A Reduced Order Observer for Underwater Manipulators Using Potentiometers for Position Measurements

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Abstract — In this paper an improved manipulator joint velocity observer is presented which is reduced in its order. The observer provides a smooth velocity estimate to be used by a trajectory tracking controller. The observer, controller and manipulator form a system where the observer error as well as the position and velocity tracking errors tend to zero asymptotically. The proposed observer is compared to numerical differentiation plus filtering as a means to provide a velocity signal which is used in control. When considering joint position signals corrupted with measurement noise, it is found that the proposed observer is superior in its performance compared to numerical differentiation plus filtering.

I. INTRODUCTION

In this paper a reduced order manipulator joint velocity observer is advocated for use in underwater robot manipulators. The need for velocity measurements or velocity estimates has been a long established premise for successful manipulator trajectory tracking control. A tachogenerator can provide a velocity measurement for control. But, measurement noise and implementation expense make these devices unsuitable for the underwater environment. Differentiation of optical encoder signals is another option for obtaining a velocity; however, difficulty in mounting these in oil which is needed for withstanding sub-sea pressures make this option practically infeasible as well.

Rotary potentiometers are robust devices well suited for underwater operations. They are easy to mount and have been used successfully on a variety of sub-sea manipulators such as MAGNUM, a hydraulic manipulator mounted on SCARAB IV a remotely operated vehicle manufactured by International Submarine Engineering (ISE) Ltd. of Port Coquitlam, British Columbia. Obtaining a velocity estimate from these devices with numerical differentiation is also infeasible due to position the measurement noise. Instead of using differentiation, a model based filter, i.e. an observer, can be used to obtain a velocity signal from potentiometers.

Full order manipulator velocity observers have already been presented in [9], [6] and [10]. In [10] a sliding mode observer is used which has non-continuous gains. Hence, observer chattering might excite the manipulator's unmodeled dynamics leading to instability. In [9] and [6] smooth observers were proposed which produce stable observation errors. These use an approach which yields an observer with second order dynamics.

In this paper an improved velocity observer is proposed. A major distinction between the existing velocity observers and the observer proposed here is that in the latter the error dynamics of the observer is formulated as first order differential equation. This is, in spirit, a nonlinear counterpart of the reduced-order observer which is a well understood matter for linear systems [5, p. 28]. When combined with a model based controller, it is shown that the observer errors as well as the position and velocity tracking errors tend to zero asymptotically.

II. NOTATION AND PRELIMINARIES

The well known dynamic equation of an n degree-of-freedom robot manipulator is given by

$$H(q)\ddot{q}+C(q,\dot{q})\dot{q}+F\dot{q}+g(q)=\tau \tag{1}$$

In this paper it is assumed that the joint position measurements q are available but that joint velocities $\dot{q}$ are not. It is also assumed that the parameters of $H(q)$, $C(q,\dot{q})$, F and $g(q)$ are known. Presently, several well known properties of the manipulator dynamic equation will be summarized:

Property 1: The manipulator inertia matrix is positive definite symmetric and bounded $m_l \le \|H(q)\| \le m_u$, $m_l, m_u > 0,\ \forall q \in \Re^n$.

Property 2: The 2-norm of $C(q,x)$ is bounded by $\|C(q,x)\| \le m_c\|x\|$ where $m_c > 0$ is a positive constant. If the joint velocity x is bounded i.e. $\|x\| \le m_v < \infty \quad \forall t \ge 0$, then $\|C(q,x)\| \le m_c m_v$.

Property 3: The matrix of Christoffel symbols $C(q,x)$ satisfies $C(q,x)\xi = C(q,\xi)x$ for any $q, x, \xi \in \Re^n$ where $C(q,x)$ is uniquely defined in [8].

Property 4: $\dot{H}(q) - 2C(q,\dot{q})$ is skew symmetric, i.e. $\xi^T\left(\frac{1}{2}\dot{H}(q) - C(q,\dot{q})\right)\xi = 0$ for any $\xi \in \Re^n$.

III. THE PROPOSED VELOCITY OBSERVER

The manipulator dynamic equation (1) can be rearranged to give

$$\dot{x}=H^{-1}(q)\left[\tau - C(q,x)x - Fx - g(q)\right] \tag{2}$$

where $x = \dot{q}$. This is a first order nonlinear differential equation in x where it is assumed that q is known, τ is the input and x is the output. The proposed velocity observer has a structure similar to (2) and is given by

$$\dot{\hat{x}} = \psi + K\tilde{x} \qquad (3)$$
$$\psi = H^{-1}(q)\left[\tau - C(q,\hat{x})\hat{x} - F\hat{x} - g(q)\right]$$

where $\hat{x}$ is the estimated velocity, $\tilde{x} = x - \hat{x}$ is the observer error and $K > 0$ is a diagonal gain matrix. The observer error dynamics is obtained by subtracting (3) from (2) and applying property 3. Then

$$\dot{\tilde{x}} = -H^{-1}(q)\left[C(q,x)\tilde{x} + C(q,\hat{x})\tilde{x} + F\tilde{x} + H(q)K\tilde{x}\right] \qquad (4)$$

The proposed observer is realized by the integration of (4), i.e.

$$\hat{x}(t) = \hat{x}(t_0) + K\left[q(t) - q(t_0)\right] + \int_{t_0}^{t} \psi - K\hat{x}\, dt \qquad (5)$$

where t_0 is the time when the estimation begins.

The implementation of (5) on a digital computer is done as follows. Let Δ be the sampling interval over which a velocity estimation is made, then discretization of (5) at $t = i\Delta$ leads to

$$\hat{x}(i) \approx \hat{x}(i-1) + K\left[q(i) - q(i-1)\right]$$
$$+\Delta\left[\psi(i-1) - K\hat{x}(i-1)\right]$$

IV. TRAJECTORY TRACKING CONTROL USING THE VELOCITY OBSERVER

The controller in this section uses a velocity estimated with the proposed observer. It is shown that the position and velocity tracking errors that result are locally asymptotically stable. The controller is given by

$$\tau = H(q)\ddot{q}_d + C(q,\hat{x})\dot{q}_d + F\dot{q}_d + g(q)$$
$$-K_d(\hat{x} - \dot{q}_d) - K_p\tilde{q} \qquad (6)$$

where $\tilde{q} = q - q_d$ is the position tracking error. The error dynamics is obtained by subtracting the control torque (6) from the system dynamics (1) and using property 3. The result is

$$H(q)\ddot{\tilde{q}} = -\left[C(q,x) + K_d + F\right]\dot{\tilde{q}}$$
$$-\left[C(q,\dot{q}_d) - K_d\right]\tilde{x} - K_p\tilde{q} \qquad (7)$$

A stability theorem for the position and velocity tracking errors will now be given.

Theorem. $e^T = \left[\tilde{x}^T \quad \dot{\tilde{q}}^T \quad \tilde{q}^T\right]$ is a locally asymptotically stable equilibrium point of (4) and (7) where $\|e\|$ belongs to the region of attraction defined by

$$B = \left\{ \begin{array}{l} \|e\| \in \Re^n : \|e\| \\ < \sqrt{\dfrac{p_u}{2p_l}}\left(\dfrac{\underline{\sigma} + m_f - m_c m_d - \beta_1}{m_c} - \dfrac{(k_d + m_c m_d)}{4\varepsilon^2 m_c}\right) \end{array} \right\}$$

provided that for fixed $\beta_1, \beta_2 > 0$, $K_d = k_d I$ is designed such that $k_d > \left(\beta_2 - m_f + m_c m_d \varepsilon^2\right)/\left(1 - \varepsilon^2\right)$ with $\varepsilon \in (0,1)$, and $K > 0$ is chosen such that $\underline{\sigma} > \beta_1 - m_f + m_c\left(m_d + \|\tilde{x}\| + \|\dot{\tilde{q}}\|\right) + (k_d + m_c m_d)/4\varepsilon^2$.

Proof. Consider the Lyapunov function candidate

$$v = \frac{1}{2}e^T P(q) e \qquad (8)$$

where $e^T = \left[\tilde{x}^T \quad \dot{\tilde{q}}^T \quad \tilde{q}^T\right]$ and $P(q) =$ diag $(H(q), H(q), K_p)$. By using property 1 and $K_p > 0$ we conclude that v satisfies

$$\frac{1}{2}p_l\|e\|^2 \le v \le \frac{1}{2}p_u\|e\|^2 \qquad (9)$$

where $p_l = \lambda_{\min}(P(q))$ and $p_u = \lambda_{\max}(P(q))$. The derivative of (8) is given by

$$\dot{v} = \tilde{x}^T\left(H(q)\dot{\tilde{x}} + \frac{1}{2}\dot{H}(q)\tilde{x}\right)$$
$$+\dot{\tilde{q}}^T\left(H(q)\ddot{\tilde{q}} + \frac{1}{2}\dot{H}(q)\dot{\tilde{q}} + K_p\tilde{q}\right) \qquad (10)$$

Substituting (4) and (7) into (10) and applying properties 3 and 4 we obtain

$$\dot{v} = -\tilde{x}^T\left(C(q,\hat{x}) + F + H(q)K\right)\tilde{x}$$
$$-\dot{\tilde{q}}^T(F + K_d)\dot{\tilde{q}} + \dot{\tilde{q}}^T K_d\tilde{x} - \dot{\tilde{q}}^T C(q,\dot{q}_d)\tilde{x} \qquad (11)$$

By taking the 2-norm of (11) and observing $\hat{x} = \dot{q}_d + \dot{\tilde{q}} - \tilde{x}$, we obtain

$$\dot{v} \le -\left(\underline{\sigma} + m_f - m_c\left(m_d + \|\tilde{x}\| + \|\dot{\tilde{q}}\|\right)\right)\|\tilde{x}\|^2$$
$$-\left(k_d + k_v\right)\|\dot{\tilde{q}}\|^2 + \left(k_d + m_c m_d\right)\|\tilde{x}\|\|\dot{\tilde{q}}\| \qquad (12)$$

Then by using $\left(\varepsilon\|\dot{\tilde{q}}\|\right)^2 + (\|\tilde{x}\|/2\varepsilon)^2 \ge \left(\varepsilon\|\dot{\tilde{q}}\|\right)(\|\tilde{x}\|/\varepsilon) = \|\dot{\tilde{q}}\|\|\tilde{x}\|$ equation (12) can be rewritten as

$$\dot{v} \le -\begin{pmatrix} \underline{\sigma} + m_f - m_c m_v - m_c\left(m_d + \|\tilde{x}\| + \|\dot{\tilde{q}}\|\right) \\ -\dfrac{\left(k_d + m_c m_d\right)}{4\varepsilon^2} \end{pmatrix}\|\tilde{x}\|^2$$
$$-\left(k_d + m_f - \varepsilon^2\left(k_d + m_c m_d\right)\right)\|\dot{\tilde{q}}\|^2$$
$$< -\beta_1\|\tilde{x}\|^2 - \beta_2\|\dot{\tilde{q}}\|^2 \qquad (13)$$

We can now fix an $\varepsilon \in (0 \;\; 1)$ and choose $k_d > \left(\beta_2 - m_f + m_c m_d \varepsilon^2\right)/\left(1 - \varepsilon^2\right)$ for a selected $\beta_2 > 0$. Then for fixed ε, k_d and β_2, a proper $K > 0$ can be chosen so $\underline{\sigma} > \beta_1 - m_f + m_c\left(m_d + \|\tilde{x}\| + \|\dot{\tilde{q}}\|\right) + \left(k_d + m_c m_d\right)/4\varepsilon^2$ for a fixed $\beta_1 > 0$. This guarantees $\dot{v} < 0$. Furthermore, from (13) we see that $\tilde{x}, \dot{\tilde{q}} \in L_2$. Also, (9) implies $\tilde{x}, \dot{\tilde{q}}, \tilde{q} \in L_\infty$ which in conjunction with (4) and (7) implies the boundedness of $\dot{\tilde{x}}$ and $\ddot{\tilde{q}}$. The use of Barbalat's Lemma [7, p. 19] then concludes $\tilde{x} \to 0$ and $\dot{\tilde{q}} = \dot{q} \to 0$ as $t \to \infty$. Further, application of Lasalle's theorem, [3, p. 110], implies $q \to q_d$ as $t \to \infty$.

The region of attraction is determined by applying the Barbashin and Krasovski theorem [4, pp. 108], then

$$\|e(0)\| < \sqrt{\frac{p_u}{2p_l}}\left(\frac{\underline{\sigma} + m_f - m_c m_d - \beta_1}{m_c} - \frac{\left(k_d + m_c m_d\right)}{4\varepsilon^2 m_c}\right)$$

which can be made as large as required by arbitrary selection of $\underline{\sigma}$.

V. EXPERIMENTS AND SIMULATIONS

A. The Model

The manipulator model considered for this paper uses the kinematics and dynamics of the second and third link of PUMA-560. The characteristics of the joint position noise to be used, however, was obtained from measurements from a General Electric (GE) Arm. To characterize the noise, tests were performed at International Submarine Engineering (ISE) Ltd. The GE Arm was under subcontract to be retrofitted with a digital controller.

Presently, the dynamic equation of the two degree of freedom manipulator is given as

$$H(q) = \begin{bmatrix} i_{m1} + h_0 + 2h_1\cos(q_2) & h_2 + h_1\cos(q_2) \\ h_2 + h_1\cos(q_2) & i_{m2} + h_2 \end{bmatrix}$$

$$C(q,\dot{q}) = h_1\sin(q_2)\begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1 + \dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix},\; g(q) = \begin{bmatrix} g_1\cos(q_1) + g_2\cos(q_1 + q_2) \\ g_2\cos(q_1 + q_2) \end{bmatrix}$$

The numerical values of the dynamic parameters were obtained from [1] and are given as: $h_0 = 2.082$, $h_1 = 0.373$, $h_2 = 0.336$, $g_1 = 37.19$, $g_2 = 8.47$, $i_{m1} = 4.71$ and $i_{m2} = 0.83$. The viscous friction coefficients f_1=27.6 and f_2=4.54 were obtained with an integral recursive least squares identification algorithm during experiments on the PUMA-560.

B. The Position Signal

To characterize the typical attributes of position measurement noise found in rotary potentiometers mounted on robots, a GE hydraulic sub-sea arm was tested. The data was collected at ISE Ltd. of Port Coquitlam, British Columbia where the arm's old analog controller was being replaced by a new digital controller.

Data logging was made possible with the aid of ISE's real-time system control software, Control System Probe (CSP). The position measurements were taken during unconstrained manipulator motions. A typical position trajectory for joint one of the manipulator is shown in Fig. 1a. Twenty ensemble sets of data were logged during the testing. A sampling frequency of 150 Hz was used for a duration of 11 seconds leading to more than 1600 samples per data set. The sampling of the potentiometer signal was done using a 12 bit analog to digital converter board which plugged into an 80386 IBM-PC compatible computer.

The measurement noise was separated from the signal with off-line low pass filtering. The filter which was used had a cutoff frequency of 30 Hz. A plot of one ensemble set of measurement noise is given in Fig. 1c. It is interesting to note that although the position trajectory in Fig. 1a seems smooth, differentiation of the position signal yields a noisy velocity as shown in Fig. 1b. Obviously, this velocity estimate could not be used in control. The distribution density of the position noise is shown in Fig. 1d as the dashed curve. A normal distribution density is shown by the solid curve for comparison. The discrepancy between the two curves has not been quantified. This requires more research.

The average noise power spectrum for the twenty ensemble sets of data is shown in Fig 1e. Note the flat spectrum above the filter cutoff frequency of 30 Hz. The standard deviation of the measurement noise was found to be $\sigma_n = 0.0818$ degrees.

In light of the previous characterization, a model of the measurement noise was assumed to be zero mean, normally distributed with a standard deviation $\sigma_n = 0.0818$ degrees. Simulations of trajectory tracking which is based on this noise model shall be described presently.

C. Control using Numerical Differentiation with Filter

In this section, trajectory tracking simulation results will be described for a computed torque controller [2] which uses joint velocities obtained with differentiation of filtered position data. The position control gains were chosen as $k_{p1} = 100$ and $k_{p2} = 500$ for joints one and two respectively. Also, a controller update frequency of 250 Hz was used.

Each joint of the manipulator tracked a fifth order polynomial trajectory as shown in Fig. 2a for joint one. The position tracking is shown in Fig 2a with associated tracking error shown in Fig. 2b. Note that the noise model characterized in the previous section was used in this simulation. The velocity tracking for joint one is given in 3c and the associated velocity tracking error is shown in Fig. 3c. The applied joint torque for joint one is shown in Fig. 2a. Similar results were obtained for the second joint.

By not filtering the position signals prior to differentiation, no control would have been possible due to excessive chattering in the torque signal. The filtering was done with first order filters having gains $a_1 = 20$ and $a_2 = 30$ for joints one and two respectively. Filters with larger bandwidths caused excessive chattering in the control torque On the other hand, filters with smaller bandwidths produced smoother velocity estimates but caused unstable tracking.

D. Control using the Proposed Observer

Results for the proposed observer-controller will be given presently where control gains equivalent to those for the computed torque controller were used. The observer gains were chosen as $k_1 = 20$ and $k_2 = 30$, yielding a model based filter with the same delay characteristics as the non-model based filter used in the previous section to smooth the position signals.

The results of trajectory tracking for joint one are shown in Fig. 4. and the applied control torque for joint one is shown in Fig. 2b. Comparison of Figures 2 and 3 shown that the tracking results of the observer-controller system are superior to those of the computed torque controller which uses joint velocities produced with the numerical differentiation of filtered joint position signals.

An additional benefit of the observer-controller is the reduction in control torque chattering as shown in Fig. 2.

VI. CONCLUSIONS

A reduced-order joint velocity observer has been presented for use in robot control. One of the main advantages of this observer over previous ones is that it exhibits first order dynamics rather than second order dynamics. The velocity signal generated by the observer was used by a trajectory tracking controller. The result is locally asymptotically stable position and velocity trajectory tracking errors.

The proposed observer-controller was compared to the well known computed torque controller through simulations on the second and third joints of PUMA-560. The joint position measurements were modeled to have zero mean, normally distributed noise with standard deviation $\sigma_n = 0.0818$ degrees. This model was obtained from measurements taken from a sub-sea hydraulic manipulator manufactured by the General Electric Company. In the presence of this noise, it was found that the proposed observer-controller had better trajectory tracking results than the computed torque controller which obtained its joints velocity estimates by differentiating filtered position data.

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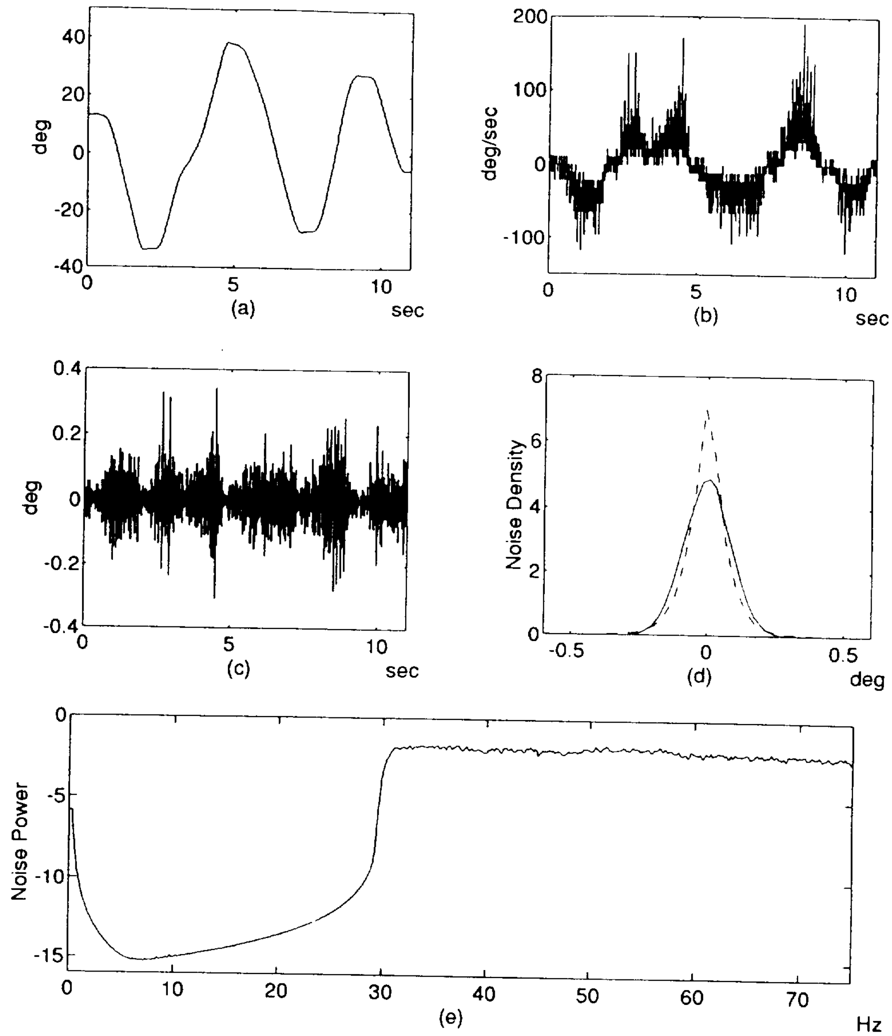

Fig. 1. Experimental data from the GE Arm. (a) Measured position trajectory, joint one. (b) Joint velocity using differentiation. (c) Noise in the position signal. (d) Distribution density of the noise. (e) Spectral noise power (dB).

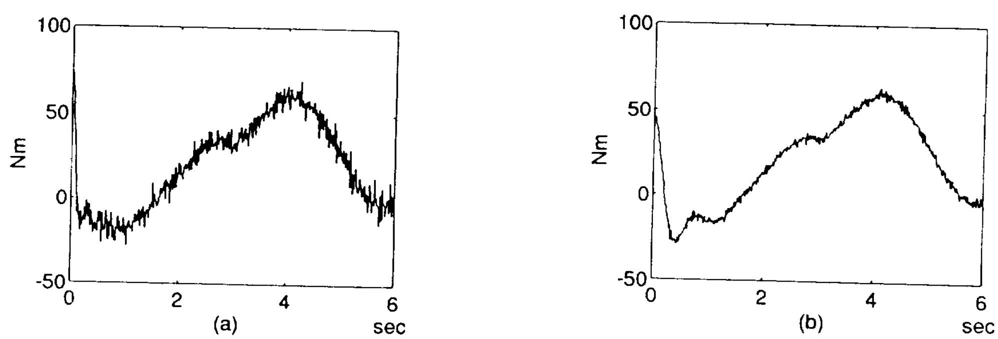

Fig. 2. Applied torques. (a) Control torque for case using numerical differentiation. (b) Control torquė for case using observer.

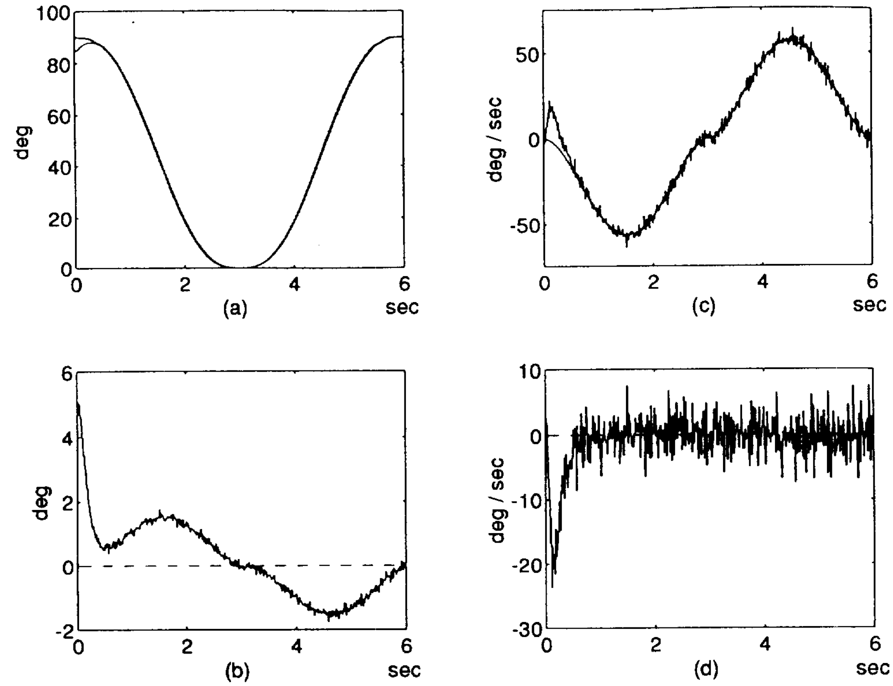

Fig. 3. Trajectory tracking for case using numerical differentiation, joint one. (a) Desired vs. measured position. (b) Position error. (c) Desired vs. measured velocity. (d) Velocity error.

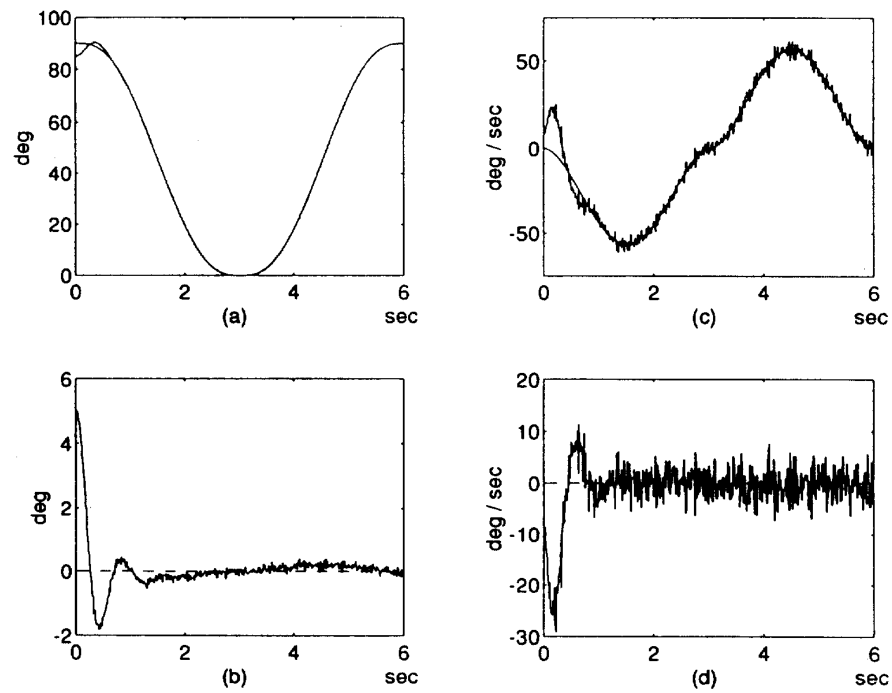

Fig. 4. Trajectory tracking for case using observer, joint one. (a) Desired vs. measured position. (b) Position error. (c) Desired vs. measured velocity. (d) Velocity error.