\ \overline{\boldsymbol{W}}_o &= \boldsymbol{T}^T\boldsymbol{W}_o\boldsymbol{T}, \qquad \overline{\boldsymbol{K}}_c = \boldsymbol{T}^{-1}\boldsymbol{K}_c\boldsymbol{T}^{-T} \end{aligned}$$

with $\boldsymbol{T} = \boldsymbol{T}_1 \oplus \boldsymbol{T}_4$. If the L_2-norm dynamic-range scaling constraints are imposed on $\overline{\boldsymbol{x}}(i, j)$, then

$$\begin{aligned} (\overline{\boldsymbol{K}}_{1c})_{ii} &= (\boldsymbol{T}_1^{-1}\boldsymbol{K}_{1c}\boldsymbol{T}_1^{-T})_{ii} = 1 \\ (\overline{\boldsymbol{K}}_{4c})_{jj} &= (\boldsymbol{T}_4^{-1}\boldsymbol{K}_{4c}\boldsymbol{T}_4^{-T})_{jj} = 1 \end{aligned} \tag{13}$$

are required for $i = 1, 2, \cdots, m$ and for $j = 1, 2, \cdots, n$. Here, $m \times m$ matrix $\boldsymbol{K}_{1c}$ and $n \times n$ matrix $\boldsymbol{K}_{4c}$ are symmetric and positive-definite, and obtained by partitioning matrix $\boldsymbol{K}_c$ as

$$\boldsymbol{K}_c = \begin{bmatrix} \boldsymbol{K}_{1c} & \boldsymbol{K}_{2c} \\ \boldsymbol{K}_{3c} & \boldsymbol{K}_{4c} \end{bmatrix}.$$

The problem here is to obtain $(m + n) \times (m + n)$ nonsingular matrix $\boldsymbol{T} = \boldsymbol{T}_1 \oplus \boldsymbol{T}_4$ which minimizes (12) subject to the scaling constraints in (13).

III. L_2-SENSITIVITY MINIMIZATION

Since (1) is stable and separately locally controllable, $\boldsymbol{K}_{ic}$ for $i = 1, 4$ are symmetric and positive-definite [15]. Thus, $\boldsymbol{K}_{ic}^{1/2}$ satisfying $\boldsymbol{K}_{ic} = \boldsymbol{K}_{ic}^{1/2}\boldsymbol{K}_{ic}^{1/2}$ are also symmetric and positive-definite for $i = 1, 4$. Defining

$$\begin{aligned}\hat{\boldsymbol{T}} &= \hat{\boldsymbol{T}}_1 \oplus \hat{\boldsymbol{T}}_4 \\ &= (\boldsymbol{T}_1 \oplus \boldsymbol{T}_4)^T(\boldsymbol{K}_{1c} \oplus \boldsymbol{K}_{4c})^{-\frac{1}{2}}\end{aligned} \tag{14}$$

it is follows that

$$\overline{\boldsymbol{K}}_c = \hat{\boldsymbol{T}}^{-T}\begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{K}_{1c}^{-\frac{1}{2}}\boldsymbol{K}_{2c}\boldsymbol{K}_{4c}^{-\frac{1}{2}} \\ \boldsymbol{K}_{4c}^{-\frac{1}{2}}\boldsymbol{K}_{3c}\boldsymbol{K}_{1c}^{-\frac{1}{2}} & \boldsymbol{I}_n \end{bmatrix}\hat{\boldsymbol{T}}^{-1}. \tag{15}$$

Thus, the scaling constraints in (13) can be written as

$$\begin{aligned}(\hat{\boldsymbol{T}}_1^{-T}\hat{\boldsymbol{T}}_1^{-1})_{ii} = 1, &\qquad i = 1, 2, \cdots, m \\ (\hat{\boldsymbol{T}}_4^{-T}\hat{\boldsymbol{T}}_4^{-1})_{jj} = 1, &\qquad j = 1, 2, \cdots, n.\end{aligned} \tag{16}$$

Note that the conditions in (16) are always satisfied by assuming $\hat{\boldsymbol{T}}_i^{-1}$ for $i = 1, 4$ as the forms

$$\begin{aligned}\hat{\boldsymbol{T}}_1^{-1} &= \left[\frac{\boldsymbol{t}_{11}}{||\boldsymbol{t}_{11}||}, \frac{\boldsymbol{t}_{12}}{||\boldsymbol{t}_{12}||}, \cdots, \frac{\boldsymbol{t}_{1m}}{||\boldsymbol{t}_{1m}||}\right] \\ \hat{\boldsymbol{T}}_4^{-1} &= \left[\frac{\boldsymbol{t}_{41}}{||\boldsymbol{t}_{41}||}, \frac{\boldsymbol{t}_{42}}{||\boldsymbol{t}_{42}||}, \cdots, \frac{\boldsymbol{t}_{4n}}{||\boldsymbol{t}_{4n}||}\right].\end{aligned} \tag{17}$$

It follows from (14) that (12) is changed to

$$\begin{aligned}J(\hat{\boldsymbol{T}}) = \mathrm{tr}&\left[\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\hat{\boldsymbol{T}}\hat{\boldsymbol{H}}^T(i,j)\hat{\boldsymbol{T}}^{-1}\hat{\boldsymbol{T}}^{-T}\hat{\boldsymbol{H}}(i,j)\hat{\boldsymbol{T}}^T\right] \\ &+\mathrm{tr}[\hat{\boldsymbol{T}}\hat{\boldsymbol{W}}_o\hat{\boldsymbol{T}}^T] + \mathrm{tr}[\hat{\boldsymbol{T}}^{-T}\hat{\boldsymbol{K}}_c\hat{\boldsymbol{T}}^{-1}]\end{aligned} \tag{18}$$

where

$$\begin{aligned}\hat{\boldsymbol{T}} &= \hat{\boldsymbol{T}}_1 \oplus \hat{\boldsymbol{T}}_4 \\ \hat{\boldsymbol{H}}(i,j) &= (\boldsymbol{K}_{1c} \oplus \boldsymbol{K}_{4c})^{-\frac{1}{2}}\boldsymbol{H}(i,j)(\boldsymbol{K}_{1c} \oplus \boldsymbol{K}_{4c})^{\frac{1}{2}} \\ \hat{\boldsymbol{W}}_o &= (\boldsymbol{K}_{1c} \oplus \boldsymbol{K}_{4c})^{\frac{1}{2}}\boldsymbol{W}_o(\boldsymbol{K}_{1c} \oplus \boldsymbol{K}_{4c})^{\frac{1}{2}} \\ \hat{\boldsymbol{K}}_c &= \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{K}_{1c}^{-\frac{1}{2}}\boldsymbol{K}_{2c}\boldsymbol{K}_{4c}^{-\frac{1}{2}} \\ \boldsymbol{K}_{4c}^{-\frac{1}{2}}\boldsymbol{K}_{3c}\boldsymbol{K}_{1c}^{-\frac{1}{2}} & \boldsymbol{I}_n \end{bmatrix}.\end{aligned}$$

The constrained optimization problem of obtaining $(m+n) \times (m+n)$ nonsingular matrix $\boldsymbol{T} = \boldsymbol{T}_1 \oplus \boldsymbol{T}_4$ which minimizes (12) subject to the constraints in (13) is therefore converted into an unconstrained one of obtaining $(m+n) \times (m+n)$ nonsingular matrix $\hat{\boldsymbol{T}} = \hat{\boldsymbol{T}}_1 \oplus \hat{\boldsymbol{T}}_4$ given by (17) which minimizes (18).

We apply the quasi-Newton algorithm to minimize (18) with respect to matrix $\hat{\boldsymbol{T}} = \hat{\boldsymbol{T}}_1 \oplus \hat{\boldsymbol{T}}_4$ given by (17). Define an $(m^2+n^2) \times 1$ vector $\boldsymbol{x} = (\boldsymbol{x}_1^T, \boldsymbol{x}_4^T)^T$ where $\boldsymbol{x}_1$ and $\boldsymbol{x}_4$ are the column vectors that collect the variables in matrices $\hat{\boldsymbol{T}}_1$ and $\hat{\boldsymbol{T}}_4$, respectively. Then, $J(\hat{\boldsymbol{T}})$ is a function of $\boldsymbol{x}$, denoted by $J(\boldsymbol{x})$. The algorithm starts with a trivial initial point $\boldsymbol{x}_0$ obtained from an initial assignment $\hat{\boldsymbol{T}} = \boldsymbol{I}_{m+n}$ and updates in the kth iteration the most recent point $\boldsymbol{x}_k$ to point $\boldsymbol{x}_{k+1}$ as

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k\boldsymbol{d}_k \tag{19}$$

where [13]

$$\begin{aligned}\boldsymbol{d}_k &= -\boldsymbol{S}_k\nabla J(\boldsymbol{x}_k) \\ \alpha_k &= arg\ \min_{\alpha}\ J(\boldsymbol{x}_k + \alpha\boldsymbol{d}_k) \\ \boldsymbol{S}_{k+1} &= \boldsymbol{S}_k + \left(1 + \frac{\boldsymbol{\gamma}_k^T\boldsymbol{S}_k\boldsymbol{\gamma}_k}{\boldsymbol{\gamma}_k^T\boldsymbol{\delta}_k}\right)\frac{\boldsymbol{\delta}_k\boldsymbol{\delta}_k^T}{\boldsymbol{\gamma}_k^T\boldsymbol{\delta}_k} \\ &\quad -\frac{\boldsymbol{\delta}_k\boldsymbol{\gamma}_k^T\boldsymbol{S}_k + \boldsymbol{S}_k\boldsymbol{\gamma}_k\boldsymbol{\delta}_k^T}{\boldsymbol{\gamma}_k^T\boldsymbol{\delta}_k} \\ \boldsymbol{S}_0 &= \boldsymbol{I}, \qquad \boldsymbol{\delta}_k = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k \\ \boldsymbol{\gamma}_k &= \nabla J(\boldsymbol{x}_{k+1}) - \nabla J(\boldsymbol{x}_k).\end{aligned}$$

Here, $\nabla J(\boldsymbol{x})$ is the gradient of $J(\boldsymbol{x})$ with respect to $\boldsymbol{x}$, and $\boldsymbol{S}_k$ is a positive-definite approximation of the inverse Hessian matrix of $J(\boldsymbol{x})$. This iteration process continues until

$$|J(\boldsymbol{x}_{k+1}) - J(\boldsymbol{x}_k)| < \varepsilon \tag{20}$$

where $\varepsilon > 0$ is a prescribed tolerance. If the iteration is terminated at step k, $\boldsymbol{x}_k$ is viewed as a solution point.

IV. NUMERICAL EXAMPLE

Let a 2-D stable filter with order (2,2) be modeled by

$$\boldsymbol{A} = \begin{bmatrix} 1.888990 & -0.912170 & -0.113998 & 0.0 \\ 1.000022 & 0.0 & 0.0 & 0.0 \\ 0.243074 & -0.226314 & 1.888900 & 0.926310 \\ -0.244323 & 0.230208 & -0.984757 & 0.0 \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 0.023503 & 0.0 & -0.027186 & 0.092669 \end{bmatrix}^T$$

$$\boldsymbol{c} = \begin{bmatrix} 2.692940 & -0.850295 & -0.232816 & 0.0 \end{bmatrix}$$

$$d = 0.870210$$

which satisfies the L_2-norm scaling constraints in (13) and takes the L_2-sensitivity: $S_{2D} = 2.953212 \times 10^5$.

Choosing $\hat{\boldsymbol{T}} = \boldsymbol{I}_m \oplus \boldsymbol{I}_n$ (therefore $\boldsymbol{T}_i = \boldsymbol{K}_{ic}^{1/2}$ for $i = 1, 4$ in (14)) as the initial estimate and $\varepsilon = 10^{-3}$, the proposed quasi-Newton algorithm took 8 iterations to yield the solution

$$\hat{\boldsymbol{T}}_1^{opt} = \begin{bmatrix} 0.989295 & -0.626422 \\ 0.145928 & 0.779484 \end{bmatrix}$$

$$\hat{\boldsymbol{T}}_4^{opt} = \begin{bmatrix} 0.907595 & -0.625332 \\ 0.419846 & 0.780358 \end{bmatrix}$$

and the L_2-sensitivity $J(\hat{\boldsymbol{T}}^{opt}) = 8392.549286$.

If the L_2-sensitivity measure (12) is minimized (without considering the scaling constraints in (13)) by applying the existing method reported in [8],[9] and if the optimal realization is scaled to satisfy (13), then we arrived at the L_2-sensitivity $S_{2D}(\boldsymbol{T}) = 11033.912683$.

VI. CONCLUSION

The minimization problem of L_2-sensitivity of 2-D state-space digital filters subject to L_2-norm scaling constraints has been investigated. It has been shown that the L_2-sensitivity minimization problem subject to the scaling constraints can be converted into an unconstrained one by using linear algebraic techniques. An efficient quasi-Newton algorithm has been applied to solve the unconstrained optimization problem. Our computer simulation results have demonstrated the effectiveness of the proposed technique compared with the existing methods.

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