DESIGN OF SIGNAL-ADAPTED BIORTHOGONAL FILTER BANKS USING SECOND-ORDER CONE PROGRAMMING

Wu-Sheng Lu
Dept. of Electrical and Computer Engineering, University of Victoria,
P.O.Box 3055, Victoria, BC, CANADA V8W 3P6
Tel: +1 250 721 8692; fax: +1 250 721 6052
e-mail: wslu@ece.uvic.ca

ABSTRACT

This paper is concerned with the design of M-channel, biorthogonal filter banks that are adapted to input statistics in terms of a certain coding gain criterion. Using a linear approximation of the coding gain together with a norm constraint on the parameter perturbation vector which validates such an approximation, and parameterization of a first-order approximation of the perfect reconstruction condition, we show that the design problem at hand can be formulated as a second-order cone programming problem. Simulation results are presented to illustrate the proposed design method.

1 Instruction

There has been a great deal of interest in orthogonal and biorthogonal filter banks that are optimal in terms of some coding gain criterion [1]–[6]. Biorthogonal filter banks offer improved performance over orthogonal filter banks but the optimal design of a biorthogonal filter banks involves some sophisticated constrained optimization problem [4][6].

In [6], the design of M-channel signal-adapted biorthogonal filters banks of finite length is formulated as a constrained optimization problem and is solved by converting it into an iterative line-search problem through a first-order parameterization of the perfect reconstruction condition. A problem encountered in the algorithmic implementation of the method in [6] is that numerical instability may occur due to lack of a mechanism to control the magnitude of the parameter perturbation vector. Controlling the magnitude of the parameter perturbations is of critical importance because it validates a key first-order parameterization step in the design algorithm. In this paper, we take a different approach to the problem at hand using second-order cone programming (SOCP). By viewing a bound condition on the Euclidean norm of the parameter perturbation vector as a second-order cone constraint, the design problem is shown to well fit into an SOCP setting. Simulation results are presented to illustrate the proposed design method.

2 A Brief Review of SOCP

Second-order cone programming, which is sometimes called the conic quadratic programming [7][8], is a subclass of convex programming problems where a linear function is minimized subject to a set of second-order cone constraints [7][9]:

$$\text{minimize} \quad \boldsymbol{f}^T\boldsymbol{x} \tag{1a}$$

$$\text{subject to:} \quad \|\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{b}_i\| \le \boldsymbol{c}_i^T\boldsymbol{x}+h_i, \quad i=1,\,\ldots,\,N \tag{1b}$$

where $\boldsymbol{f} \in \mathcal{R}^{n\times 1}$, $\boldsymbol{A}_i \in \mathcal{R}^{(n_i-1)\times n}$, $\boldsymbol{b}_i \in \mathcal{R}^{(n_i-1)\times 1}$, $\boldsymbol{c}_i \in \mathcal{R}^{n\times 1}$, and $h_i \in \mathcal{R}$. The term "second-order cone" here reflects the fact that each constraint in (1b) is equivalent to a conic constraint

$$\begin{bmatrix} \boldsymbol{c}_i^T \\ \boldsymbol{A}_i \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} h_i \\ \boldsymbol{b}_i \end{bmatrix} \in \mathcal{C}_i$$

where $\mathcal{C}_i$ is the second-order cone in $\mathcal{R}^{n_i}$, i.e.,

$$\mathcal{C}_i = \left\{ \begin{bmatrix} t \\ \boldsymbol{u} \end{bmatrix} : \boldsymbol{u} \in \mathcal{R}^{(n_i-1)\times 1},\ t \ge 0,\ \|\boldsymbol{u}\| \le t \right\}$$

From (1), it is evident that CQP includes linear programming and convex quadratic programming as special cases. On the other hand, since each constraint in (1b) can be expressed as

$$\begin{bmatrix} (\boldsymbol{c}_i^T\boldsymbol{x}+h_i)\boldsymbol{I} & \boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{b}_i \\ (\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{b}_i)^T & \boldsymbol{c}_i^T\boldsymbol{x}+h_i \end{bmatrix} \succeq \boldsymbol{0} \tag{2}$$

where $\boldsymbol{M} \succeq \boldsymbol{0}$ denotes that $\boldsymbol{M}$ is positive semidefinite, SOCP is a subclass of semidefinite programming (SDP) [9][10]. Commercial and public domain software based on interior-point optimization algorithms for SOCP and SDP are available [11]–[13]. It is important to stress, however, that in general the problem in (1) can be solved more efficiently as a SOCP problem than solving it in an equivalent SDP setting [7].

3 The Design Problem

We are concerned with the design of M-channel, biorthogonal filter banks illustrated in Fig. 1 that are adapted to input statistics in terms of a certain coding gain criterion, where the input signal $x(n)$ is wide-sense stationary (WSS) with power density $S_{xx}(\omega)$ and variance σ_x^2, and each of the blocks labeled with Q represents a quantizer. The performance of the subband system in Fig. 1 can be measured in terms of the coding gain, $G_{\text{SBC}}(M)$, which is defined as the ratio of the mean-square value of the roundoff quantization error to the average variance of the reconstruction error, and can be expressed as [4] $G_{\text{SBC}}(M) = \sigma_x^2/\Phi^{1/M}$, where

$$\Phi = \prod_{i=1}^{M-1} \int_0^{2\pi} S_{xx}(\omega)|H_i(\omega)|^2 \frac{d\omega}{2\pi} \int_0^{2\pi} |F_i(\omega)|^2 \frac{d\omega}{2\pi} \quad (3)$$

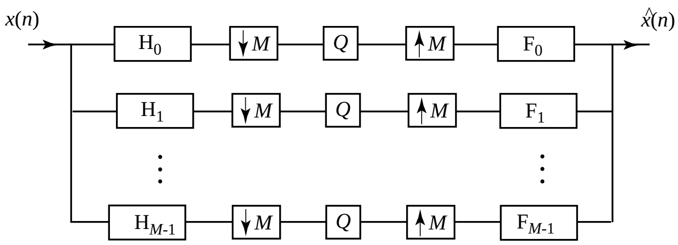

Figure 1: M-channel maximally decimated uniform filter bank.

It follows that for a given WSS input, maximizing the coding gain is equivalent to minimizing Φ subject to that the filter bank holds the perfect reconstruction property (PR). Therefore the design problem at hand can be formulated as the constrained optimization problem

$$\begin{aligned} &\text{minimize } \Phi && (4a)\\ &\text{subject to: the filter bank is PR} && (4b)\end{aligned}$$

Assume for the sake of simplicity that all filters involved in the subband system are of FIR and have the same length, i.e.,

$$H_i(z) = \sum_{k=0}^{N-1} h_{i,k} z^{-k} \qquad \text{for } 0 \le i \le M-1 \quad (5a)$$

$$F_i(z) = \sum_{k=0}^{N-1} f_{i,k} z^{-k} \qquad \text{for } 0 \le i \le M-1 \quad (5b)$$

and define

$$\boldsymbol{h}_i = [h_{i,0} \ \cdots \ h_{i,N-1}]^T \quad (6a)$$

$$\boldsymbol{f}_i = [f_{i,0} \ \cdots \ f_{i,N-1}]^T \quad (6b)$$

$$\boldsymbol{x} = [\boldsymbol{h}_0^T \ \cdots \ \boldsymbol{h}_{M-1}^T \boldsymbol{f}_0^T \ \cdots \ \boldsymbol{f}_{M-1}^T]^T \quad (6c)$$

Function Φ in (3) can then be expressed as

$$\Phi(\boldsymbol{x}) = \hat{\Phi}^2(\boldsymbol{x}) \quad \text{with} \quad \hat{\Phi}(\boldsymbol{x}) = \prod_{i=0}^{M-1} \left(\left\| \boldsymbol{R}^{1/2} \boldsymbol{h}_i \right\| \cdot \|\boldsymbol{f}_i\| \right) \quad (7)$$

where $\boldsymbol{R}$ is a symmetric, positive-definite Toeplitz matrix whose first row is given by $[r_0 \ r_1 \ \cdots \ r_{N-1}]$ with

$$r_i = \frac{1}{2\pi} \int_0^{2\pi} S_{xx}(\omega) \cos(i\omega)\, d\omega \quad (8)$$

Suppose we are in the kth iteration and seek to find an increment vector $\boldsymbol{\delta}$ such that $\Phi(\boldsymbol{x}_k + \boldsymbol{\delta}) < \Phi(\boldsymbol{x}_k)$ and, at $\boldsymbol{x}_k + \boldsymbol{\delta}$, the system is near PR.

At $\boldsymbol{x} = \boldsymbol{x}_k + \boldsymbol{\delta}$, we can write

$$\hat{\Phi}(\boldsymbol{x}) \approx \hat{\Phi}(\boldsymbol{x}_k) + \boldsymbol{g}_k^T \boldsymbol{\delta} \quad (9)$$

provided that

$$\|\boldsymbol{\delta}\| \text{ is small} \quad (10)$$

where $\boldsymbol{g}_k = \nabla \hat{\Phi}(\boldsymbol{x}_k)$. In this case we have

$$\Phi(\boldsymbol{x}) \approx [\hat{\Phi}(\boldsymbol{x}_k) + \boldsymbol{g}_k^T \boldsymbol{\delta}]^2 \quad (11)$$

and the above $\boldsymbol{\delta}$ can be determined by solving the constrained problem

$$\begin{aligned} &\text{minimize} && \eta && (12a)\\ &\text{subject to:} && (\hat{\Phi}(\boldsymbol{x}_k) + \boldsymbol{g}_k^T \boldsymbol{\delta})^2 \le \eta && (12b)\\ & && \|\boldsymbol{\delta}\|^2 \le \beta && (12c)\\ & && \boldsymbol{x}_k + \boldsymbol{\delta} \text{ is near PR} && (12d)\end{aligned}$$

where (12a) with (12b) and (12d) are aimed at reducing $\Phi(\boldsymbol{x})$ while (12c) implements (10) so as to validate (9).

4 Gradient of $\hat{\Phi}(\boldsymbol{x})$

Let $\hat{\boldsymbol{R}} = \boldsymbol{R}^{1/2}$ and write $\hat{\Phi}(\boldsymbol{x})$ in (7) as

$$\hat{\Phi}(\boldsymbol{x}) = \prod_{i=0}^{M-1} \|\hat{\boldsymbol{R}} \boldsymbol{h}_k\| \cdot \|\boldsymbol{f}_i\| \quad (13)$$

We compute

$$\nabla \hat{\Phi}(\boldsymbol{x}) = \begin{bmatrix} \frac{\partial \hat{\Phi}}{\partial \boldsymbol{h}_0} \\ \vdots \\ \frac{\partial \hat{\Phi}}{\partial \boldsymbol{h}_{M-1}} \\ \frac{\partial \hat{\Phi}}{\partial \boldsymbol{f}_0} \\ \vdots \\ \frac{\partial \hat{\Phi}}{\partial \boldsymbol{f}_{M-1}} \end{bmatrix} \quad (14a)$$

with

$$\frac{\partial \hat{\Phi}}{\partial \mathbf{h}_i} = \hat{\Phi}(\boldsymbol{x}) \frac{\sum_{k=1}^{N} (\hat{\boldsymbol{R}} \boldsymbol{h}_k)_k \boldsymbol{r}_k}{\|\hat{\boldsymbol{R}} \boldsymbol{h}_i\|^2} \quad (14b)$$

$$\frac{\partial \hat{\Phi}}{\partial \boldsymbol{f}_i} = \hat{\Phi}(\boldsymbol{x}) \frac{\boldsymbol{f}_i}{\|\boldsymbol{f}_k\|^2} \quad (14c)$$

where $\boldsymbol{r}_k$ denotes the kth column of $\hat{\boldsymbol{R}}$.

5 Parameterization of the PR Condition

Let $\boldsymbol{P}$ and $\boldsymbol{Q}$ be the matrices that comprise the coefficients of the analysis and synthesis filters, respectively, i.e.,

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{h}_0^T \\ \vdots \\ \boldsymbol{h}_{M-1}^T \end{bmatrix}, \ \boldsymbol{Q} = \begin{bmatrix} \boldsymbol{f}_0^T \\ \vdots \\ \boldsymbol{f}_{M-1}^T \end{bmatrix} \quad (15)$$

and assume $N = ML$ for some integer L, and partition each of $\boldsymbol{P}$ and $\boldsymbol{Q}$ into L blocks as

$$\boldsymbol{P} = [\boldsymbol{P}_0 \ \cdots \ \boldsymbol{P}_{L-1}],\ \boldsymbol{Q} = [\boldsymbol{Q}_0 \ \cdots \ \boldsymbol{Q}_{L-1}] \tag{16}$$

with each $\boldsymbol{P}_i$ and $\boldsymbol{Q}_i$ an $M \times M$ matrix. It is known [6] that the PR condition can then be expressed for $k = 0,\ 1\ \ldots,\ 2L-2$ as

$$\boldsymbol{S}_k \ = \begin{cases} \boldsymbol{J} & \text{if } k = L-1 \\ \boldsymbol{0} & \text{elsewhere} \end{cases} \tag{17a}$$

where

$$\boldsymbol{S}_k \ = \sum_{i=0}^{L-1} \boldsymbol{P}_i^T \boldsymbol{Q}_{k-i} \tag{17b}$$

with the understanding that $\boldsymbol{P}_i$ and $\boldsymbol{Q}_i$ for $i < 0$ or $i > L-1$ are zero matrices, and

$$\boldsymbol{J} = \frac{\hat{\boldsymbol{I}}}{M}, \hat{\boldsymbol{I}} = \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix} \tag{17c}$$

For the sake of notation simplicity, we introduce two matrix sequences $\mathcal{P} = \{\boldsymbol{P}_0,\ \boldsymbol{P}1,\ \ldots, \boldsymbol{P}_L\}$ and $\mathcal{Q} = \{\boldsymbol{Q}_0,\ \boldsymbol{Q}_1,\ \ldots,\ \boldsymbol{Q}_L\}$ and define the matrix convolution of $\mathcal{P}$ and $\mathcal{Q}$ as

$$\mathcal{S} = \text{conv}(\mathcal{P}, \mathcal{Q}) = \{\boldsymbol{S}_0,\ \ldots,\ \boldsymbol{S}_{2L-2}\} \tag{18}$$

The PR condition in (17) can be expressed as

$$\text{conv}(\mathcal{P}, \mathcal{Q}) \ = \mathcal{J} \tag{19a}$$

where

$$\mathcal{J} \ = \{\boldsymbol{0},\ \ldots,\ \boldsymbol{0},\ \boldsymbol{J},\ \boldsymbol{0},\ \ldots,\ \boldsymbol{0}\} \tag{19b}$$

In the kth iteration, point $\boldsymbol{x}_k$ is updated to $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{\delta}$ where

$$\boldsymbol{\delta} = [\Delta \boldsymbol{h}_0^T \ \cdots \ \Delta \boldsymbol{h}_{M-1}^T \ \Delta \boldsymbol{f}_0^T \ \cdots \Delta \boldsymbol{f}_{M-1}^T]^T$$

such that, in addition to the requirement imposed Sec. III, $\boldsymbol{x}_k + \boldsymbol{\delta}$ is a better approximate solution of the equation in (19a) subject to a normalization condition

$$\sum_{i=0}^{M-1} \sum_{k=0}^{N-1} h_{ik} = 1 \tag{19c}$$

Now if we let $(\mathcal{P}_k, \mathcal{Q}_k)$ and $(\mathcal{P}_{k+1}, \mathcal{Q}_{k+1})$ be the matrix sequences $(\mathcal{P}, \mathcal{Q})$ associated with $\boldsymbol{x}_k$ and $\boldsymbol{x}_{k+1}$, respectively, then we have

$$\begin{aligned} \mathcal{P}_{k+1} \ &= \mathcal{P}_k + \Delta\mathcal{P} \\ \mathcal{Q}_{k+1} \ &= \mathcal{Q}_k + \Delta\mathcal{Q} \end{aligned}$$

where $\Delta\mathcal{P}$ and $\Delta\mathcal{Q}$ are two perturbation sequences that are linearly dependent on $\boldsymbol{\delta}$. It can be readily verified that

$$\text{conv}(\Delta\mathcal{P}, \mathcal{Q}_k) + \text{conv}(\mathcal{P}_k, \Delta\mathcal{Q}) = \hat{\mathcal{J}} \tag{20}$$

where

$$\hat{\mathcal{J}} = \mathcal{J} - \text{conv}(\mathcal{P}_k, \mathcal{Q}_k) - \text{conv}(\Delta\mathcal{P}, \Delta\mathcal{Q})$$

Under the constraint in (10), a reasonable linear approximation of the equation in (20) is given by

$$\text{conv}(\Delta\mathcal{P}, \mathcal{Q}_k) + \text{conv}(\mathcal{P}_k, \Delta\mathcal{Q}) = \hat{\mathcal{J}}_0 \tag{21a}$$

with

$$\hat{\mathcal{J}}_0 = \mathcal{J} - \text{conv}(\mathcal{P}_k, \mathcal{Q}_k) \tag{21b}$$

In addition, the constraint in (19c) at $\boldsymbol{x}_k + \boldsymbol{\delta}$ remains linear:

$$\sum_{i=0}^{M-1} \sum_{k=0}^{N-1} \Delta h_{ik} = 0$$

i.e.,

$$\boldsymbol{e}^T \boldsymbol{\delta} = 0 \tag{22}$$

where $\boldsymbol{e} = [1 \ \cdots \ 1 \ 0 \ \cdots \ 0]^T$ is the $2MN$-vector whose first half of the components are unity and the remaining half equal to zero. The linearized equation in (21a) is now combined with (22) to form a linear equality constraint

$$\boldsymbol{\Gamma}\boldsymbol{\delta} = \boldsymbol{\gamma}_k \tag{23}$$

In the above formula, $\boldsymbol{\Gamma} \in \mathcal{R}^{(2MN-M^2+1)\times 2MN}$ and $\boldsymbol{\gamma}_k \in \mathcal{R}^{(2MN-M^2+1)\times 1}$. Clearly, for $M > 1$, the linear system in (23) is underdetermined, and its solutions can be parameterized by an (M^2-1)-dimensional free parameter vector:

$$\boldsymbol{\delta} = \boldsymbol{\delta}_0 + \boldsymbol{V}_e \boldsymbol{\xi} \tag{24}$$

where $\boldsymbol{\delta}_0 = \boldsymbol{\Gamma}^\dagger \boldsymbol{\gamma}_k$ with $\boldsymbol{\Gamma}^\dagger$ being the Moore-Penrose pseudo-inverse of $\boldsymbol{\Gamma}$, $\boldsymbol{V}_e$ consists of the M^2-1 basis vectors in the null space of $\boldsymbol{\Gamma}$, and $\boldsymbol{\xi} \in \mathcal{R}^{(M^2-1)\times 1}$ is a free parameter vector. Note that matrix $\boldsymbol{V}_e$ can be obtained, for example, using the singular value decomposition of $\boldsymbol{\Gamma} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$ and taking $\boldsymbol{V}_e$ as the submatrix of $\boldsymbol{V}$ consisting of its last M^2-1 columns.

6 An SDP/SOCP Formulation

The analysis of our design problem now leads to the constrained optimization problem

$$\begin{aligned} &\text{minimize} && \eta && (25a) \\ &\text{subject to:} && [\hat{\Phi}(\boldsymbol{x}_k) + \boldsymbol{g}_k^T(\boldsymbol{\delta}_0 + \boldsymbol{V}_e\boldsymbol{\xi})]^2 \le \eta && (25b) \\ & && \|\boldsymbol{\delta}_0 + \boldsymbol{V}_e\boldsymbol{\xi}\|^2 \le \beta && (25c) \end{aligned}$$

It is straightforward to verify that the constraint in (25b) is equivalent to

$$\begin{bmatrix} \eta & \hat{\boldsymbol{g}}_k^T\boldsymbol{\xi} + b_k \\ \hat{\boldsymbol{g}}_k^T\boldsymbol{\xi} + b_k & 1 \end{bmatrix} \succeq \boldsymbol{0} \tag{26}$$

where $\hat{\boldsymbol{g}}_k = \boldsymbol{V}_e^T \boldsymbol{g}_k$ and $b_k = \hat{\Phi}(\boldsymbol{x}_k) + \boldsymbol{g}_k^T \boldsymbol{\delta}_0$, and the constraint in (25c) is equivalent to

$$\begin{bmatrix} \beta - c_0 & (\boldsymbol{\xi} - \boldsymbol{d})^T \\ \boldsymbol{\xi} - \boldsymbol{d} & \boldsymbol{I} \end{bmatrix} \succeq \boldsymbol{0} \tag{27}$$

where $c_0 = \boldsymbol{\delta}_0^T(\boldsymbol{I} - \boldsymbol{V}_e\boldsymbol{V}_e^T)\boldsymbol{\delta}_0$ and $\boldsymbol{d} = \boldsymbol{V}_e^T\boldsymbol{\delta}_0$. By defining

$$\hat{\boldsymbol{\xi}} = \begin{bmatrix} \eta \\ \boldsymbol{\xi} \end{bmatrix} \qquad \text{and} \qquad \hat{\boldsymbol{c}} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The problem in (25) can be written as

$$\begin{aligned} &\text{minimize} && \hat{\boldsymbol{c}}^T\hat{\boldsymbol{\xi}} && (28a) \\ &\text{subject to:} && \text{(26) and (27)} && (28b) \end{aligned}$$

Since (26) and (27) are linear matrix inequalities with respect to $\hat{\boldsymbol{\xi}}$, (28) is an SDP problem.

We note that (25b) and (25c) can also be expressed as

$$|\hat{\boldsymbol{g}}_k^T\boldsymbol{\xi} + b_k| \leq \hat{\eta} \tag{29}$$

and

$$\|\boldsymbol{V}_e\boldsymbol{\xi} + \boldsymbol{\delta}_0\| \leq \hat{\beta}$$

respectively, where $\hat{\eta} = \eta^{1/2}$ and $\hat{\beta} = \beta^{1/2}$. If we re-define vector $\hat{\boldsymbol{\xi}}$ as

$$\hat{\boldsymbol{\xi}} = \begin{bmatrix} \hat{\eta} \\ \boldsymbol{\xi} \end{bmatrix} \tag{30}$$

then the problem in (25) can be converted into

$$\begin{aligned} &\text{minimize} && \hat{\boldsymbol{c}}^T\hat{\boldsymbol{\xi}} && (31a) \\ &\text{subject to:} && \|[0\ \ \hat{\boldsymbol{g}}_k^T]\hat{\boldsymbol{\xi}} + b_k\| \leq \hat{\boldsymbol{c}}^T\hat{\boldsymbol{\xi}} && (31b) \\ & && \|[\boldsymbol{0}\ \ \boldsymbol{V}_e]\hat{\boldsymbol{\xi}} + \boldsymbol{\delta}_0\| \leq \hat{\beta} && (31c) \end{aligned}$$

where (31b) is the same as (29) because we are dealing with a scalar quantity. Obviously, (31) is an SOCP problem.

7 Summary of the Algorithm

We may start the algorithm by designing an M-channel maximally decimated filter bank of length N, which is not necessarily biorthogonal. The parameter vector associated with this filter bank is denoted by $\boldsymbol{x}_0$. Without loss of generality, suppose we are in the kth iteration with a known $\boldsymbol{x}_k$ which is to be updated to $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{\delta}_k$. The increment vector $\boldsymbol{\delta}_k$ is determined in to steps:

- Solve the SOCP problem (31) or the equivalent SDP problem in (28); Delete the first component of the solution and denote the remaining part of the vector as $\boldsymbol{\xi}^*$
- Use (24) to compute

$$\boldsymbol{\delta}_k = \boldsymbol{\delta}_0 + \boldsymbol{V}_e\boldsymbol{\xi}^* \tag{32}$$

Repeat the above steps until $\|\boldsymbol{x}_{k+1} - \boldsymbol{x}_k\|$ is less than a prescribed tolerance, and then claim the converging $\boldsymbol{x}^* = \boldsymbol{x}_{k+1}$ as the solution vector.

As expected, however, the filter bank associated with $\boldsymbol{x}^*$ is only *near* PR, and a "final touch" is needed to slightly adjust $\boldsymbol{x}^*$ so as to generate a practically PR filter bank. The needed adjustment can be carried out by applying the proposed algorithm with $\boldsymbol{x}^*$ as the starting point and a reduced value of β in (25c). An immediate effect of using a reduced β is that the term conv$(\Delta\mathcal{P}, \Delta\mathcal{Q})$ in (20) gets reduced accordingly, yielding a better approximate linear system (21). Consequently, the solution so obtained shall generate a filter bank not only with further improved coding gain but also a further near PR property. If necessary, the above adjustment can be repeated several times, each time using a further reduced β. In this way, the filter bank eventually becomes practically PR.

8 Design Examples

We now present two examples to illustrate the proposed design method. Each example involves a 4-channel FIR filter bank of length 8. Thus $M = 4,\ N = 8$, and $L = 2$. The input signal was an autoregressive process (AR) with poles at $0.975e^{\pm j\theta}$ where the values of θ are specified below. The algorithm starts with a 4-channel cosine modulated filter bank [14].

Example 1 With $\theta = \pi/2.8$ in the AR(2) process and $\beta = 0.1$, it took the proposed algorithm 100 iterations to generate a filter bank with coding gain increased from an initial value 2.2171 to 4.8261. Then with β being an half of its preceding value, the algorithm was applied repeatedly until the improvement in terms of coding gain as well as the PR condition became insignificant. The coding gain achieved was 6.8172 and the PR constraint was satisfied to within the Frobenius norm

$$\begin{aligned} &e_F = \\ &\|\boldsymbol{P}_0^T\boldsymbol{Q}_0\|_F + \|\frac{1}{4}\boldsymbol{J} - \boldsymbol{P}_0^T\boldsymbol{Q}_1 - \boldsymbol{P}_1^T\boldsymbol{Q}_0\|_F + \|\boldsymbol{P}_1^T\boldsymbol{Q}_0\|_F \\ &= 3.8153 \times 10^{-14} \end{aligned}$$

The frequency responses of the various filters and power spectral density of the input signal are shown in Fig. 2. It is noted that the filter bank obtained improves both the coding gain (6.6411) and the PR condition $(e_F = 2.3405 \times 10^{-9})$ for a similar system reported in [6].

Example 2 With $\theta = \pi/1.75$ in the AR(2) process and $\beta = 0.1$, it took the algorithm 50 iterations to obtain a filter bank with coding gain 3.8697 and $e_F = 0.0077$. Then with β being an half of its preceding value, the algorithm was applied repeatedly until the improvement in coding gain and e_F became insignificant. The coding gain achieved was 4.9617 and the associated $e_F = 1.3824 \times 10^{-15}$. The frequency responses of the various filters and the power spectral density of the input signal are shown in Fig. 3. We note that the filter bank obtained offers improvement over a similar system reported in [6] in terms of the coding gain (4.9174) as well as PR condition $(e_F = 6.3876 \times 10^{-10})$.

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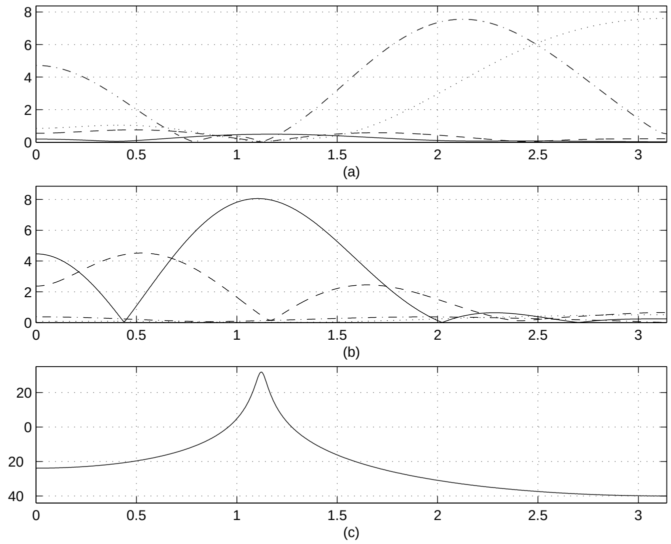

Figure 2: (a) Amplitude responses of filters H_0 to H_3; (b) Amplitude responses of filters F_0 to F_3; (c) Power spectral density $S_{xx}(e^{j\omega})$ in dB with $\theta = \pi/2.8$.

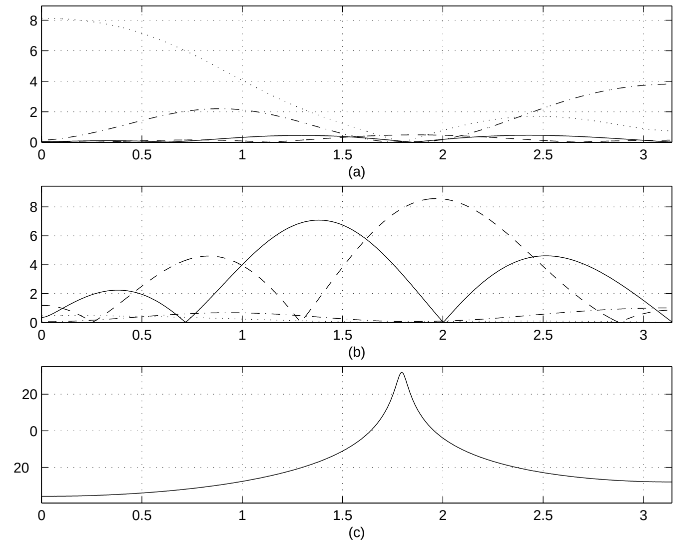

Figure 3: (a) Amplitude responses of filters H_0 to H_3; (b) Amplitude responses of filters F_0 to F_3; (c) Power spectral density $S_{xx}(e^{j\omega})$ in dB with $\theta = \pi/1.75$.

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