A Reduced-Order Adaptive Velocity Observer for Manipulator Control

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Abstract

A reduced-order adaptive velocity observer is proposed for manipulator control. The observer combined with an adaptive controller yields locally asymptotically stable observed velocity errors and locally asymptotically stable position and velocity tracking errors. Implementation of the observer-controller on the PUMA-560 yields high quality tracking results.

1. Introduction

In order to eliminate the need for tachometers or numerical differentiation in obtaining velocity estimates for use in control, one may use a velocity observer. An observer was first used to estimate manipulator joint velocities in [1] and [2] where full state estimators yielded observers with second order dynamics. In [3] a reduced-order velocity observer was proposed which has first order dynamics. In these papers an exact knowledge of the manipulator's dynamic parameters is assumed. For cases where dynamic parameters are only partly known or not known at all, an adaptive mechanism is required. In [4] an adaptive observer-controller was proposed that uses a variable structure formulation which might excite unmodelled frequencies. In this paper a *reduced-order* adaptive observer-controller is proposed which has been implemented on a PUMA-560 manipulator. The experimental results demonstrate that accurate tracking results can be achieved.

2. Notation and Preliminaries

The dynamic equation of an n degree-of-freedom manipulator is given by

$$H(q)\ddot{q}+C(q,\dot{q})\dot{q}+F\dot{q}+g(q)=Y(q,\dot{q},\ddot{q})a=\tau \qquad (1)$$

where $Y(q,\dot{q},\ddot{q})\in\Re^{n\times p}$ is a regressor matrix of known functions and $a\in\Re^{p}$ is a vector of dynamic parameters.

In this paper joint position measurements q are assumed to be available but joint velocities $\dot{q}$ are not. It is also assumed that some or all parameters of $H(q)$, $C(q,\dot{q})$, F and $g(q)$ are not precisely known.

Several properties of the manipulator dynamic equation are useful in the development of the proposed adaptive observer-controller and these are summarized as

Property 1: The manipulator inertia matrix is positive definite symmetric and bounded $m_l\le\|H(q)\|\le m_u$ where $m_l,m_u>0,\ \forall q\in\Re^n$.

Property 2: The 2-norm of $C(q,x)$ is bounded by $m_c\|x\|$ where $m_c>0$ is a positive constant.

Property 3: The matrix of Christoffel symbols, $C(q,x)$ satisfies $C(q,x)\xi=C(q,\xi)x$ for any $q,x,\xi\in\Re^n$ as defined in [5].

Property 4: $\xi^T\left(\dot{H}(q)-2C(q,x)\right)\xi=0$ for any $\xi\in\Re^n$.

Property 5: The manipulator dynamic equation is linear in its parameters of interest, i.e. $H(q)\psi+C(q,\xi)\xi+F\xi+g(q)=Y(q,\xi,\psi)a$ for any $q,\xi,\psi\in\Re^n$ and $a\in\Re^p$.

3. The Combined Adaptive Observer-Controller

In the formulation of the adaptive manipulator velocity observer it is useful to rearrange (1) to give

$$\dot{x}=H^{-1}(q)\left[\tau-C(q,x)x-Fx-g(q)\right] \qquad (2)$$

where $x=\dot{q}$. This is a first order equation in x where q is assumed to be known, τ is the input and x is the output. The proposed observer has a similar structure given by

$$\dot{\hat{x}}_a=\hat{H}^{-1}(q)\left[\tau-\hat{C}(q,\hat{x}_a)\hat{x}_a-\hat{F}\hat{x}_a-\hat{g}(q)\right]+K\tilde{x}_a \qquad (3)$$

where $\hat{x}_a$ is the *observed* velocity estimate, $\tilde{x}_a=x-\hat{x}_a$ is the observer error, $K>0$ is a diagonal gain matrix, and $(\hat{*})$ denotes the estimate of $(*)$. Concerning the realization of (3), we note that

$$\hat{x}_a(t)=\hat{x}_a(t-\Delta)+\int_{t-\Delta}^{t}\left\{\psi(q,\hat{x}_a,\tau,\hat{a})-K\hat{x}_a\right\}dt$$
$$+K\left[q(t)-q(t-\Delta)\right] \qquad (4)$$

$$\psi(q,\hat{x}_a,\tau,\hat{a})=\hat{H}^{-1}(q)\left[\tau-\hat{C}(q,\hat{x}_a)\hat{x}_a-\hat{F}\hat{x}_a-\hat{g}(q)\right] \qquad (5)$$

where $\hat{x}_a(0)$ is a reasonable initial guess of $\dot{q}(0)$ and Δ is the integration interval. By subtracting (3) from (2) and applying properties 3 and 5 the observation error can be written as

$$H(q)\dot{\tilde{x}}_a = -C(q,\hat{x}_a)\tilde{x}_a - C(q,x)\tilde{x}_a - F\tilde{x}_a - H(q)K\tilde{x}_a + Y(q,\hat{x}_a,\psi)\tilde{a} \tag{6}$$

where $\tilde{H} = \hat{H} - H$, $\tilde{C} = \hat{C} - C$, $\tilde{F} = \hat{F} - F$, $\tilde{g} = \hat{g} - g$, $\tilde{a} = \hat{a} - a$ and $Y(q,\hat{x}_a,\psi)$ is the regressor matrix.

The adaptive observer presented above can be combined with an adaptive controller to give locally asymptotically stable tracking and observation errors. The controller is given by

$$\tau = \hat{H}(q)[\dot{x}_d - (\hat{x}_a - x_d)] + \hat{C}(q,\hat{x}_a)x_r + \hat{F}x_r + \hat{g}(q) - K_d\hat{s} - K_p\tilde{q} \tag{7}$$

where $\hat{s} = \hat{x}_a - x_r$. The reference velocity is defined by $x_r = x_d - \tilde{q}$ where $x_d(t)$ is the desired velocity trajectory and $\tilde{q} = q - q_d$ is the position tracking error with desired position trajectory $q_d(t)$. The proportional and derivative gain matrices are K_p and K_d respectively. The controller's error dynamics is found by equating (1) and (7) and applying properties 3 and 5. Thus we have

$$H(q)\dot{s} = -C(q,x)s - (K_d + f)s - K_p\tilde{q} + K_d\tilde{x}_a + H(q)\tilde{x}_a - C(q,x_r)\tilde{x}_a + Y(q,\hat{x}_a,x_r,\dot{x}_d)\tilde{a} \tag{8}$$

where $s = x - x_r$. Presently a theorem regarding system stability will be developed.

Theorem. $\tilde{x}_a$, $\tilde{q}$, $\dot{\tilde{q}} \to 0$ as $t \to \infty$ as long as the initial conditions for the state vector $e^T = [s^T \quad \tilde{x}_a^T \quad \tilde{q}^T \quad \tilde{a}^T]$ satisfy $B = \{e(0) : \|e(0)\| < \min(B_1 , B_2)\}$ where

$$B_1 = \frac{1}{2}\sqrt{\frac{p_l}{p_u}}\left[\frac{(1-\varepsilon^2)k_d + m_f - \beta_1 - \varepsilon^2(m_u + m_c m_d)}{m_c\varepsilon^2}\right] \tag{9}$$

$$B_2 = \frac{2\varepsilon^2}{8\varepsilon^2+1}\sqrt{\frac{p_l}{p_u}}\left[\frac{\underline{\sigma} + m_f - \beta_2 - \frac{1}{4\varepsilon^2}(k_d + m_u + m_c m_d)}{m_c}\right] \tag{10}$$

and gains K and K_d are designed such that $k_d > (\beta_1 - m_f + \varepsilon^2(m_u + m_c m_d)) / (1-\varepsilon^2)$ and $\underline{\sigma} > \beta_2 - m_f + (k_d + m_u + m_c m_d)/4\varepsilon^2$ for $\varepsilon \in (0,1)$ where $\underline{\sigma} = \lambda_{\min}(KH(q) + H(q)K)/2$ and $K_d = k_d I$.

Proof. Consider the candidate Lyapunov function

$$v(t) = \frac{1}{2}e(t)^T P(q(t))e(t) \tag{11}$$

where $e^T = [s^T \quad \tilde{x}_a^T \quad \tilde{q}^T \quad \tilde{a}^T]$ and $P(q) = \mathrm{diag}(H(q), H(q), K_p, \Gamma^{-1})$ with $K_p = K_p^T > 0, \Gamma > 0$. Then (11) satisfies

$$\frac{1}{2}p_l\|e(t)\|^2 \le v(t) \le \frac{1}{2}p_u\|e(t)\|^2 \tag{12}$$

where $p_l = \lambda_{\min}(P(q))$ and $p_u = \lambda_{\max}(P(q))$. Since $\dot{\tilde{a}} = \dot{\hat{a}}$ and $\dot{\tilde{q}} = s - \tilde{q}$, the derivative of (11) can be expressed as

$$\dot{v} = s^T\left(H(q)\dot{s} + K_p\tilde{q} + \frac{\dot{H}(q)}{2}s\right) - \tilde{q}^T K_p\tilde{q} + \tilde{x}_a^T\left(H(q)\dot{\tilde{x}}_a + \frac{\dot{H}(q)}{2}\tilde{x}_a\right) + \tilde{a}^T\Gamma^{-1}\dot{\hat{a}} \tag{13}$$

Substituting observer and controller error dynamics (6) and (8) into (13) and using property 4 gives us

$$\dot{v} = -s^T(K_d + F)s - \tilde{q}^T K_p\tilde{q} - \tilde{x}_a^T(H(q)K + F + C(q,\hat{x}_a))\tilde{x}_a + s^T(K_d + H(q) - C(q,x_r))\tilde{x}_a + s^T Y(q,\hat{x}_a,x_r,\dot{x}_d)\tilde{a} + \tilde{x}_a^T Y(q,\hat{x}_a,\psi)\tilde{a} + \tilde{a}^T\Gamma^{-1}\dot{\hat{a}}$$

If the parameter adaptation law is chosen as

$$\dot{\hat{a}} = -\Gamma[Y^T(q,\hat{x}_a,x_r,\dot{x}_d)s + Y^T(q,\hat{x}_a,\psi)\tilde{x}_a] \tag{14}$$

then

$$\dot{v} = -s^T(K_d + F)s - \tilde{q}^T K_p\tilde{q} - \tilde{x}_a^T(H(q)K + F + C(q,\hat{x}_a))\tilde{x}_a + s^T(K_d + H(q) - C(q,x_r))\tilde{x}_a$$

Hence, since $x_r = x_d - \tilde{q}$ and $\hat{x}_a = x_d + s - \tilde{x}_a - \tilde{q}$ we obtain

$$\dot{v} \le -(k_d + m_f)\|s\|^2 - (\underline{\sigma} + m_f - m_c m_d - m_c(\|\tilde{x}_a\| + \|s\| + \|\tilde{q}\|))\|\tilde{x}_a\|^2 - k_p\|\tilde{q}\|^2 + (k_d + m_u + m_c(m_d + \|\tilde{q}\|))\|\tilde{x}_a\|\|s\| \tag{15}$$

where $\|x_d\| \le m_d$, $K_d = k_d I$, $k_p = \lambda_{\min}(K_p)$ and $m_f = \lambda_{\min}(F)$. Using $\|s\|^2\varepsilon^2 + \|\tilde{x}_a\|^2/4\varepsilon^2 \ge \|s\|\|\tilde{x}_a\|$, (15) becomes

$$\dot{v} \le -\left[\left(1-\varepsilon^2\right)k_d + m_f - \varepsilon^2\left(m_u + m_c m_d + m_c\|\tilde{q}\|\right)\right]\|s\|^2$$
$$-k_p\|\tilde{q}\|^2 - [\underline{\sigma} + m_f - \frac{1}{4\varepsilon^2}\left(k_d + m_u + m_c m_d\right)$$
$$-m_c\left(\|\tilde{x}_a\| + \|s\| + \left(1+\frac{1}{4\varepsilon^2}\right)\|\tilde{q}\|\right)]\|\tilde{x}_a\|^2 \tag{16}$$

If at time $t = 0$ we have

$$k_d > \frac{\beta_1 - m_f + \varepsilon^2\left(m_u + m_c m_d + m_c\|\tilde{q}(0)\|\right)}{\left(1-\varepsilon^2\right)} \tag{17}$$

$$\underline{\sigma} > \beta_2 - m_f + \frac{1}{4\varepsilon^2}\left(k_d + m_u + m_c m_d\right)$$
$$+m_c\left(\|\tilde{x}_a(0)\| + \|s(0)\| + \left(1+\frac{1}{4\varepsilon^2}\right)\|\tilde{q}(0)\|\right) \tag{18}$$

for $\varepsilon \in (0,1)$, then $\dot{v}(0) < 0$. Furthermore, since $2\|e(0)\| \ge \|s(0)\| + \|\tilde{x}_a(0)\| + \|\tilde{q}(0)\| + \|\tilde{a}(0)\|$ we see that (17) and (18) hold if

$$k_d > \frac{\beta_1 - m_f + \varepsilon^2\left(m_u + m_c m_d + 2m_c\sqrt{\frac{p_u}{p_l}}\|e(0)\|\right)}{\left(1-\varepsilon^2\right)} \tag{19}$$

$$\underline{\sigma} > \beta_2 - m_f + \frac{1}{4\varepsilon^2}\left(k_d + m_u + m_c m_d\right)$$
$$+m_c\left(2\sqrt{\frac{p_u}{p_l}}\|e(0)\| + \left(1+\frac{1}{4\varepsilon^2}\right)2\sqrt{\frac{p_u}{p_l}}\|e(0)\|\right) \tag{20}$$

are satisfied. So, if (19) and (20) hold, then $\exists\, t < \delta$ for which $\dot{v}(t) < 0$. This implies that $v(t) < v(0)$ for $t < \delta$. As a consequence (12) tells us that $\|e(t)\| \le \sqrt{p_u/p_l}\,\|e(0)\|$. Then

$$\|s(t)\| + \|\tilde{x}_a(t)\| \le \sqrt{\frac{p_u}{p_l}}\|e(0)\| \le 2\sqrt{\frac{p_u}{p_l}}\|e(0)\| \tag{21}$$

$$\|\tilde{q}(t)\| \le \sqrt{\frac{p_u}{p_l}}\|e(0)\| \le 2\sqrt{\frac{p_u}{p_l}}\|e(0)\| \tag{22}$$

So, as t goes on, (16) implies $\dot{v}(t) < 0$ as long as

$$k_d > \frac{\beta_1 - m_f + \varepsilon^2\left(m_u + m_c m_d + m_c\|\tilde{q}(t)\|\right)}{\left(1-\varepsilon^2\right)} \tag{23}$$

$$\underline{\sigma} > \beta_2 - m_f + \frac{1}{4\varepsilon^2}\left(k_d + m_u + m_c m_d\right)$$
$$+m_c\left(\|\tilde{x}_a(t)\| + \|s(t)\| + \left(1+\frac{1}{4\varepsilon^2}\right)\|\tilde{q}(t)\|\right) \tag{24}$$

for $\varepsilon \in (0,1)$ and $\beta_1, \beta_2 > 0$. But (23) is satisfied by virtue of (22) and (19). Likewise, (24) is satisfied by virtue of (22), (21) and (20). Therefore, it follows that

$$\dot{v}(t) \le -\beta_1\|s\|^2 - \beta_2\|\tilde{x}_a\|^2 - k_p\|\tilde{q}\|^2 \quad , \quad \forall\, t \ge 0 \tag{25}$$

where β_1, β_2, $k_p > 0$. The region of stability, the minimum of (9) and (10), is obtained by rewriting equations (19) and (20) explicitly for $\|e(0)\|$.

For stability, we note from (12) and (25) that s, $\tilde{x}_a$, $\tilde{q}$, $\tilde{a} \in L_\infty$. In addition, from (25) we conclude s, $\tilde{x}_a$, $\tilde{q} \in L_2$. To show $\dot{s}, \dot{\tilde{x}}_a \in L_\infty$, express (8) and (6) explicitly for $\dot{s}$ and $\dot{\tilde{x}}_a$ and note that all terms on the right hand side are bounded since $x = x_d + s - \tilde{q} \in L_\infty$, $x_r = x_d - \tilde{q} \in L_\infty$ and $\hat{x}_a = x_d + s - \tilde{x}_a - \tilde{q} \in L_\infty$.

For the observer we have $\tilde{x}_a, \dot{\tilde{x}}_a \in L_\infty$ and $\tilde{x}_a \in L_2$. Proceeding with Barbalat's lemma we conclude $\tilde{x}_a \to 0$ as $t \to \infty$. Similarly, for the controller we have $s, \dot{s} \in L_\infty$ and $s \in L_2$ which implies $s \to 0$ as $t \to \infty$. Finally, since $s = \dot{\tilde{q}} + \tilde{q}$ is a stable filter $s \to 0$ implies that both $\tilde{q} \to 0$ and $\dot{\tilde{q}} \to 0$ as $t \to \infty$.

The implementation of the velocity observer (4) and adaptation rule (14) will be considered presently. Let Δ be the sampling interval, then discretization of (4) and (5) at $t = i\Delta$ leads to

$$\hat{x}_a(i) \approx \hat{x}_a(i-1) + \Delta\psi(i-1) + K\left[q(i) - q(i-1)\right]$$
$$-\Delta K\hat{x}_a(i-1)$$

$$\psi(i-1) = \hat{H}^{-1}(i-1)\left[\tau(i-1) - \hat{C}(i-1)\hat{x}_a(i-1)\right]$$
$$+\hat{H}^{-1}(i-1)\left[-\hat{F}\hat{x}_a(i-1) - \hat{g}(i-1)\right]$$

where i represents the time at $t = i\Delta$. □

Remark: A problem with continuity could arise in the calculation of ψ if $\hat{a}$ adapted in such a way that $\hat{H}$ underwent rank reduction. However, using an adjustment technique given in [6], positive definitness of $\hat{H}$ can be maintained.

The controller can be implemented by calculating (7) at each time instant $t = i\Delta$. The implementation of the parameter update law, however, is more involved and it is accomplished by numerically integrating (14) using the trapezoidal rule to yield

$$\hat{a}(i) = \hat{a}(i-1) - \Gamma Y_o^T(i-1)\left[\frac{1}{2}\big(q(i) - q(i-1)\big) - \Delta\hat{x}_a(i-1)\right]$$
$$-\Gamma Y_c^T(i-1)\left[\frac{1}{2}\big(q(i) - q(i-1)\big) + \big(q(i) - q(i-1) - x_d(i-1)\big)\right]$$

where $Y_o^T(i-1)$ is the value of $Y^T(q, \hat{x}_a, \psi)$ at time $t = (i-1)\Delta$ and $Y_c^T(i-1)$ is the value of $Y^T(q, \hat{x}_a, x_r, \dot{x}_d)$ at time $t = (i-1)\Delta$.

4. Experimental results

The second and third links of the PUMA-560 were used for the implementation of the adaptive observer-controller. To approximate continuous control more accurately the Mark II controller was modified [7], making a 250Hz sampling frequency possible. In this section, the second PUMA link angle was considered joint angle one and the third link angle was considered joint angle two. The fourth, fifth and sixth links of PUMA have been combined to represent the third link; the mass of this last link was denoted by m_3. In the experiment m_3 was assumed an unknown quantity, even though it was accurately determined in [8]. The relevant model was defined as

$$H(q) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$
$$H_{11} = a_1 + 2a_2c_2 + (p_1 + 2p_2c_2)a_9$$
$$H_{12} = H_{21} = (a_3 + a_2c_2) + (p_3 + p_2c_2)a_9$$
$$H_{22} = a_7 + p_3a_9$$
$$C(q, \dot{q}) = s_2(a_2 + p_2a_9)\begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1 + \dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}$$
$$F = \begin{bmatrix} a_4 & 0 \\ 0 & a_8 \end{bmatrix} \text{ and } g(q) = \begin{bmatrix} a_5c_1 + a_6c_{12} + (p_4c_1 + p_5c_{12})a_9 \\ a_6c_{12} + (p_5c_{12})a_9 \end{bmatrix}$$

where $a_1 = 6.33$, $a_2 = 0.14$, $a_3 = 0.11$, $a_4 = 27.6$, $a_5 = 31.9$, $a_6 = 3.30$, $a_7 = 0.94$, $a_8 = 4.54$, $a_9 = 1.25$ and $p_1 = 0.37$, $p_2 = 0.18$, $p_3 = 0.18$, $p_4 = 4.23$, and $p_5 = 4.15$. Also, note the use of the short notation $c_1 = \cos(q_1)$, $s_{12} = \sin(q_1 + q_2)$. In the regressor formulation of the model we have $Y_k a_k + Y_u a_u = \tau$ where

$$Y_k = \begin{bmatrix} \dot{x}_1 & Y_{k12} & \dot{x}_2 & x_1 & c_2 & c_{12} & 0 & 0 \\ 0 & c_2\dot{x}_1 + s_2x_1^2 & \dot{x}_1 & 0 & 0 & c_{12} & \dot{x}_2 & x_2 \end{bmatrix}$$
$$Y_{k12} = 2c_2\dot{x}_1 + c_2\dot{x}_2 - 2s_2x_1x_2 - s_2x_2^2$$

was the regressor associated with the known parameters $a_k = [a_1 \ \ldots \ a_8]^T$ and $p = [p_1 \ \ldots \ p_5]^T$ and

$$Y_u = [Y_{u1} \ \ Y_{u2}]^T$$

$$Y_{u1} = (p_1 + 2p_2c_2)\dot{x}_1 + (p_3 + p_2c_2)\dot{x}_2 - 2p_2s_2x_1x_2 - p_2s_2x_2^2$$
$$+ p_4c_1 + p_5c_{12}$$
$$Y_{u2} = (p_3 + p_2c_2)\dot{x}_1 + p_3\dot{x}_2 + p_2s_2x_1^2 + p_5c_{12}$$

was the regressor associated with the unknown parameter $a_9 = m_3$.

Presently, the implementation of the observer, controller and adaptation law on PUMA for a fifth order trajectory (see Fig. 1) and update rate of $\Delta = 4$ msec. will be detailed.

The velocity estimate provided at the k-th time instant was calculated with

$$\hat{x}_a(k) = \hat{x}_a(k-1) + \Delta\psi(k-1) + K\big(q(k) - q(k-1)\big)$$
$$-\Delta K\hat{x}_a(k-1)$$

$$\psi(k-1) = \hat{H}^{-1}\big(q(k-1)\big)\left[\tau - \hat{C}\big(q(k-1), \hat{x}_a(k-1)\big)\hat{x}_a(k-1)\right]$$
$$-\hat{H}^{-1}\big(q(k-1)\big)\left[F\hat{x}_a(k-1) - \hat{g}\big(\hat{x}_a(k-1)\big)\right]$$

The observer gain was made $K = \text{diag}(100, 100)$, the initial velocity estimate was set at $\hat{x}_a(0) = 0$ and the estimates

$$\hat{H}(q) = \begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} \\ \hat{H}_{21} & \hat{H}_{22} \end{bmatrix}$$
$$\hat{H}_{11} = a_1 + 2a_2c_2 + (p_1 + 2p_2c_2)\hat{a}_9$$
$$\hat{H}_{12} = \hat{H}_{21} = (a_3 + a_2c_2) + (p_3 + p_2c_2)\hat{a}_9$$
$$\hat{H}_{22} = a_7 + p_3\hat{a}_9$$
$$\hat{C}(q, \hat{x}_a) = s_2(a_2 + p_2\hat{a}_9)\begin{bmatrix} -\hat{x}_{a2} & -(\hat{x}_{a1} + \hat{x}_{a2}) \\ \hat{x}_{a1} & 0 \end{bmatrix}$$
$$\hat{g}(q) = \begin{bmatrix} a_5c_1 + a_6c_{12} + (p_4c_1 + p_5c_{12})\hat{a}_9 \\ a_6c_{12} + (p_5c_{12})\hat{a}_9 \end{bmatrix}$$

were created by replacing the third link's mass $a_9 = m_3$ with estimate $\hat{a}_9 = \hat{m}_3$. The initial value for $\hat{a}_9$ was $\hat{a}_9(0) = 0$.

The k-th torque signal was calculated as

$$\tau(k) = Y_k\big(q(k), \hat{x}_a(k), x_r(k), \dot{x}_d(k)\big)a_k(k)$$
$$+ Y_u\big(q(k), \hat{x}_a(k), x_r(k), \dot{x}_d(k)\big)\hat{a}_9(k)$$
$$- K_d\hat{s}(k) - K_p\big(q(k) - q_d(k)\big)$$

where

$$Y_{kc} = \begin{bmatrix} \dot{x}_{d1} & Y_{kc12} & \dot{x}_{d2} & \hat{x}_{a1} & c_2 & c_{12} & 0 & 0 \\ 0 & Y_{kc22} & \dot{x}_{d1} & 0 & 0 & c_{12} & \dot{x}_{d2} & \hat{x}_{a2} \end{bmatrix}$$
$$Y_{kc12} = 2c_2\dot{x}_{d1} + c_2\dot{x}_{d2} - s_2\hat{x}_{a2}x_{r1} - s_2\hat{x}_{a1}x_{r2} - s_2\hat{x}_{a2}x_{r2}$$
$$Y_{kc22} = c_2\dot{x}_{d1} + s_2\hat{x}_{a1}x_{r1}$$

was the regressor associated with the known dynamic parameters and

$$Y_{uc} = \left[Y_{uc1} \quad Y_{uc2}\right]^T$$
$$Y_{uc1} = (p_1 + 2p_2c_2)\dot{x}_{d1} + (p_3 + p_2c_2)\dot{x}_{d2} - p_2s_2\hat{x}_{a1}x_{r2} - p_2s_2x_{r1}\hat{x}_{a2} - p_2s_2\hat{x}_{a2}x_{r2} + p_4c_1 + p_5c_{12}$$
$$Y_{uc2} = (p_3 + p_2c_2)\dot{x}_{d1} + p_3\dot{x}_{d2} + p_2s_2\hat{x}_{a1}x_{r1} + p_5c_{12}$$

was the regressor associated with the unknown dynamic parameter. The position and derivative gains were chosen as $K_p = \text{diag}(500, 500)$ and $K_d = \text{diag}\left(2\sqrt{500}, 2\sqrt{500}\right)$ respectively.

The k-th parameter estimate $\hat{a}_9(k)$ was calculated with

$$\hat{a}_9(k) = \hat{a}_9(k-1) + \gamma Y_{uc}^T(k-1)\left[\Delta x_d(k-1) - \frac{1}{2}(q(k) - q(k-2)) - \Delta(q(k) - q(k-1))\right] + \gamma Y_{uo}^T(k-1)\left[\Delta\hat{x}_a(k-1) - \frac{1}{2}(q(k) - q(k-2))\right]$$

where $\gamma = 1.5$ was selected as the adaptation gain, Y_{uc} is defined above and

$$Y_{uo} = \left[Y_{uo1} \quad Y_{uo2}\right]^T$$
$$Y_{uo1} = (p_1 + 2p_2c_2)\psi_1 + (p_3 + p_2c_2)\psi_2 - 2p_2s_2\hat{x}_{a1}\hat{x}_{a2} - p_2s_2\hat{x}_{a2}^2 + p_4c_1 + p_5c_{12}$$
$$Y_{uo2} = (p_3 + p_2c_2)\psi_1 + p_3\psi_2 + p_2s_2\hat{x}_{a1}^2 + p_5c_{12}$$

The results of the experiment are shown in Fig. 1 for joint one of the model. A plot of the desired position versus the actual position is shown in Fig. 1 (a) and the corresponding position tracking error is shown in Fig. 1 (b). Since the actual joint velocities could not be measured directly, the observed velocity was used for comparison against the desired velocity. This is shown in Fig. 1 (c). The difference is plotted in Fig. 1 (d).

The estimate of the unknown value of the end-effector mass was calculated at each sampling instant. The time history of adaptation for this mass estimate is given in Fig. 2. The actual value of the end-effector mass was known from [8] to be $m_3 = 1.25$ Kg. In order to obtain convergence to this value, the parameter estimation gain had to be selected properly.

With experimentation, it would not be possible to verify the convergence of the observer error since the actual joint velocities are not directly available. To make this verification, a simulations was performed. The planar manipulator described in the experimental section was used with identical test parameters.

Figure 3 shows the observer simulation results for the first joint of this model. In Fig. 3 (a) the observed velocity $\hat{x}_1$ versus the actual velocity x_1 is plotted and the related error $\tilde{x}_1 = x_1 - \hat{x}_1$ is shown in Fig. 3 (b). The initial observer error was set at -10 degrees.

5. Conclusions

A reduced-order adaptive observer-controller has been proposed which has locally asymptotically stable observation errors and locally asymptotically stable position and velocity tracking errors. The proposed observer-controller structure is adaptive towards unknown dynamic parameters and does not require the switching modes such as the variable structure methods to insure stability. This results in an observer-controller which provides smooth numerical performance. The combined observer-controller was implemented on the PUMA-560 which yielded high quality tracking. A simulation was also made to support the theory and show that the observer error tends to zero asymptotically.

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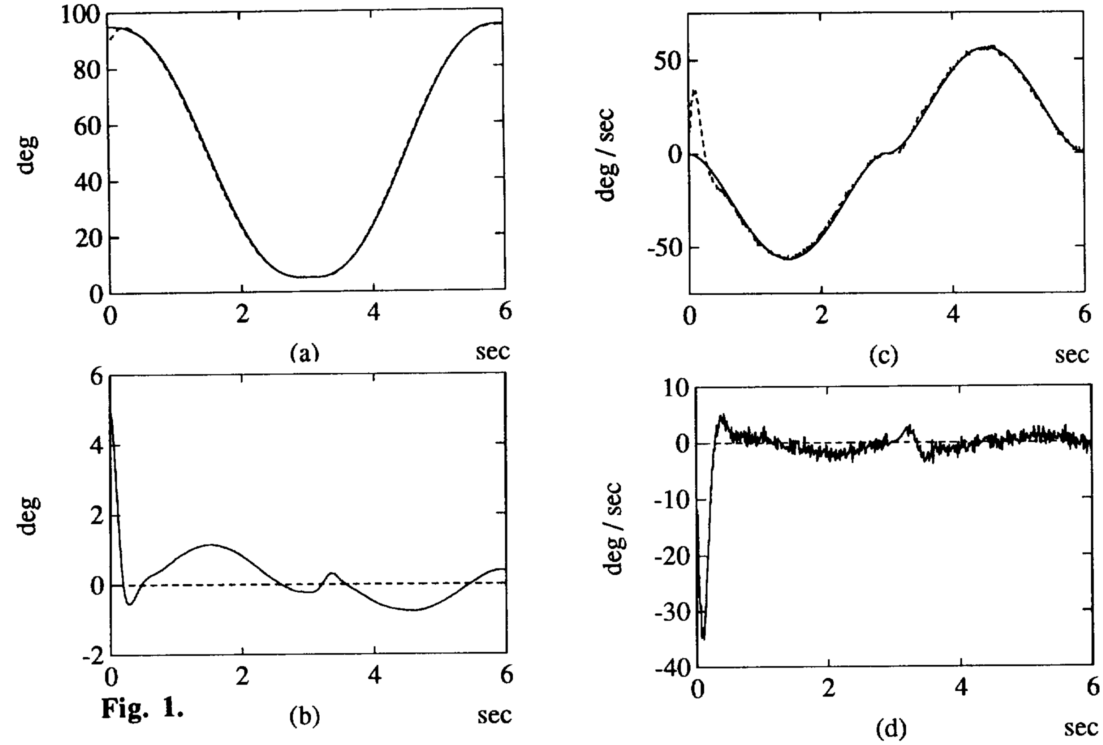

Fig. 1.

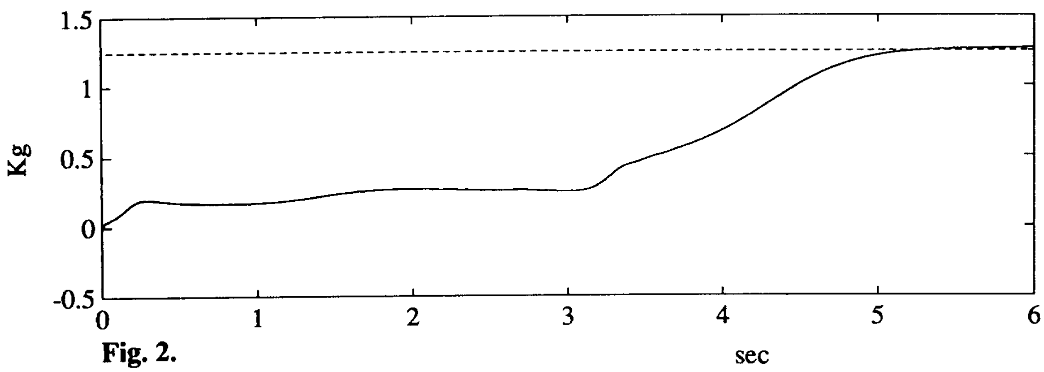

Fig. 2.

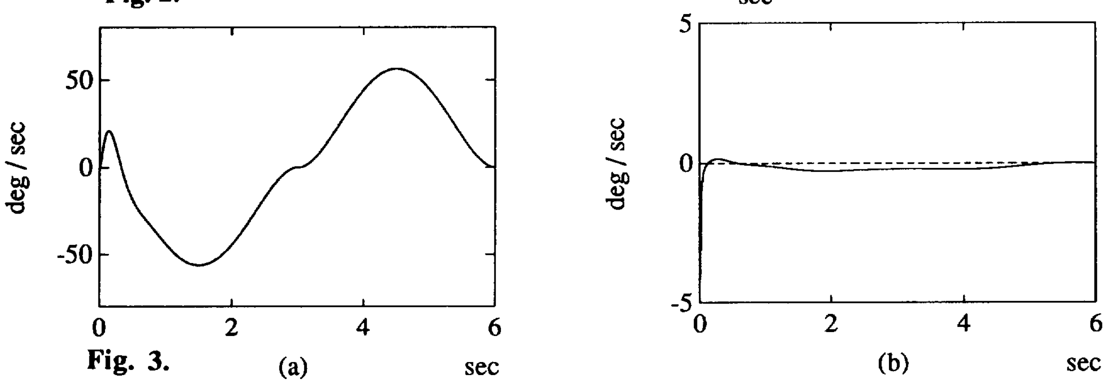

Fig. 3.