WEIGHTED L_2 SENSITIVITY MINIMIZATION OF 2-D DISCRETE SYSTEMS

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ABSTRACT

In this paper, the problem of synthesizing the finite-word-length 2-D state-space filter structures with minimum weighted L_2 sensitivity is considered. First, a frequency-weighted sensitivity measure is defined in place of a usual sensitivity measure. This is based on a pure L_2-norm. Next, an iterative procedure is developed for obtaining the optimal coordinate transformation that minimizes the weighted L_2 sensitivity measure. Applying the proposed procedure, the estimate at each iteration can be calculated analytically. Finally, a numerical example is given to illustrate the utility of the proposed technique.

1. INTRODUCTION

Undesirable finite-word-length effects occur in fixed-point implementation of 2-D recursive digital filters. One of them is the deviation of the actual transfer function from the ideal transfer function which is caused by truncation or rounding of the filter coefficients. Coefficient sensitivity of 2-D state-space digital filters has been investigated [1]. Several techniques have been proposed to synthesize 2-D filter structures with low sensitivity [2]–[6]. Some of them have treated all the frequency regions uniformly [2], [3], [6], while the others have studied coefficient sensitivity of a transfer function within a specified frequency range [4], [5]. However, the sensitivity measures used in [4], [5] are based on a mixture of L_1/L_2 norms. In particular, there exists a specific relationship between the weights of the various terms of the measure in [4].

This paper is interested in the sensitivity performance of a 2-D transfer function within a specified range. A frequency weighted sensitivity measure is defined on the basis of a pure L_2 norm. An iterative procedure is developed to obtain the optimal coordinate transformation that minimizes the weighted L_2-sensitivity measure. This procedure is advantageous since the estimate is calculated analytically at each iteration. A numerical example is presented to demonstrate the validity of the proposed technique.

2. PROBLEM FORMULATION

Let a 2-D local state-space (LSS) digital filter be described by

$$\begin{aligned}\begin{bmatrix}\mathbf{x}^h(i+1,j)\\ \mathbf{x}^v(i,j+1)\end{bmatrix} &= \mathbf{A}\begin{bmatrix}\mathbf{x}^h(i,j)\\ \mathbf{x}^v(i,j)\end{bmatrix} + \mathbf{b}u(i,j)\\ y(i,j) &= \mathbf{c}\begin{bmatrix}\mathbf{x}^h(i,j)\\ \mathbf{x}^v(i,j)\end{bmatrix} + du(i,j)\end{aligned} \tag{1}$$

where $\mathbf{x}^h(i,j)$ is an $m\times 1$ horizontal state vector, $\mathbf{x}^v(i,j)$ is an $n\times 1$ vertical state vector, $u(i,j)$ is a scalar input, $y(i,j)$ is a scalar output, and $\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}, d$ are real constant matrices of appropriate dimensions. The LSS model (1) is assumed to be BIBO stable, separately locally controllable, and separately locally observable. Define

$$\begin{aligned}\mathbf{F}(z_1,z_2) &= (\mathbf{S}-\mathbf{A})^{-1}\mathbf{b}\\ \mathbf{G}(z_1,z_2) &= \mathbf{c}(\mathbf{S}-\mathbf{A})^{-1}\\ H(z_1,z_2) &= \mathbf{c}(\mathbf{S}-\mathbf{A})^{-1}\mathbf{b}+d\end{aligned} \tag{2}$$

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where $\mathbf{S} = z_1\mathbf{I}_m \oplus z_2\mathbf{I}_n$, and let the coordinate transformation be specified as

$$\bar{\mathbf{x}}(i,j) = \mathbf{T}^{-1}\mathbf{x}(i,j) \tag{3}$$

where $\mathbf{T} = \mathbf{T}_1 \oplus \mathbf{T}_4$ and $\mathbf{T}_1(\mathbf{T}_4)$ is an $m\times m(n\times n)$ nonsingular matrix. Then an algebraically equivalent realization $(\bar{\mathbf{A}}, \bar{\mathbf{b}}, \bar{\mathbf{c}}, d)_{m,n}$ is given by

$$\bar{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}, \quad \bar{\mathbf{b}} = \mathbf{T}^{-1}\mathbf{b}, \quad \bar{\mathbf{c}} = \mathbf{c}\mathbf{T}. \tag{4}$$

Definition 1: Let $\mathbf{X}$ be an $m \times n$ real matrix and let $f(\mathbf{X})$ be a scalar complex function of $\mathbf{X}$, differentiable w.r.t. all the entries of $\mathbf{X}$. The sensitivity function of f w.r.t. $\mathbf{X}$ is then defined as

$$\mathbf{S}_X = \frac{\partial f}{\partial \mathbf{X}} \quad \text{with} \quad (\mathbf{S}_X)_{ij} = \frac{\partial f}{\partial x_{ij}} \tag{5}$$

where x_{ij} denotes the (i, j)th entry of matrix $\mathbf{X}$.

It follows that

$$\begin{aligned}\frac{\partial H(z_1,z_2)}{\partial \mathbf{A}} &= \mathbf{G}^t(z_1,z_2)\mathbf{F}^t(z_1,z_2)\\ \frac{\partial H(z_1,z_2)}{\partial \mathbf{b}} &= \mathbf{G}^t(z_1,z_2)\\ \frac{\partial H(z_1,z_2)}{\partial \mathbf{c}^t} &= \mathbf{F}(z_1,z_2)\end{aligned} \tag{6}$$

The weighted sensitivity functions are defined as

$$\begin{aligned}\frac{\delta H(z_1,z_2)}{\delta \mathbf{A}} &= W_A(z_1,z_2)\frac{\partial H(z_1,z_2)}{\partial \mathbf{A}}\\ \frac{\delta H(z_1,z_2)}{\delta \mathbf{b}} &= W_B(z_1,z_2)\frac{\partial H(z_1,z_2)}{\partial \mathbf{b}}\\ \frac{\delta H(z_1,z_2)}{\delta \mathbf{c}^t} &= W_C(z_1,z_2)\frac{\partial H(z_1,z_2)}{\partial \mathbf{c}^t}\end{aligned} \tag{7}$$

where $W_A(z_1,z_2)$, $W_B(z_1,z_2)$ and $W_C(z_1,z_2)$ are three stable, causal, scalar rational functions of the complex variables z_1 and z_2.

Definition 2: Let $\mathbf{X}(z_1,z_2)$ be any $m\times n$ complex-valued matrix function of the complex variables z_1 and z_2. The L_p-norm of $\mathbf{X}(z_1,z_2)$ is then defined as

$$||\mathbf{X}||_p = \left[\frac{1}{(2\pi j)^2}\oint\oint_{\Gamma^2} ||\mathbf{x}(z_1,z_2)||_F^p \frac{dz_1dz_2}{z_1z_2}\right]^{\frac{1}{p}} \tag{8}$$

where $\Gamma^2 = \{(z_1,z_2) : |z_1| = 1, |z_2| = 1\}$ and $||\mathbf{X}(z_1,z_2)||_F$ is the Frobenius norm of $\mathbf{X}(z_1,z_2)$:

$$||\mathbf{X}(z_1,z_2)||_F = \left[\sum_{p=1}^{m}\sum_{q=1}^{n}|x_{pq}(z_1,z_2)|^2\right]^{\frac{1}{2}}.$$

By Definition 2, the overall weighted sensitivity measure is defined as

$$m_2 = \left|\left|\frac{\delta H(z_1,z_2)}{\delta \mathbf{A}}\right|\right|_2^2 + \left|\left|\frac{\delta H(z_1,z_2)}{\delta \mathbf{b}}\right|\right|_2^2 + \left|\left|\frac{\delta H(z_1,z_2)}{\delta \mathbf{c}^t}\right|\right|_2^2 \tag{9}$$

From Eqs. (6)–(9) it is easy to show that

$$m_2 = \mathrm{tr}\mathbf{K}_A + \mathrm{tr}\mathbf{K}_{oB} + \mathrm{tr}\mathbf{K}_{cC} \tag{10}$$

where $\mathbf{K}_A$, $\mathbf{K}_{oB}$ and $\mathbf{K}_{cC}$ can be obtained by the following general expression:

$$\mathbf{K} = \frac{1}{(2\pi j)^2}\oint\oint_{\Gamma^2}\mathbf{Y}(z_1,z_2)\mathbf{Y}^*(z_1,z_2)\frac{dz_1dz_2}{z_1z_2}$$

with $\mathbf{Y}(z_1,z_2) = W_A(z_1,z_2)\mathbf{G}^t(z_1,z_2)\mathbf{F}^t(z_1,z_2)$, $W_B^*(z_1,z_2)\mathbf{G}^*(z_1,z_2)$, and $W_C(z_1,z_2)\mathbf{F}(z_1,z_2)$, respectively. Referring to (2), (10) is written in the form

$$m_2 = \mathrm{tr}\left[\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\mathbf{M}_A(i,j)\mathbf{M}_A^t(i,j)\right] + \mathrm{tr}\mathbf{K}_{oB} + \mathrm{tr}\mathbf{K}_{cC} \tag{11}$$

where

$$\begin{aligned}\mathbf{M}_A(i,j) &= \sum\sum_{(0,0)\le(k,r)<(i,j)} w_A(k,r)\mathbf{M}(i-k,j-r)\\ \mathbf{M}(i,j) &= \sum\sum_{(0,0)\le(k,r)<(i,j)} \mathbf{g}^t(k,r)\mathbf{f}^t(i-k,j-r)\\ \mathbf{f}(i,j) &= \mathbf{A}^{(i-1,j)}\begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{b} + \mathbf{A}^{(i,j-1)}\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_n\end{bmatrix}\mathbf{b}\\ \mathbf{g}(i,j) &= \mathbf{c}\mathbf{A}^{(i-1,j)}\begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + \mathbf{c}\mathbf{A}^{(i,j-1)}\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_n\end{bmatrix}\\ \mathbf{A}^{(1,0)} &= \begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{A}, \ \mathbf{A}^{(0,1)} = \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_n\end{bmatrix}\mathbf{A}\\ \mathbf{A}^{(0,0)} &= \mathbf{I}_{m+n}, \ \mathbf{A}^{(-i,j)} = \mathbf{A}^{(i,-j)} = \mathbf{0} \ (i,j \ge 1)\\ \mathbf{A}^{(i,j)} &= \mathbf{A}^{(1,0)}\mathbf{A}^{(i-1,j)} + \mathbf{A}^{(0,1)}\mathbf{A}^{(i,j-1)}\\ &= \mathbf{A}^{(i-1,j)}\mathbf{A}^{(1,0)} + \mathbf{A}^{(i,j-1)}\mathbf{A}^{(0,1)}\\ &\qquad (i,j) > (0,0)\end{aligned}$$

with $w_A(k,r)$ being the unit-sample response of $W_A(z_1,z_2)$. Applying the coordinate transformation defined by (3) to the original filter, (11) becomes

$$\begin{aligned}\bar{m}_2(\mathbf{P}) \;=\; &\mathrm{tr}\left[\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\mathbf{M}_A(i,j)\mathbf{P}^{-1}\mathbf{M}_A^t(i,j)\mathbf{P}\right]\\ &+\mathrm{tr}[\mathbf{K}_{oB}\mathbf{P}] + \mathrm{tr}[\mathbf{K}_{cC}\mathbf{P}^{-1}]\end{aligned} \tag{12}$$

where

$$\mathbf{P} = \mathbf{T}\mathbf{T}^t, \quad \mathbf{T} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_4 \end{bmatrix}.$$

The problem being considered here is to find a matrix $\mathbf{P}$ which minimizes (12).

3. WEIGHTED L_2-SENSITIVITY MINIMIZATION

In this section, an iterative algorithm is proposed to obtain a matrix $\mathbf{P}$ that minimizes $\bar{m}_2(\mathbf{P})$ given by (12). According to the partition

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_4 \end{bmatrix} \qquad \mathbf{P}_i = \mathbf{T}_i\mathbf{T}_i^t, \; i = 1,4 \quad (13)$$

$\mathbf{M}_A(i,j)$ can be represented as

$$\mathbf{M}_A(i,j) = \begin{bmatrix} \mathbf{M}_A^{(1)}(i,j) & \mathbf{M}_A^{(2)}(i,j) \\ \mathbf{M}_A^{(3)}(i,j) & \mathbf{M}_A^{(4)}(i,j) \end{bmatrix}. \quad (14)$$

Using the formula for evaluating matrix gradient

$$\frac{\partial[\mathrm{tr}(\mathbf{MX})]}{\partial \mathbf{X}} = \mathbf{M}^t$$
$$\frac{\partial[\mathrm{tr}(\mathbf{MX}^{-1})]}{\partial \mathbf{X}} = -(\mathbf{X}^{-1}\mathbf{M}\mathbf{X}^{-1})^t,$$

(12) implies that

$$\begin{aligned} \frac{\partial \bar{m}_2(\mathbf{P})}{\partial \mathbf{P}_1} &= \mathbf{F}_1(\mathbf{P}) - \mathbf{P}_1^{-1}\mathbf{F}_2(\mathbf{P})\mathbf{P}_1^{-1} \\ \frac{\partial \bar{m}_2(\mathbf{P})}{\partial \mathbf{P}_4} &= \mathbf{F}_3(\mathbf{P}) - \mathbf{P}_4^{-1}\mathbf{F}_4(\mathbf{P})\mathbf{P}_4^{-1} \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mathbf{F}_1(\mathbf{P}) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[\mathbf{M}_A^{(1)}(i,j)\mathbf{P}_1^{-1}\mathbf{M}_A^{(1)}(i,j)^t \\ &\quad +\mathbf{M}_A^{(2)}(i,j)\mathbf{P}_4^{-1}\mathbf{M}_A^{(2)}(i,j)^t] + \mathbf{K}_{oB}^{(1)} \\ \mathbf{F}_2(\mathbf{P}) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[\mathbf{M}_A^{(1)}(i,j)^t\mathbf{P}_1\mathbf{M}_A^{(1)}(i,j) \\ &\quad +\mathbf{M}_A^{(3)}(i,j)^t\mathbf{P}_4\mathbf{M}_A^{(3)}(i,j)] + \mathbf{K}_{cC}^{(1)} \\ \mathbf{F}_3(\mathbf{P}) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[\mathbf{M}_A^{(3)}(i,j)\mathbf{P}_1^{-1}\mathbf{M}_A^{(3)}(i,j)^t \\ &\quad +\mathbf{M}_A^{(4)}(i,j)\mathbf{P}_4^{-1}\mathbf{M}_A^{(4)}(i,j)^t] + \mathbf{K}_{oB}^{(4)} \\ \mathbf{F}_4(\mathbf{P}) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[\mathbf{M}_A^{(2)}(i,j)^t\mathbf{P}_1\mathbf{M}_A^{(2)}(i,j) \\ &\quad +\mathbf{M}_A^{(4)}(i,j)^t\mathbf{P}_4\mathbf{M}_A^{(4)}(i,j)] + \mathbf{K}_{cC}^{(4)} \end{aligned}$$

Letting the two equations in (15) be null yields

$$\begin{aligned} \mathbf{P}_1\mathbf{F}_1(\mathbf{P})\mathbf{P}_1 &= \mathbf{F}_2(\mathbf{P}) \\ \mathbf{P}_4\mathbf{F}_3(\mathbf{P})\mathbf{P}_4 &= \mathbf{F}_4(\mathbf{P}) \end{aligned} \quad (16)$$

Note that for given $\mathbf{W} > 0$ and $\mathbf{M} \geq 0$, $\mathbf{PWP} = \mathbf{M}$ has the unique solution [7]

$$\mathbf{P} = \mathbf{W}^{-\frac{1}{2}}[\mathbf{W}^{-\frac{1}{2}}\mathbf{M}\mathbf{W}^{\frac{1}{2}}]^{\frac{1}{2}}\mathbf{W}^{-\frac{1}{2}}$$

It follows from (16) that matrices $\mathbf{P}_1^{(i+1)}$ and $\mathbf{P}_4^{(i+1)}$ satisfying

$$\begin{aligned} \mathbf{P}_1^{(i+1)}\mathbf{F}_1(\mathbf{P}^{(i)})\mathbf{P}_1^{(i+1)} &= \mathbf{F}_2(\mathbf{P}^{(i)}) \\ \mathbf{P}_4^{(i+1)}\mathbf{F}_3(\mathbf{P}^{(i)})\mathbf{P}_4^{(i+1)} &= \mathbf{F}_4(\mathbf{P}^{(i)}) \end{aligned} \quad (17)$$

respectively, are given by

$$\begin{aligned} &\mathbf{P}_1^{(i+1)} = \\ &\mathbf{F}_1^{-\frac{1}{2}}(\mathbf{P}^{(i)})[\mathbf{F}_1^{\frac{1}{2}}(\mathbf{P}^{(i)})\mathbf{F}_2(\mathbf{P}^{(i)})\mathbf{F}_1^{\frac{1}{2}}(\mathbf{P}^{(i)})]^{\frac{1}{2}}\mathbf{F}_1^{-\frac{1}{2}}(\mathbf{P}^{(i)}) \\ &\mathbf{P}_4^{(i+1)} = \\ &\mathbf{F}_3^{-\frac{1}{2}}(\mathbf{P}^{(i)})[\mathbf{F}_3^{\frac{1}{2}}(\mathbf{P}^{(i)})\mathbf{F}_4(\mathbf{P}^{(i)})\mathbf{F}_3^{\frac{1}{2}}(\mathbf{P}^{(i)})]^{\frac{1}{2}}\mathbf{F}_3^{-\frac{1}{2}}(\mathbf{P}^{(i)}) \end{aligned} \quad (18)$$

where $\mathbf{P}^{(i)}$ is the solution of the ith iteration. The initial estimate $\mathbf{P}^{(0)}$ in the above iteration may be obtained by the method reported in [5], for example. This iteration process continues until

$$|\bar{m}_2(\mathbf{P}^{(i+1)}) - \bar{m}_2(\mathbf{P}^{(i)})| < \varepsilon \quad (19)$$

where $\varepsilon > 0$ is a prescribed tolerance.

Given the L_2-optimal matrix $\mathbf{P} = \mathbf{P}_1 \oplus \mathbf{P}_4$ which is positive-definite and symmetric, the corresponding L_2-optimal transformation matrix can be constructed as

$$\mathbf{T} = [\mathbf{P}_1^{\frac{1}{2}} \oplus \mathbf{P}_4^{\frac{1}{2}}][\mathbf{U}_1 \oplus \mathbf{U}_4] \quad (20)$$

where $\mathbf{U}_1$ and $\mathbf{U}_4$ are arbitrary $m \times m$ and $n \times n$ orthogonal matrices, respectively.

4. AN ILLUSTRATIVE EXAMPLE

Consider the LSS model (1) specified by

$$\mathbf{A} = \begin{bmatrix} 1.88899 & -0.91219 & -1.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 \\ 0.02771 & -0.02580 & 1.88899 & 1.0 \\ -0.02580 & 0.02431 & -0.91219 & 0.0 \end{bmatrix}$$
$$\mathbf{b} = [\,0.219089 \quad 0.0 \quad -0.028889 \quad 0.091219\,]^t$$
$$\mathbf{c} = [\,0.28889 \quad -0.091219 \quad -0.219089 \quad 0.0\,]$$

where $m = n = 2$. Suppose the frequency weighting functions are given by

$$\mathbf{W}_A(z_1, z_2) = \sum_{i=0}^{20}\sum_{j=0}^{20} w_a(i,j) z_1^{-i} z_2^{-j}$$
$$\mathbf{W}_B(z_1, z_2) = \sum_{i=0}^{20}\sum_{j=0}^{20} w_b(i,j) z_1^{-i} z_2^{-j}$$
$$\mathbf{W}_C(z_1, z_2) = \frac{N(z_1, z_2)}{D_1(z_1)D_2(z_2)}$$

where

$$\begin{aligned} w_a(i,j) &= 0.256322 \exp[-0.103203\{(i-4)^2 \\ &\quad +(j-4)^2\}] \\ w_b(i,j) &= 0.256322 \exp[-0.103203\{(i-5)^2 \\ &\quad +(j-i)^2\}] \\ N(z_1, z_2) &= \sum_{i=0}^{4}\sum_{j=0}^{4} b_{ij} z_1^{-i} z_2^{-j} \\ D(z_k) &= 1 - 1.11425 z_k^{-1} + 0.75745 z_k^{-2} \\ &\quad -0.34255 z_k^{-3} + 0.10171 z_k^{-4}, \ k = 1, 2 \end{aligned}$$

$$\{b_{ij}\} = 10^{-2} \times \begin{bmatrix} 0.12814 & 0.64232 & 0.74979 & 0.64232 & 0.12814 \\ 0.64232 & 0.33077 & 0.68889 & 0.33077 & 0..64232 \\ 0.74979 & 0.68889 & 1.34339 & 0.68889 & 0.74979 \\ 0.64232 & 0.33077 & 0.68889 & 0.33077 & 0.64232 \\ 0.12814 & 0.64232 & 0.74979 & 0.64232 & 0.12814 \end{bmatrix}$$

The simulation results of the above example are summarized in Table 1.

TABLE 1
WEIGHTED L_2-SENSITIVITY ANALYSIS

Realization	Weighted L_2-Sensitivity
Original	$\bar{m}_2 = 7.8189 \times 10^8$
Suboptimal	$\bar{m}_2 = 1.6301 \times 10^5$
L_2-Optimal	$\bar{m}_2 = 8.4194 \times 10^4$

In Table 1, the suboptimal realization was derived from the technique reported in [5] and was used as the initial estimate in the iterative process. The L_2-optimal realization was constructed from the result obtained after 20 iterations.

5. CONCLUSION

A novel iterative procedure has been developed for obtaining the optimal coordinate transformation that minimizes the weighted L_2 sensitivity. This procedure allows one to obtain the estimate in a closed form at each iteration. The simulation results of a numerical example have demonstrated the validity of the proposed technique.

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